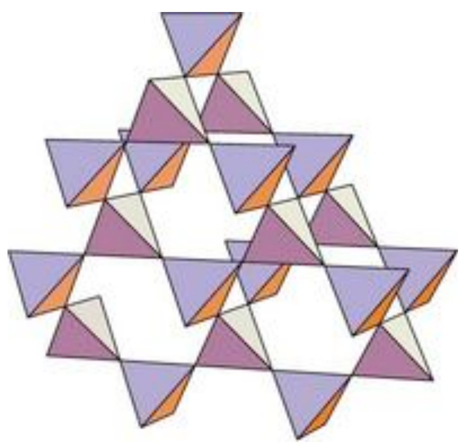
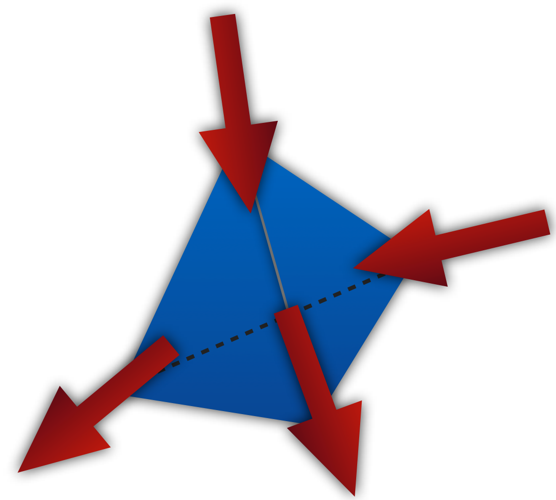
Classical realization: spin ice

- Rare earth pyrochlores $Ho_2Ti_2O_7$, $Dy_2Ti_2O_7$: spins form *Ising doublets*, behaving like classical vectors of fixed length, oriented along *local* easy axes

$$\vec{S}_i = \hat{e}_i \sigma_i$$





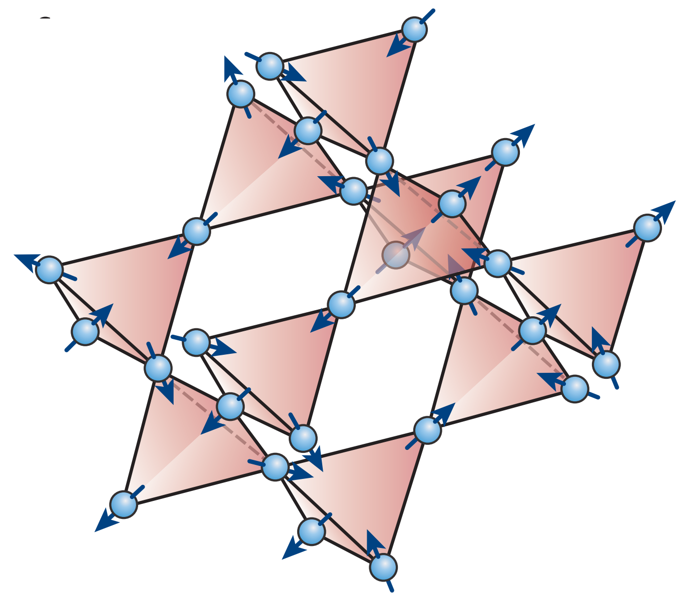
$\hat{e}_0 = (1,1,1)/\sqrt{3}$ $\hat{e}_1 = (1,-1,-1)/\sqrt{3}$ $\hat{e}_2 = (-1,1,-1)/\sqrt{3}$ $\hat{e}_3 = (-1,-1,1)/\sqrt{3}$

Spin Ice (simplified)

- Exchange (due largely to dipolar interactions) is *ferromagnetic*
  - Prefers "2 in - 2 out" states

