Physics 220: Advanced Statistical Mechanics

Spring 2012

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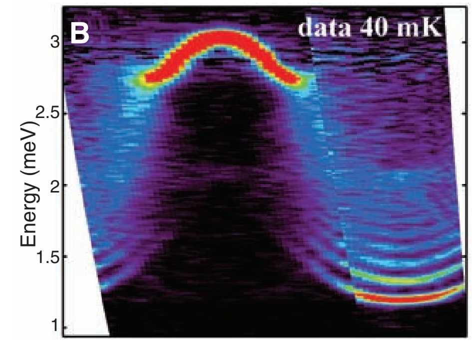
Plan

- General subject: statistical methods and phenomena in many-body systems
  - Phases and phase transitions
  - Critical phenomena - classical and quantum
  - Elementary excitations and topological defects
  - Models
  - Statistical field theory
  - Monte Carlo methods

Plan

- Cover subjects through illustrative topical examples from recent research such as
  - Quantum criticality in an Ising chain
  - Spin ice
  - Order by disorder

Ising Chain

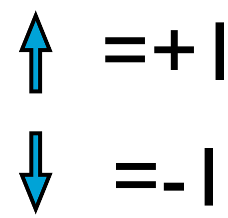


- Very beautiful paper from R. Coldea (Oxford), experimentally studying the *quantum transverse field Ising chain*, a canonical model of statistical mechanics
- We can learn about:
  - Ising models
  - Ordered and paramagnetic phases
  - Quantum and classical phase transitions
  - Elementary excitations and domain walls

Ising model

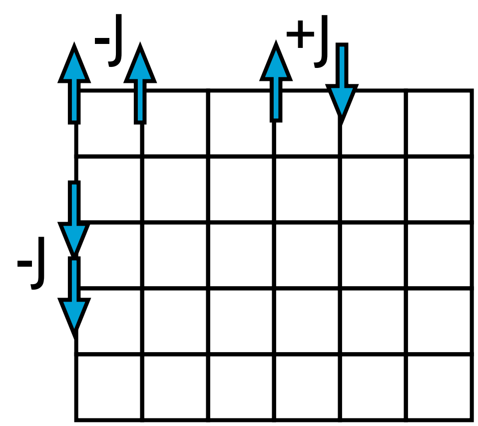
- Classical model of “spins” $\sigma_i = \pm 1$ which interact

$$H = -\frac{1}{2}\sum_{ij} J_{ij}\sigma_i\sigma_j$$



- Usually put them on a regular lattice and make them couple *locally*, e.g. by nearest-neighbors

$$H = -J\sum_{\langle ij\rangle} \sigma_i\sigma_j$$



J>0: “ferromagnetic”
J<0: “antiferromagnetic”

Thermal fluctuations

- Boltzmann

$$p[\sigma_1, \sigma_2, \cdots, \sigma_N] = \frac{1}{Z} e^{-\beta H} \qquad \beta = 1/k_{\mathrm{B}}T$$

- High temperature $\beta J \ll 1$
  - Spins are basically random and equally likely to take any value: *paramagnetic* phase
- Low temperature $\beta J \gg 1$
  - Spins are highly correlated and neighbors are almost always parallel: ?? *ordered, ferromagnetic* phase??

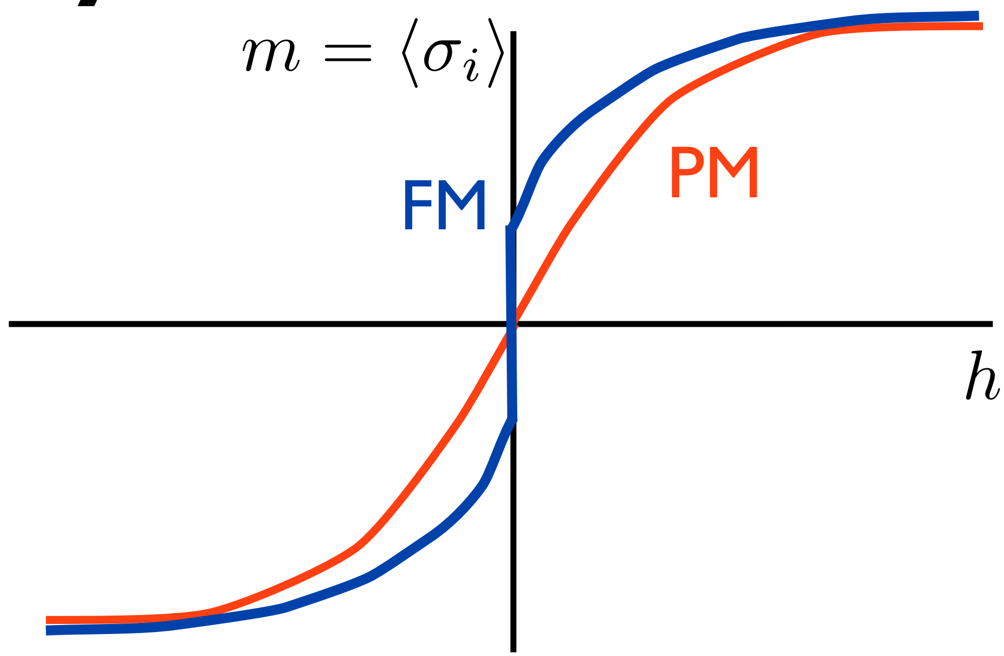
Phases

- A *phase* is a set of states of a system whose properties vary *smoothly* when varying control parameters continuously
  - Usually we say that the free energy is analytic within a phase
- Two systems are in the same phase if all their properties are *qualitatively* the same
- Distinct phases exist only in systems with (1) an infinite number of degrees of freedom and/or (2) at zero temperature
  - Why??? fluctuations etc.

Symmetry Breaking

- The difference between the paramagnetic and ferromagnetic phases is *broken Ising symmetry*
    - High T: paramagnetic $\langle \sigma_i \rangle = 0$
        - What does this mean (guaranteed by symmetry?)
        - Consider infinitesimal applied field
    - Low T: ferromagnetic $\langle \sigma_i \rangle \neq 0$
        - Infinitesimal field
        - Long range order

Susceptibility and LRO



- Susceptibility

$$\chi = \left. \frac{\partial \langle \sigma_i \rangle}{\partial h} \right|_{h=0}$$

- Linear response

$$\frac{\partial \langle \sigma_i \rangle}{\partial h} = \beta \sum_j \left(\langle \sigma_i \sigma_j \rangle - \langle \sigma_i \rangle \langle \sigma_j \rangle \right)$$

  - diverges when spins become long-range correlated

Define magnetization

- Infinitesimal field

$$m = \lim_{h \to 0^+} \langle \sigma_i \rangle_h$$

- Long-range order

$$m^2 = \lim_{|i-j| \to \infty} \langle \sigma_i \sigma_j \rangle_{h=0}$$