RG

- e.g. Correlation function

$$C(x_i - x_j) = \langle S_i^z S_j^z \rangle \sim \langle \sigma(x_i)\sigma(x_j) \rangle$$

$$C(x, h_\perp, h_\parallel) = b^{-2/8} C(x/b, b\, h_\perp, b^{15/8} h_\parallel)$$

- Can choose *b*=*x*

$$C(x, h_\perp, h_\parallel) = x^{-1/4} C(1, h_\perp\, x, h_\parallel\, x^{15/8})$$

- Or b=1/$h_\perp$= ξ

$$C(x, h_\perp, h_\parallel) = \xi^{-1/4} C(x/\xi, 1, h_\parallel\, \xi^{15/8})$$

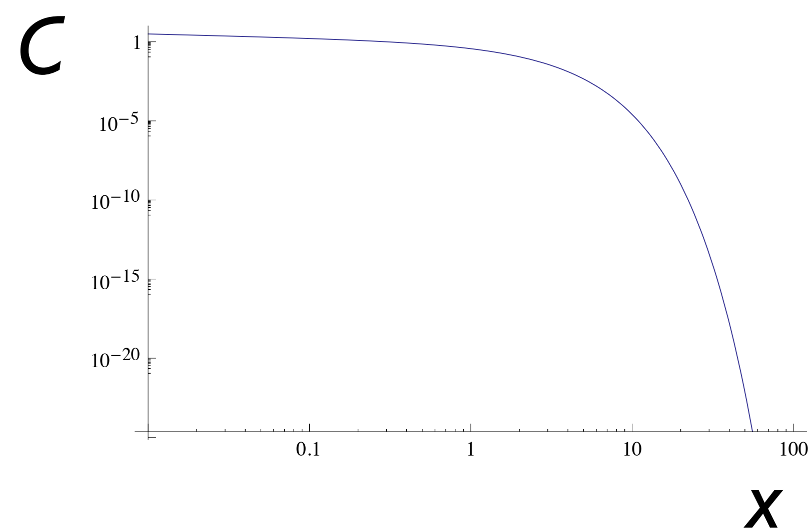
Correlation function

- In zero longitudinal field ($h_{\parallel}=0$)

$$C(x, h_\perp) = x^{-1/4}\mathsf{C}(x/\xi)$$

$$\mathsf{C}(0) = \mathsf{C}_0$$

$h_\perp > h_\perp^c$



$$\mathsf{C}(X) \sim X^{-\alpha} e^{-X} \qquad h_\perp > h_\perp^c \quad X \gg 1$$

$$\sim X^{1/4} \qquad h_\perp < h_\perp^c \quad X \gg 1$$

$\Rightarrow$ $\beta = 1/8$

Summary

- 1d TFIM has a QCP (like *all* continuous phase transitions) described by a *scale invariant continuum field theory*
  - The critical point is characterized by scaling operators (ε,σ) with scaling dimensions d_σ etc., and by a dynamical critical exponent z
- Perturbations to the QCP can be analyzed by RG, or scaling theory
  - Usually the *relevant* ones (which grow under rescaling) are most important
  - Scaling analysis can be applied to correlation functions, free energy, excitation energies,...you name it!

Back to Coldea

- Coldea studies $CoNb_2O_6$ via *inelastic neutron scattering*

$E = E_{in} - E_{out}$

$k = k_{in} - k_{out}$

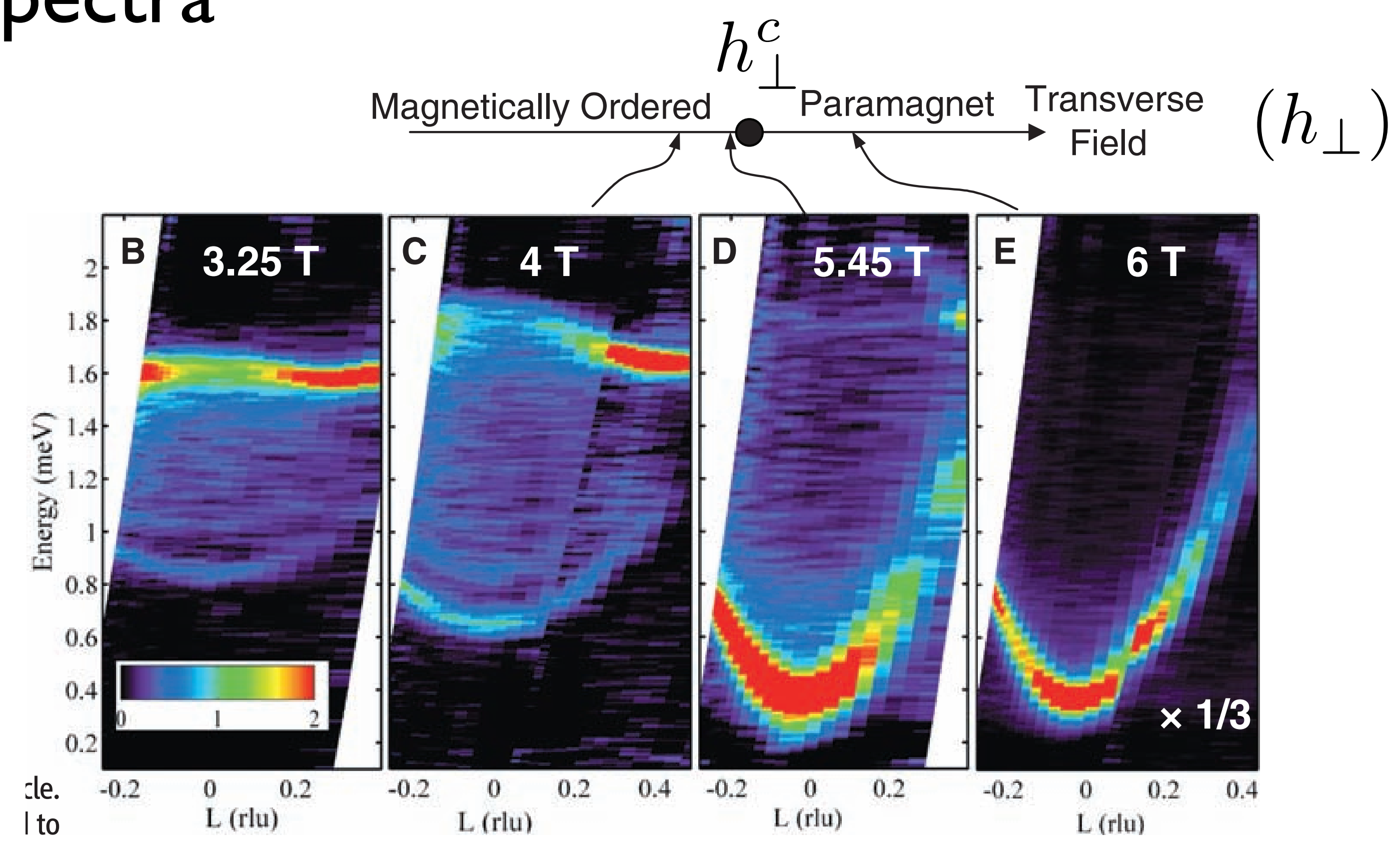
$\Delta S = 1$

measure

$$A(k,E) \sim \sum_n |\psi_n|^2 \delta(E - \epsilon_n(k))$$

Coldea

- Spectra



continuum broken into many small dispersion curves

sharply peaked dispersion