$$-J\vec{S}_i \cdot \vec{S}_j = \frac{J}{3}\sigma_i\sigma_j$$

same as Ising antiferromagnet

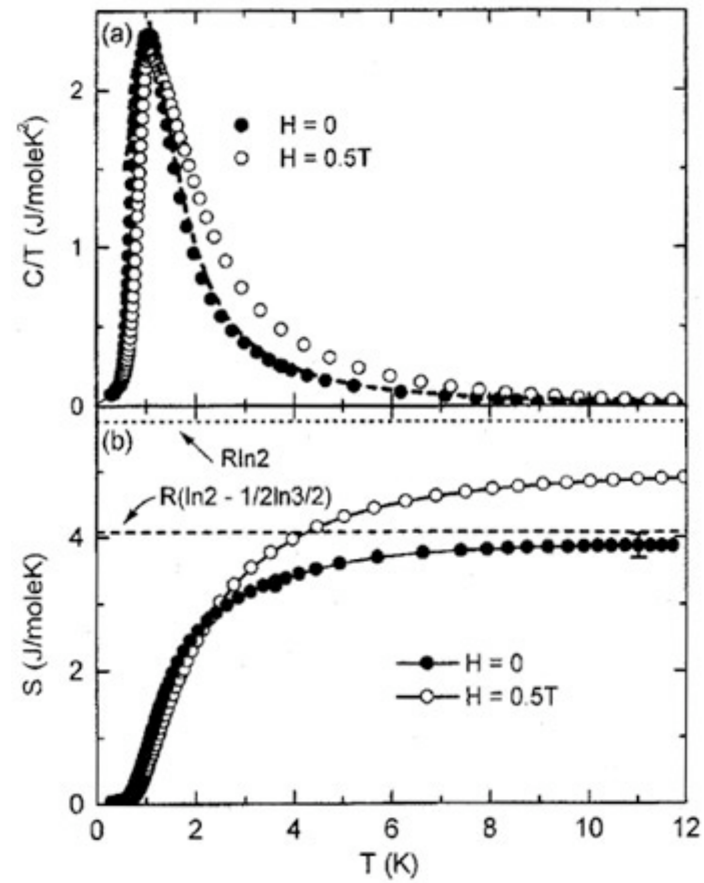


"ice rules"

$\hat{e}_i \cdot \hat{e}_j = -1/3 \quad i \neq j$

Entropy

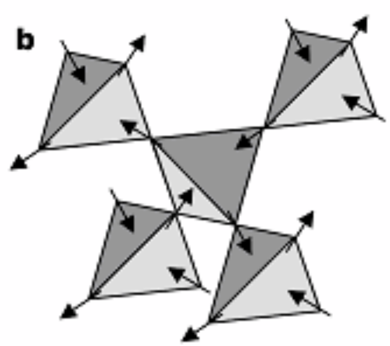
- The integrated specific heat of $Dy_2Ti_2O_7$ showed explicitly that the entropy did not vanish at low temperature
  - quantitative agreement with Pauling's 1935 estimate
- We call this situation, with spins fluctuating for kT<<J, a classical *spin liquid*



A.P. Ramirez *et al*, 1999

Spin liquid physics

- The spin liquid fluctuations are a form of "artificial magnetostatics" (classical)
  - ice rules: divergence free condition



Spin ice

$$\vec{S} \sim \vec{b}$$

$$\vec{\nabla} \cdot \vec{b} = 0$$

Spin liquid physics

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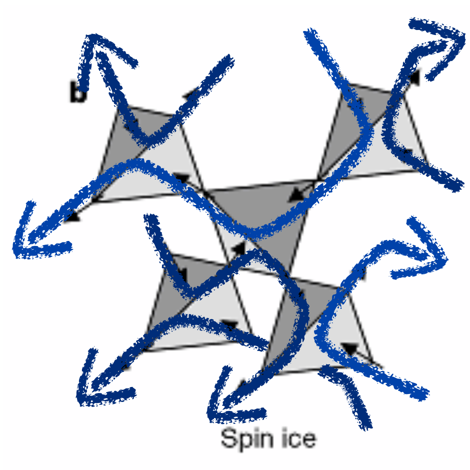

$$\vec{S} \sim \vec{b}$$

$$\vec{\nabla} \cdot \vec{b} = 0$$

field lines = loops or strings tracing spin configurations
Can we see effects in long-distance correlations?

Structure Factor

- Static neutron structure factor

$$\mathcal{S}_{\mu\nu}(k) = \sum_{i,j} \langle S_i^\mu S_j^\nu \rangle e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)}$$

- Typically, S(k) is used to distinguish *ordered* and paramagnetic states via *Bragg peak*
    - Long range order: |i-j|≫ξ

$$\langle \vec{S}_i \cdot \vec{S}_j \rangle \to \langle \vec{S}_i \rangle \cdot \langle \vec{S}_j \rangle \sim |M_Q|^2 \cos[\mathbf{Q} \cdot (\mathbf{r}_i - \mathbf{r}_j)]$$

$$S(k) \sim |M_Q|^2 \delta(k - Q)$$

    - Short range order

$$S(k) \sim \frac{A}{(k-Q)^2 + \xi^{-2}}$$

Structure Factor

- In spin ice, there is no incipient ordered state: feature in correlations is more subtle than a peak
- Coarse-graining argument: correlations are governed by *effective free energy*

$$H_{\text{eff}} = \int d^3r \, \frac{c}{2} |\vec{b}|^2$$

- Need to calculate

$$\langle b_\mu(r) b_\nu(r') \rangle = \frac{1}{Z} \int [d\vec{b}(r)] \, \delta[\vec{\nabla} \cdot \vec{b}] \, b_\mu(r) b_\nu(r') \, e^{-\beta H_{\text{eff}}[\vec{b}]}$$