QUESTIONS

1. In 1969, three Apollo astronauts left Cape Canaveral, went to the Moon and back, and splashed down at a selected landing site in the Pacific Ocean; see Fig. 21. An admiral bid them goodbye at the Cape and then sailed to the Pacific Ocean in an aircraft carrier to pick them up. Compare the displacements of the astronauts and the admiral.

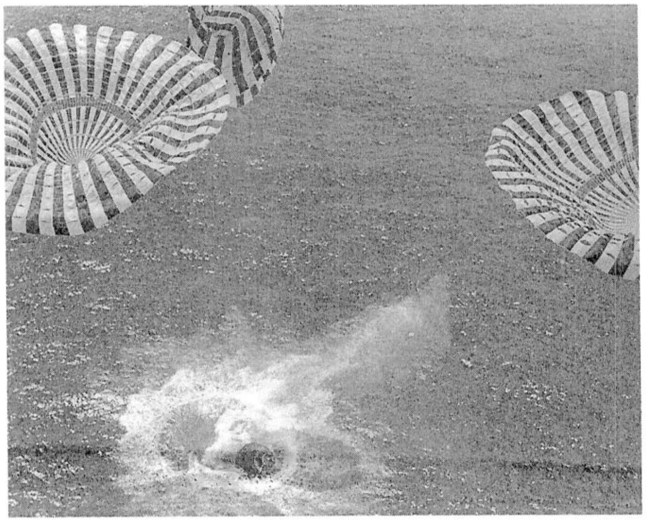

Figure 21 Question 1.

2. A dog runs 100 m south, 100 m east, and 100 m north, ending up at his starting point, his displacement for the entire trip being zero. Where is his starting point? One clear answer is the North Pole but there is another solution, located near the South Pole. Describe it.
3. Can two vectors having different magnitudes be combined to give a zero resultant? Can three vectors?
4. Can a vector have zero magnitude if one of its components is not zero?
5. Can the sum of the magnitudes of two vectors ever be equal to the magnitude of the sum of these two vectors?
6. Can the magnitude of the difference between two vectors ever be greater than the magnitude of either vector? Can it be greater than the magnitude of their sum? Give examples.
7. Suppose that $\mathbf{d} = \mathbf{d}_1 + \mathbf{d}_2$. Does this mean that we must have either $d \geq d_1$ or $d \geq d_2$? If not, explain why.
8. If three vectors add up to zero, they must all be in the same plane. Make this plausible.
9. Do the unit vectors **i**, **j**, and **k** have units?
10. Explain in what sense a vector equation contains more information than a scalar equation.
11. Name several scalar quantities. Does the value of a scalar quantity depend on the coordinate system you choose?
12. You can order events in time. For example, event b may precede event c but follow event a, giving us a time order of events a, b, c. Hence, there is a sense of time, distinguishing past, present, and future. Is time a vector therefore? If not, why not?
13. Do the commutative and associative laws apply to vector subtraction?
14. Can a scalar product be a negative quantity?
15. (*a*) If $\mathbf{a} \cdot \mathbf{b} = 0$, does it follow that **a** and **b** are perpendicular to one another? (*b*) If $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$, does it follow that $\mathbf{b} = \mathbf{c}$?
16. If $\mathbf{a} \times \mathbf{b} = 0$, must **a** and **b** be parallel to each other? Is the converse true?
17. A vector **a** lies parallel to the Earth's rotation axis, pointing from south to north. A second vector **b** points vertically upward at your location. What is the direction of the vector $\mathbf{a} \times \mathbf{b}$? At what locations on the Earth's surface is the magnitude of the vector $\mathbf{a} \times \mathbf{b}$ a maximum? A minimum?
18. Must you specify a coordinate system when you (*a*) add two vectors, (*b*) form their scalar product, (*c*) form their vector product, or (*d*) find their components?
19. (*a*) Show that if all the components of a vector are reversed in direction, then the vector itself is reversed in direction. (*b*) Show that if the components of the two vectors forming a vector product are all reversed, then the vector product is not changed. (*c*) Is a vector product, then, a vector?
20. We have discussed addition, subtraction, and multiplication of vectors. Why do you suppose that we do not discuss division of vectors? Is it possible to define such an operation?
21. It is conventional to use, as we did, the right-hand rule for vector algebra. What changes would be required if a left-handed convention were adopted instead?
22. (*a*) Convince yourself that the vector product of two polar vectors is an axial vector. (*b*) What is the vector product of a polar vector with an axial vector?

