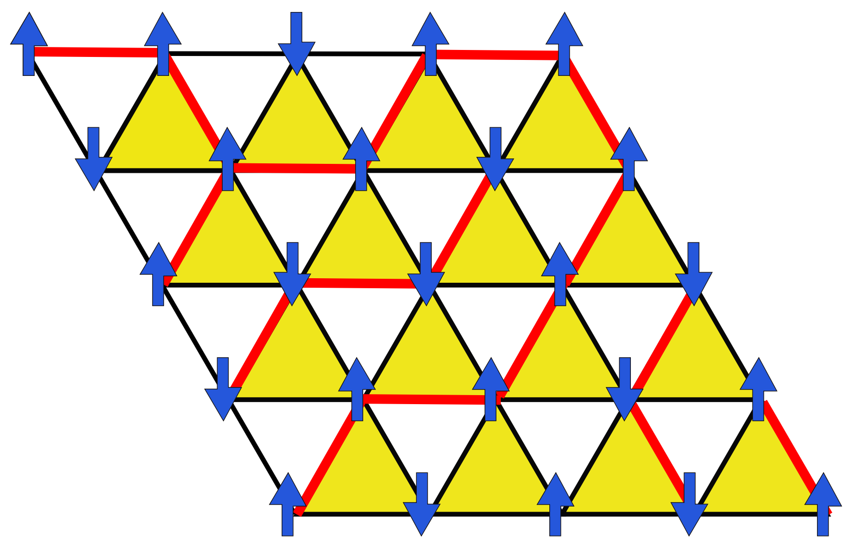
Degeneracy

- Ideally: frustration induces ground state degeneracy, and spins fluctuate amongst those ground states down to low temperature
- e.g. triangular lattice Ising antiferromagnet

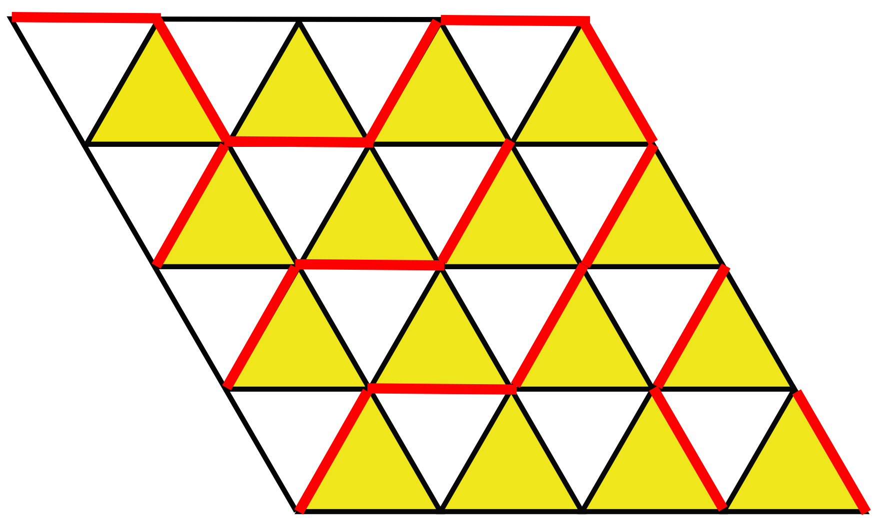


1 frustrated bond per triangle

Wannier (1950): $\Omega = e^{S/k_B}$ $S \approx 0.34 N k_B$

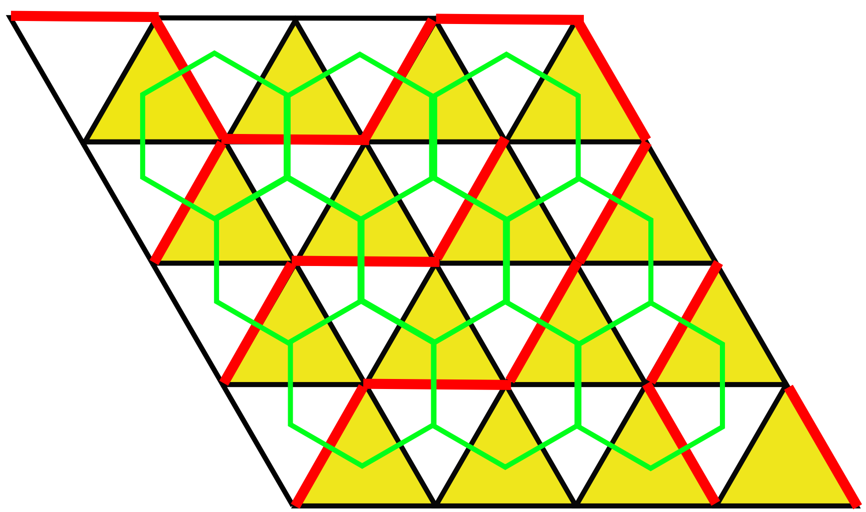
Estimate degeneracy?

