Spectral Representation

By inserting a complete set of energy eigenstates $\sum_n |n\rangle \langle n| = 1$ into $G_{BA}^{\text{ret}}(t)$, one can show that

$$G_{BA}^{\text{ret}}(\omega) = \sum_n \left\{ \frac{\langle 0 | B | n \rangle \langle n | A | 0 \rangle}{\omega - (E_n - E_0) + i\delta} - \frac{\langle 0 | A | n \rangle \langle n | B | 0 \rangle}{\omega + (E_n - E_0) + i\delta} \right\}.$$

This is called the spectral representation of $G_{BA}^{\text{ret}}$. It is useful in determining the analytic properties of $G$ and relating $G$ to various experiments, e.g.,

1. $G_{BA}^{\text{ret}}(\omega)$ is analytic in the uhp. It has a cut just below the real $\omega$-axis.

2. The neutron spin flip ($\uparrow \rightarrow \downarrow$) scattering cross-section for momentum transfer $q$ and energy transfer $\omega$ is proportional to

$$S(q, \omega) = \frac{1}{\pi} \text{Im} \chi^{+-}(q, \omega)$$

with

$$\chi^{+-}(q, \omega) = \text{F.T.} \left\{ i \langle [m_q^-(t), m_q^+(0)] \rangle \theta(t) \right\}.$$

3. Some properties of the single-particle time-ordered Green’s function will be discussed.