PROBLEMS

Section 3-2 Adding Vectors: Graphical Method

1. Consider two displacements, one of magnitude 3 m and another of magnitude 4 m. Show how the displacement vectors may be combined to get a resultant displacement of magnitude (*a*) 7 m, (*b*) 1 m, and (*c*) 5 m.
2. What are the properties of two vectors **a** and **b** such that (*a*) $\mathbf{a} + \mathbf{b} = \mathbf{c}$ and $a + b = c$; (*b*) $\mathbf{a} + \mathbf{b} = \mathbf{a} - \mathbf{b}$; (*c*) $\mathbf{a} + \mathbf{b} = \mathbf{c}$ and $a^2 + b^2 = c^2$?

3. A woman walks 250 m in the direction 35° east of north, then 170 m directly east. (*a*) Using graphical methods, find her final displacement from the starting point. (*b*) Compare the magnitude of her displacement with the distance she walked.

4. A person walks in the following pattern: 3.1 km north, then 2.4 km west, and finally 5.2 km south. (*a*) Construct the vector diagram that represents this motion. (*b*) How far and in what direction would a bird fly in a straight line to arrive at the same final point?

5. Two vectors **a** and **b** are added. Show graphically with vector diagrams that the magnitude of the resultant cannot be greater than $a + b$ or smaller than $|a - b|$, where the vertical bars signify absolute value.

6. A car is driven east for a distance of 54 km, then north for 32 km, and then in a direction 28° east of north for 27 km. Draw the vector diagram and determine the total displacement of the car from its starting point.

7. Vector **a** has a magnitude of 5.2 units and is directed east. Vector **b** has a magnitude of 4.3 units and is directed 35° west of north. By constructing vector diagrams, find the magnitudes and directions of (*a*) **a** + **b**, and (*b*) **a** − **b**.

8. A golfer takes three putts to get his ball into the hole once he is on the green. The first putt displaces the ball 12 ft north, the second 6.0 ft southeast, and the third 3.0 ft southwest. What displacement was needed to get the ball into the hole on the first putt? Draw the vector diagram.

9. A bank in downtown Boston is robbed (see the map in Fig. 22). To elude police, the thieves escape by helicopter, making three successive flights described by the following displacements: 20 mi, 45° south of east; 33 mi, 26° north of west; 16 mi, 18° east of south. At the end of the third flight they are captured. In what town are they apprehended? (Use the graphical method to add these displacements on the map.)

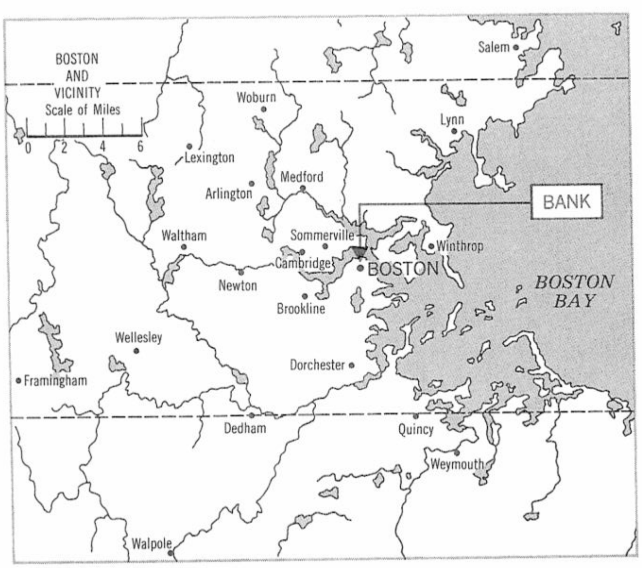

Figure 22 Problem 9.

Section 3-3 Components of Vectors

10. (*a*) What are the components of a vector **a** in the xy plane if its direction is 252° counterclockwise from the positive x axis and its magnitude is 7.34 units? (*b*) The x component of a certain vector is −25 units and the y component is +43 units. What are the magnitude of the vector and the angle between its direction and the positive x axis?