- Dual representation
  - honeycomb lattice



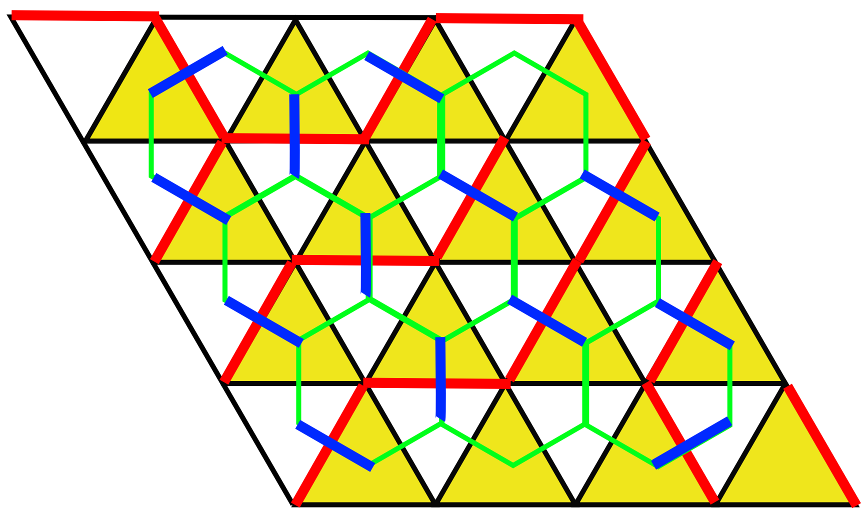
Estimate degeneracy?

- Dual representation
  - focus on the frustrated bonds



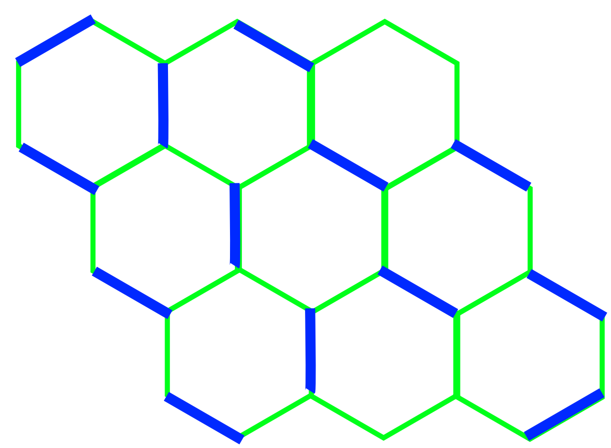
Estimate degeneracy?

- Dual representation
  - color "dimers" corresponding to frustrated bonds
    - "hard core" dimer covering



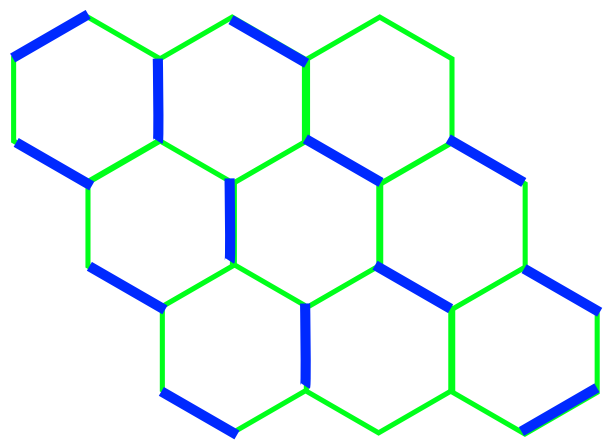
Estimate degeneracy?

- Dual representation
  - A 2:1 mapping from Ising ground states to dimer coverings

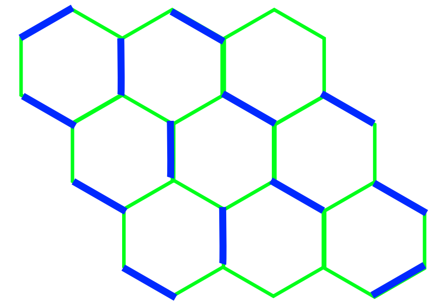


Dimer states

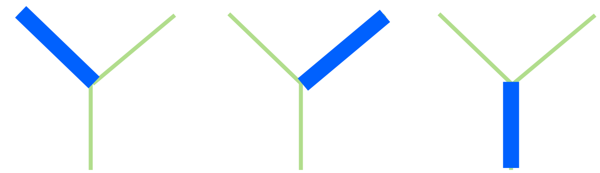
- First exercise: can we understand Wannier's result?
  - count the dimer coverings



Dimer states



- Consider the "Y" dual sites
  - each has 3 configurations
  - this choice fully determines the dimer covering



