Correlation Length

- In the paramagnet, there is a finite length beyond which spins are uncorrelated

$$\langle \sigma_i \sigma_j \rangle \sim e^{-|i-j|/\xi} \qquad\qquad |i-j| \gg \xi$$

- The correlation length must go from finite to infinite to enter the FM: defines *critical temperature* T_c
  - Either it jumps to infinity: "first order transition"
  - Or it diverges continuously: "second order" or "continuous" transition
    - In the latter case, there can be non-analytic features on approaching T_c (why??)

Mean field theory

- The simplest approximation to describe a phase transition is MFT
  - There are many types of MFT, and if one wants to be more precise, this is "Curie-Weiss MFT"
- Idea: replace interaction between spins by an effective "exchange field"
  - Then solve the stat. mech. of this spin, and make the field self-consistent

MFT

- Decoupling

$$J_{ij}\sigma_i\sigma_j \to J_{ij}\left[\langle\sigma_i\rangle\sigma_j + \sigma_i\langle\sigma_j\rangle - \langle\sigma_i\rangle\langle\sigma_j\rangle\right]$$

- Exchange field

$$-\frac{1}{2}\sum_{ij} J_{ij}\sigma_i\sigma_j \to -\sum_i h_i^{\mathrm{eff}}\sigma_i + \mathrm{const.}$$

$$h_i^{\mathrm{eff}} = \sum_j J_{ij}\langle\sigma_j\rangle$$

- Self-consistency (for a classical Ising spin)

$$\langle\sigma_j\rangle = \tanh\beta h_j^{\mathrm{eff}}$$

MFT: solution

- For Ising Ferromagnet, on lattice with *z* nearest neighbors

$$m = \tanh z\beta J m$$

- For $k_B T > zJ$, only solution is m=0 (PM)
- For $k_B T < zJ$, get spontaneous m≠0 (FM)

$$m \sim (T_c - T)^{1/2}\Theta(T_c - T) \qquad |T - T_c|/T_c \ll 1$$

  - non-analytic behavior characteristic of *continuous* transition

Other MFT predictions

- Susceptibility

$$\chi \sim \frac{A}{T - T_c} \qquad\qquad T > T_c$$

- Specific heat

$$c_v \sim A - B\Theta(T - T_c)$$

- These kinds of predictions often work qualitatively and sometimes semi-quantitatively
  - We expect MFT works best when z is large

Ising Chain

- Coldea: $$H = \sum_i \left[-J S_i^z S_{i+1}^z - h_\perp S_i^x \right]$$

  $\boxed{\mathbf{S}_i = \boldsymbol{\sigma}_i/2 \text{ Pauli matrices}}$

- Zero transverse field: effectively classical

$$H = -J_{\text{eff}} \sum_i \sigma_i^z \sigma_{i+1}^z$$

- What is the transition like?
  - exactly solvable by "transfer matrix"

Transfer matrix

- Partition function

$$Z = \sum_{\{\sigma_i\}} e^{\beta J_{\text{eff}} \sum_{i=1}^{N} \sigma_i \sigma_{i+1}} \qquad \text{(PBCs)}$$

$$= \sum_{\{\sigma_i\}} \prod_{i=1}^{N} e^{K \sigma_i \sigma_{i+1}}$$

$$\equiv \sum_{\{\sigma_i\}} \prod_{i=1}^{N} \langle \sigma_i | \hat{T} | \sigma_{i+1} \rangle = \mathrm{Tr}\left(\hat{T}^N \right)$$

- Transfer matrix

$$\hat{T} = \begin{pmatrix} e^{K} & e^{-K} \\ e^{-K} & e^{K} \end{pmatrix} \qquad K = \beta J$$

Transfer Matrix (2)

- Solution

$$Z = \lambda_1^N + \lambda_2^N$$

$$\lambda_1 = 2\cosh K \;\; > \;\; \lambda_2 = 2\sinh K$$

- Large system

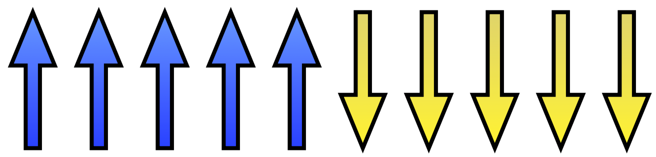
$$Z \approx (2\cosh K)^N$$

$$F = -\beta^{-1} \ln Z \approx -N\beta^{-1} \ln(2\cosh K)$$

- This is a smooth function with no singularity at finite, non-zero K = J/k_BT: no phase transition!

Why no transition?

- This is because of *domain walls*



ΔE=2J

- Correlation length = distance between domain walls: finite for any T>0

$$\xi \sim e^{2\beta J}$$

- Can verify this from transfer matrix

Fluctuations

- So *thermal* fluctuations have a drastic effect in 1d - destroy the phase transition entirely
  - In fact this is a general phenomena: d=1 is the "lower critical dimension" for discrete symmetry breaking at T>0
  - more on this theme later
- What about quantum fluctuation effects at T=0, or thermal fluctuations for d>1?
  - Even when they do not destroy the ordered phase, they alter critical properties and lead to other effects