11. A heavy piece of machinery is raised by sliding it 13 m along a plank oriented at 22° to the horizontal, as shown in Fig. 23. (*a*) How high above its original position is it raised? (*b*) How far is it moved horizontally?

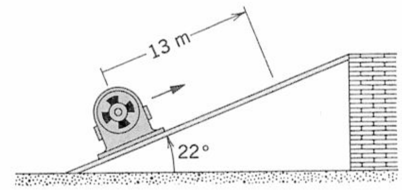

Figure 23 Problem 11.

12. The minute hand of a wall clock measures 11.3 cm from axis to tip. What is the displacement vector of its tip (*a*) from a quarter after the hour to half past, (*b*) in the next half hour, and (*c*) in the next hour?

13. A person desires to reach a point that is 3.42 km from her present location and in a direction that is 35.0° north of east. However, she must travel along streets that go either north–south or east–west. What is the minimum distance she could travel to reach her destination?

14. A ship sets out to sail to a point 124 km due north. An unexpected storm blows the ship to a point 72.6 km to the north and 31.4 km to the east of its starting point. How far, and in what direction, must it now sail to reach its original destination?

15. Rock *faults* are ruptures along which opposite faces of rock have moved past each other, parallel to the fracture surface. Earthquakes often accompany this movement. In Fig. 24 points A and B coincided before faulting. The component of the net displacement AB parallel to the horizontal surface fault line is called the *strike-slip* (AC). The component of the net displacement along the steepest line of the fault plane is the *dip-slip* (AD). (*a*) What is the net shift if the strike-slip is 22 m and the dip-slip is 17 m? (*b*) If the fault plane is inclined 52° to the horizontal, what is the net *vertical* displacement of B as a result of the faulting in (*a*)?

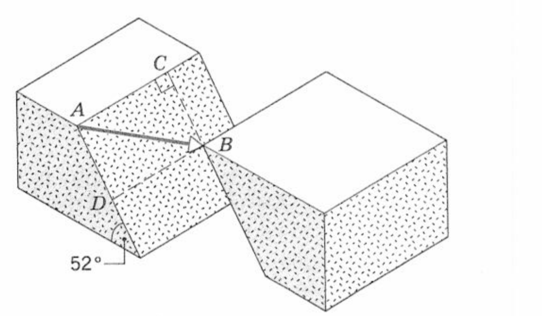

Figure 24 Problem 15.

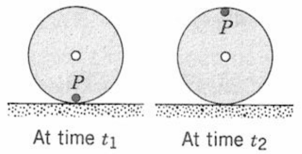

Figure 25 Problem 16.

16. A wheel with a radius of 45 cm rolls without slipping along a horizontal floor, as shown in Fig. 25. P is a dot painted on the rim of the wheel. At time t_1, P is at the point of contact between the wheel and the floor. At a later time t_2, the wheel has rolled through one-half of a revolution. What is the displacement of P during this interval?

17. A room has the dimensions 10 ft × 12 ft × 14 ft. A fly starting at one corner ends up at a diametrically opposite corner. (*a*) Find the displacement vector in a frame with coordinate axes parallel to the edges of the room. (*b*) What is the magnitude of the displacement? (*c*) Could the length of the path traveled by the fly be less than this distance? Greater than this distance? Equal to this distance? (*d*) If the fly walks rather than flies, what is the length of the shortest path it can take?

Section 3-4 Adding Vectors: Component Method

18. (*a*) What is the sum in unit vector notation of the two vectors $\mathbf{a} = 5\mathbf{i} + 3\mathbf{j}$ and $\mathbf{b} = -3\mathbf{i} + 2\mathbf{j}$? (*b*) What are the magnitude and direction of $\mathbf{a} + \mathbf{b}$?

19. Two vectors are given by $\mathbf{a} = 4\mathbf{i} - 3\mathbf{j} + \mathbf{k}$ and $\mathbf{b} = -\mathbf{i} + \mathbf{j} + 4\mathbf{k}$. Find (*a*) $\mathbf{a} + \mathbf{b}$, (*b*) $\mathbf{a} - \mathbf{b}$, and (*c*) a vector $\mathbf{c}$ such that $\mathbf{a} - \mathbf{b} + \mathbf{c} = 0$.

