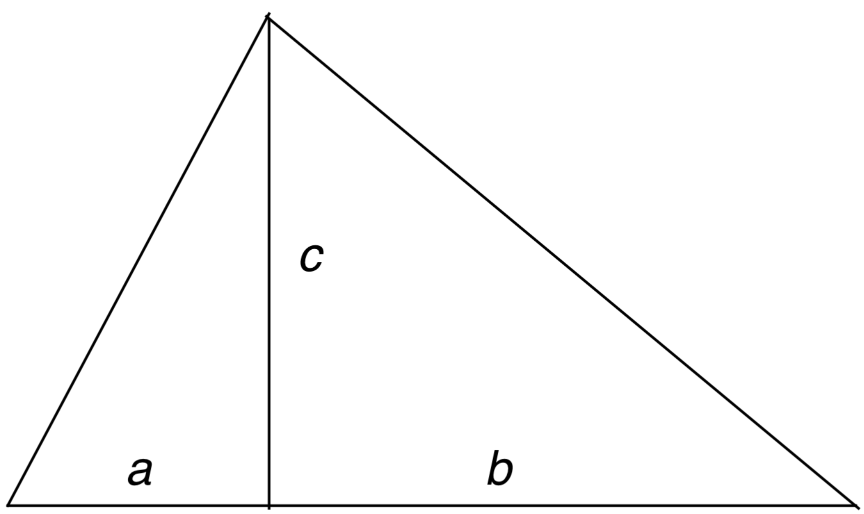
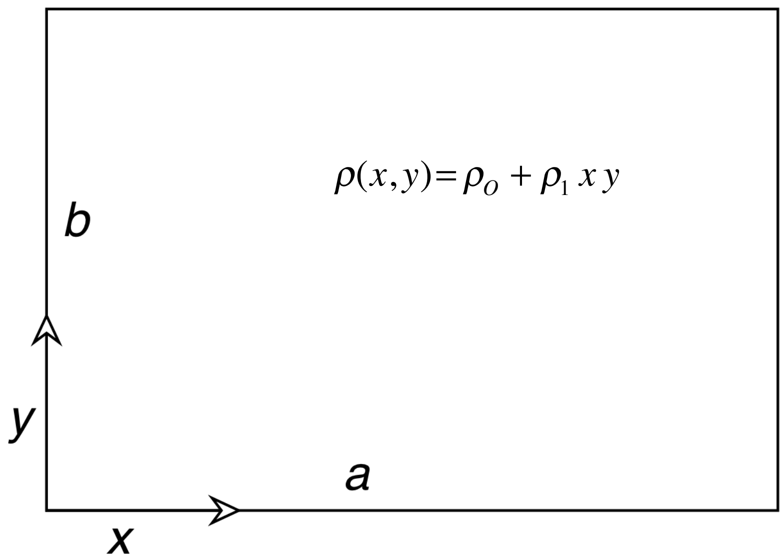
# Multiple Integral Problems

1. Use a double integral to calculate the area of the triangle shown below. Begin by drawing a small square with its lower left corner at the general point *(x,y)*, with side lengths *dx* and *dy*. You want to tell the integrals to add up all the little areas *dA=dx dy*. Primarily, this means deciding which variable to integrate first, and getting the limits on both integrals correct. By convention, the inner integral is the first one, so its differential (*dx* or *dy*) should also be first. You can look on the process as being one in which the inner integral is a function of the integration variable for the outer integral which just integrates that function over the limit for the second variable. (Don't worry, this is much easier in practice than in the explanation!)

For this problem, integrate on x first, just to make things easier to grade. That means you will need to know what the *x*-coordinates of points on the two sides having a given value of y. You will hold y constant while integrating *x* from one side to the other, and then integrate the resulting function of *y*, from $y = 0$ to $y = c$.



2. Find the mass of a plate of thickness $t$ and side lengths $a$ and $b$, as shown, but which has a non-uniform mass density $\rho(x,y) = \rho_O + \rho_1 x\, y$. (What are the units of the constant $\rho_1$?



3. Calculate the area of a circle of radius *R*, using Cartesian co-ordinates, and then repeat the calculation using polar co-ordinates to see how much easier that is. Don't forget that the area element in polar coordinates has a length in the $\phi$ direction of $r\,d\phi$, NOT $d\phi$, which does not have any dimensions at all! Be sure you draw the area element and understand why it is what it is.

4. Now lets consider the mass, moment of inertia and radius of gyration of a disk that does not have a uniform density. We want the moment of inertia about an axis through the center of the disk, perpendicular to its face, as shown. The disk has thickness t, and mass density given by $\rho(r,\phi,z) = \rho_O + \rho_1 r^2\, Sin(\phi/2)$.

a. Find the mass of the disk.

b. Find the moment of inertia about the given axis.

c. Use the definition of the radius of gyration ($I \equiv M\,R_G^2$) to find $R_G$.