- But we have to make sure the Y^{-1} sites are singly covered. Make a crude approximation:
  - Prob(dimer) = 1- Prob(no dimer)= 1/3
  - Prob(good Y^{-1}) = 2/3 * 2/3 * 1/3 * 3 = 4/9
- Hence

$$\Omega \approx 3^N \left(\frac{4}{9}\right)^N = e^{N \ln(4/3)}$$

$S \approx 0.29\ N\ k_B$

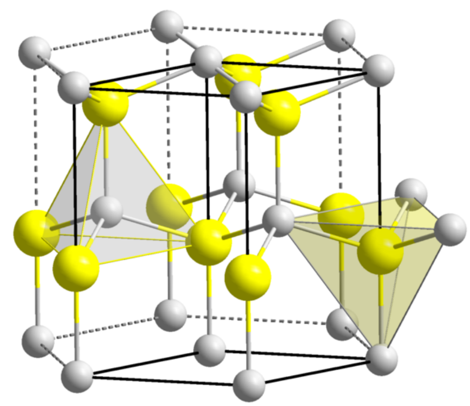
Wannier $S \approx 0.34\ N\ k_B$

Spin (and water) Ice

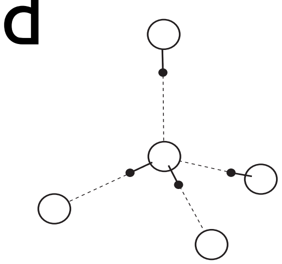
- This simple NN AF Ising model is rather idealized
  - You may expect that there are always perturbations that split this degeneracy and change the physics
- BUT...turns out that something similar happens in spin ice, which really seems to be an almost ideally simple material - by accident!

Water ice

- Common "hexagonal" ice: tetrahedrally coordinated network of O atoms - a wurtzite lattice



- Must be two protons in each H_2O molecule - but they are not ordered



Ice entropy

- Giauque 1930's measured the "entropy deficit" by integrating C/T from low T and comparing to high T spectroscopic measurements

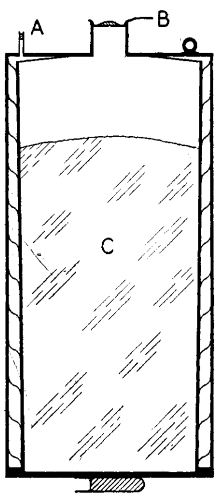


Fig. 1.—Calorimeter.

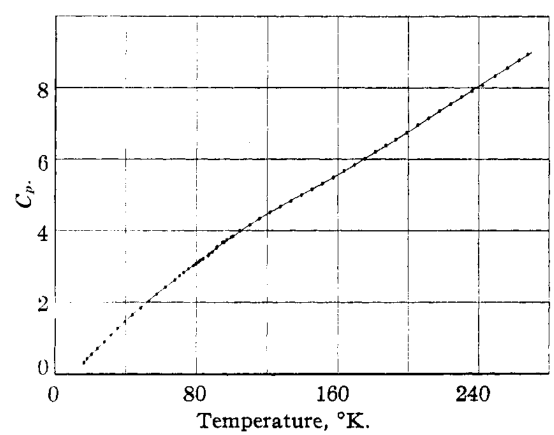


Fig. 2.—Heat capacity in calories per degree per mole of ice.

Table III

Calculation of Entropy of Water

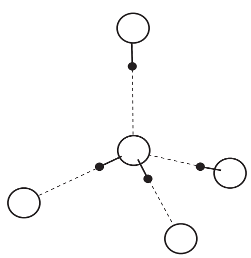
0–10°K., Debye function $h\nu/k$ = 192	0.022
10–273.10°K., graphical	9.081
Fusion 1435.7/273.10	5.257
273.10–298.10°K., graphical	1.580
Vaporization 10499/298.10	35.220
Correction for gas imperfection	0.002
Compression R ln 2.3756/760	−6.886
Cal./deg./mole	44.28 ± 0.05

cal./deg./mole. The difference between the spectroscopic and calorimetric values is 0.82 cal./deg./mole.

Pauling argument

- Pauling made a simple "mean field" estimate of the entropy due to randomness of the protons, which turns out to be quite accurate

$$\Omega = e^{S/k_B} =$$



$$2^{2N} \times \left(\frac{6}{16}\right)^N = \left(\frac{3}{2}\right)^N$$

each bond

O constraints

$$\binom{4}{2} = 6$$

allowed configurations each O

$$S = k_B \ln(3/2) = 0.81 \mathrm{Cal/deg \cdot mole}$$

$$\text{c.f. } S_{\mathrm{exp}} = 0.82 \pm 0.05 \mathrm{Cal/deg \cdot mole}$$

Classical realization: spin ice

- Rare earth pyrochlores $Ho_2Ti_2O_7$, $Dy_2Ti_2O_7$: spins form *Ising doublets*, behaving like classical vectors of fixed length, oriented along *local* easy axes

$$\vec{S}_i = \hat{e}_i \sigma_i$$