20. Given two vectors, $\mathbf{a} = 4\mathbf{i} - 3\mathbf{j}$ and $\mathbf{b} = 6\mathbf{i} + 8\mathbf{j}$, find the magnitudes and directions (with the $+x$ axis) of (*a*) $\mathbf{a}$, (*b*) $\mathbf{b}$, (*c*) $\mathbf{a} + \mathbf{b}$, (*d*) $\mathbf{b} - \mathbf{a}$, and (*e*) $\mathbf{a} - \mathbf{b}$.

21. (*a*) A man leaves his front door, walks 1400 m east, 2100 m north, and then takes a penny from his pocket and drops it from a cliff 48 m high. In a coordinate system in which the positive x, y, and z axes point east, north, and up, the origin being at the location of the penny as the man leaves his front door, write down an expression, using unit vectors, for the displacement of the penny. (*b*) The man returns to his front door, following a different path on the return trip. What is his resultant displacement for the round trip?

22. A particle undergoes three successive displacements in a plane, as follows: 4.13 m southwest, 5.26 m east, and 5.94 m in a direction 64.0° north of east. Choose the x axis pointing east and the y axis pointing north and find (*a*) the components of each displacement, (*b*) the components of the resultant displacement, (*c*) the magnitude and direction of the resultant displacement, and (*d*) the displacement that would be required to bring the particle back to the starting point.

23. Two vectors $\mathbf{a}$ and $\mathbf{b}$ have equal magnitudes of 12.7 units. They are oriented as shown in Fig. 26 and their vector sum is $\mathbf{r}$. Find (*a*) the x and y components of $\mathbf{r}$, (*b*) the magnitude of $\mathbf{r}$, and (*c*) the angle $\mathbf{r}$ makes with the $+x$ axis.

24. A radar station detects a missile approaching from the east. At first contact, the range to the missile is 12,000 ft at 40.0° above the horizon. The missile is tracked for another 123° in the east–west plane, the range at final contact being 25,800 ft; see Fig. 27. Find the displacement of the missile during the period of radar contact.

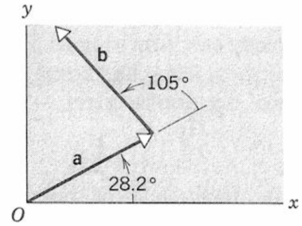

Figure 26 Problem 23.

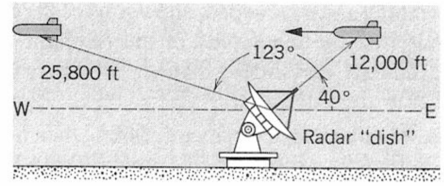

Figure 27 Problem 24.

25. Two vectors of magnitudes a and b make an angle θ with each other when placed tail to tail. Prove, by taking components along two perpendicular axes, that the magnitude of their sum is

$$r = \sqrt{a^2 + b^2 + 2ab\cos\theta}.$$

26. Prove that two vectors must have equal magnitudes if their sum is perpendicular to their difference.

27. (*a*) Using unit vectors along three cube edges, express the diagonals (the lines from one corner to another through the center of the cube) of a cube in terms of its edges, which have length a. (*b*) Determine the angles made by the diagonals with the adjacent edges. (*c*) Determine the length of the diagonals.

28. A tourist flies from Washington, DC to Manila. (*a*) Describe the displacement vector. (*b*) What is its magnitude? The latitude and longitude of the two cities are 39° N, 77° W and 15° N, 121° E. (*Hint:* See Fig. 7 and Eqs. 7. Let the z axis be along the Earth's rotation axis, so that $\theta = 90° -$ latitude and $\phi =$ longitude. The radius of the Earth is 6370 km.)

29. Let N be an integer greater than 1; then

$$\cos 0 + \cos\frac{2\pi}{N} + \cos\frac{4\pi}{N} + \cdots + \cos(N-1)\frac{2\pi}{N} = 0;$$

that is,

$$\sum_{n=0}^{n=N-1} \cos\frac{2\pi n}{N} = 0.$$

Also

$$\sum_{n=0}^{n=N-1} \sin\frac{2\pi n}{N} = 0.$$

Prove these two statements by considering the sum of N vectors of equal length, each vector making an angle of $2\pi/N$ with that preceding.

Section 3-5 Multiplication of Vectors

30. A vector $\mathbf{d}$ has a magnitude of 2.6 m and points north. What are the magnitudes and directions of the vectors (*a*) $-\mathbf{d}$, (*b*) $\mathbf{d}/2.0$, (*c*) $-2.5\mathbf{d}$, and (*d*) $5.0\mathbf{d}$?

31. Show for any vector **a** that (*a*) $\mathbf{a}\cdot\mathbf{a} = a^2$ and (*b*) $\mathbf{a}\times\mathbf{a} = 0$.

32. A vector **a** of magnitude 12 units and another vector **b** of magnitude 5.8 units point in directions differing by 55°. Find (*a*) the scalar product of the two vectors and (*b*) the vector product.

33. Two vectors, **r** and **s**, lie in the *xy* plane. Their magnitudes are 4.5 and 7.3 units, respectively, whereas their directions are 320° and 85° measured counterclockwise from the positive *x* axis. What are the values of (*a*) $\mathbf{r}\cdot\mathbf{s}$ and (*b*) $\mathbf{r}\times\mathbf{s}$?

34. Find (*a*) "north" cross "west," (*b*) "down" dot "south," (*c*) "east" cross "up," (*d*) "west" dot "west," and (*e*) "south" cross "south." Let each vector have unit magnitude.

35. Given two vectors, $\mathbf{a} = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}$ and $\mathbf{b} = b_x\mathbf{i} + b_y\mathbf{j} + b_z\mathbf{k}$, prove that the scalar product $\mathbf{a}\cdot\mathbf{b}$ is given in terms of the components by Eq. 15.

36. Given two vectors, $\mathbf{a} = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}$ and $\mathbf{b} = b_x\mathbf{i} + b_y\mathbf{j} + b_z\mathbf{k}$, prove that the vector product $\mathbf{a}\times\mathbf{b}$ is given in terms of the components by Eq. 17.

37. Show that $\mathbf{a}\times\mathbf{b}$ can be expressed by a 3 × 3 determinant as

$$\mathbf{a}\times\mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}.$$

38. Use Eqs. 13 and 15 to calculate the angle between the two vectors $\mathbf{a} = 3\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$.

39. Three vectors are given by $\mathbf{a} = 3\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$, $\mathbf{b} = -\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$, and $\mathbf{c} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$. Find (*a*) $\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})$, (*b*) $\mathbf{a}\cdot(\mathbf{b}+\mathbf{c})$, and (*c*) $\mathbf{a}\times(\mathbf{b}+\mathbf{c})$.

40. (*a*) Calculate $\mathbf{r} = \mathbf{a} - \mathbf{b} + \mathbf{c}$, where $\mathbf{a} = 5\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}$, $\mathbf{b} = -2\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$, and $\mathbf{c} = 4\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$. (*b*) Calculate the angle between **r** and the $+z$ axis. (*c*) Find the angle between **a** and **b**.

41. Three vectors add to zero, as in the right triangle of Fig. 28. Calculate (*a*) $\mathbf{a}\cdot\mathbf{b}$, (*b*) $\mathbf{a}\cdot\mathbf{c}$, and (*c*) $\mathbf{b}\cdot\mathbf{c}$.

42. Three vectors add to zero, as in Fig. 28. Calculate (*a*) $\mathbf{a}\times\mathbf{b}$, (*b*) $\mathbf{a}\times\mathbf{c}$, and (*c*) $\mathbf{b}\times\mathbf{c}$.

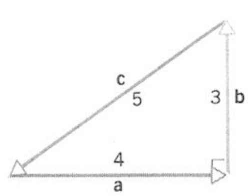

Figure 28 Problems 41 and 42.

43. Vector **a** lies in the *yz* plane 63.0° from the $+y$ axis with a positive *z* component and has magnitude 3.20 units. Vector **b** lies in the *xz* plane 48.0° from the $+x$ axis with a positive *z* component and has magnitude 1.40 units. Find (*a*) $\mathbf{a}\cdot\mathbf{b}$, (*b*) $\mathbf{a}\times\mathbf{b}$, and (*c*) the angle between **a** and **b**.

44. (*a*) We have seen that the commutative law *does not* apply to vector products; that is, $\mathbf{a}\times\mathbf{b}$ does not equal $\mathbf{b}\times\mathbf{a}$. Show that the commutative law *does* apply to scalar products; that is, $\mathbf{a}\cdot\mathbf{b} = \mathbf{b}\cdot\mathbf{a}$. (*b*) Show that the distributive law applies to both scalar products and vector products; that is, show that

$$\mathbf{a}\cdot(\mathbf{b}+\mathbf{c}) = \mathbf{a}\cdot\mathbf{b} + \mathbf{a}\cdot\mathbf{c}$$

and that

$$\mathbf{a}\times(\mathbf{b}+\mathbf{c}) = \mathbf{a}\times\mathbf{b} + \mathbf{a}\times\mathbf{c}.$$

(*c*) Does the associative law apply to vector products; that is, does $\mathbf{a}\times(\mathbf{b}\times\mathbf{c})$ equal $(\mathbf{a}\times\mathbf{b})\times\mathbf{c}$? (*d*) Does it make any sense to talk about an associative law for scalar products?

45. Show that the area of the triangle contained between the vectors **a** and **b** in Fig. 29 is $\frac{1}{2}|\mathbf{a}\times\mathbf{b}|$, where the vertical bars signify magnitude.

46. Show that the magnitude of a vector product gives numerically the area of the parallelogram formed with the two component vectors as sides (see Fig. 29). Does this suggest how an element of area oriented in space could be represented by a vector?

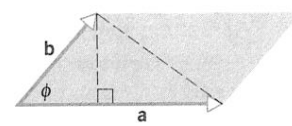

Figure 29 Problems 45 and 46.

47. Show that $\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})$ is equal in magnitude to the volume of the parallelepiped formed on the three vectors **a**, **b**, and **c** as shown in Fig. 30.

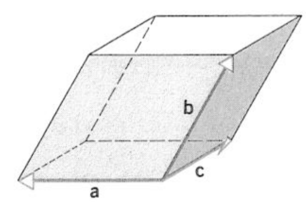

Figure 30 Problem 47.

48. Two vectors **a** and **b** have components, in arbitrary units, $a_x = 3.2$, $a_y = 1.6$; $b_x = 0.50$, $b_y = 4.5$. (*a*) Find the angle between **a** and **b**. (*b*) Find the components of a vector **c** that is perpendicular to **a**, is in the *xy* plane, and has a magnitude of 5.0 units.

49. Find the angles between the body diagonals of a cube. See Problem 27.

50. The three vectors shown in Fig. 31 have magnitudes $a = 3$, $b = 4$, $c = 10$. (*a*) Calculate the *x* and *y* components of these vectors. (*b*) Find the numbers *p* and *q* such that $\mathbf{c} = p\mathbf{a} + q\mathbf{b}$.

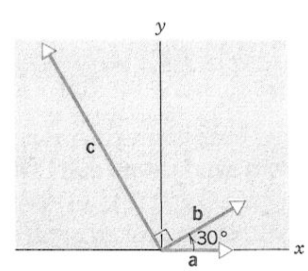

Figure 31 Problem 50.

Section 3-6 Vector Laws in Physics

51. Use Fig. 10*b* to derive Eqs. 18.

52. A vector **a** with a magnitude of 17 m is directed 56° counterclockwise from the $+x$ axis, as shown in Fig. 32. (*a*) What are the components a_x and a_y of the vector? (*b*) A second coordinate system is inclined by 18° with respect to the first. What are the components $a_{x'}$ and $a_{y'}$ in this "primed" coordinate system?

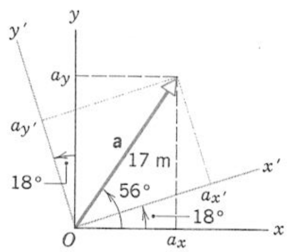

Figure 32 Problem 52.

53. Figure 33 shows two vectors **a** and **b** and two systems of coordinates which differ in that the x and x' axes and the y and y' axes each make an angle β with each other. Prove analytically that $\mathbf{a} + \mathbf{b}$ has the same magnitude and direction no matter which system is used to carry out the analysis. (*Hint:* Use Eqs. 18.)

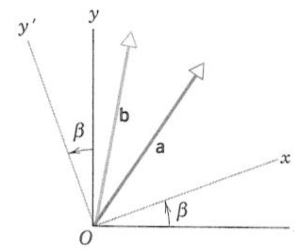

Figure 33 Problem 53.