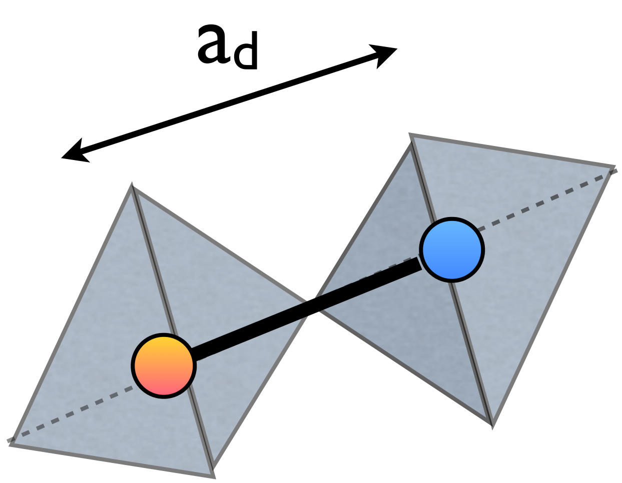
Dumbbell model



magnetic charge ±q $\quad q = \mu / a_d$

Dy, Ho $\quad \mu \approx 10\mu_B$

potential

$$V_{qq} = \frac{\mu_0}{4\pi}\frac{q_a q_b}{r_{ab}}$$
$$= \frac{\mu_0}{4\pi}\frac{\mu^2}{a_d^2}\frac{1}{r_{ab}}$$

Coulomb

$$V_{ee} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r}$$

Dumbbell model

ratio

$$V_{ee}/V_{qq} = \frac{e^2}{\mu^2}\frac{a_d^2}{\epsilon_0\mu_0} \quad = \frac{e^2c^2a_d^2}{\mu^2} \qquad \left(\frac{1}{\epsilon_0\mu_0} = c^2\right)$$

$$= \frac{e^2c^2a_d^2}{100\mu_B^2} = \frac{e^2c^2a_d^2(2m_e)^2}{100e^2\hbar^2} \qquad \left(\mu_B^2 = \frac{e\hbar}{2m_e}\right)$$

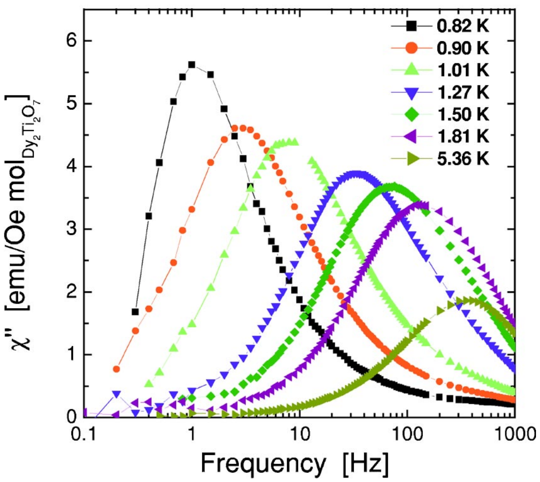
$$= \frac{a_d^2}{25\alpha^2a_0^2} \qquad \left(a_0 = \frac{\hbar}{m_e c\alpha}\right)$$

$$\approx 56000$$

Magnetic Coulomb interaction is *very* weak, but comparable to k_BT at T ~ 1K

Experiment/Theory

- Some nice evidence from magnetic relaxation



Snyder *et al*, 2004

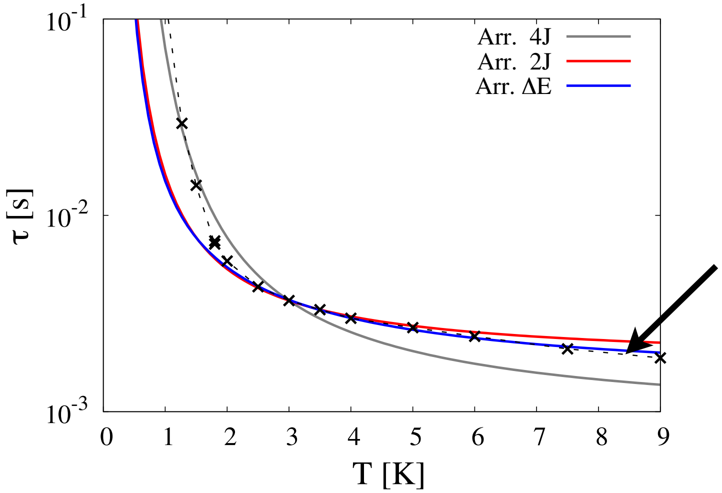


Figure from L. Jaubert's thesis

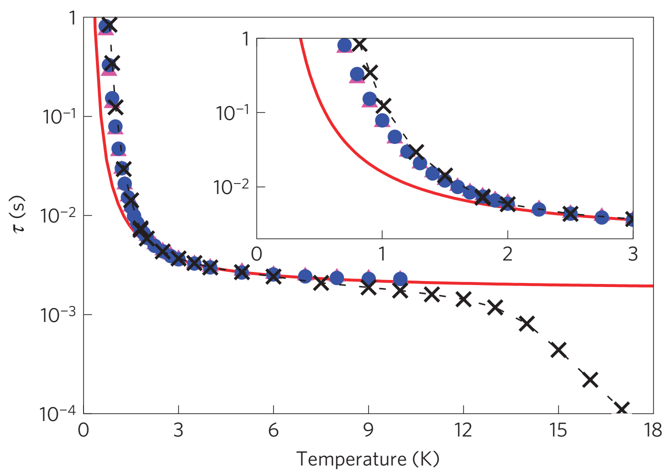
reasonable fit of activated monopoles

$$\tau \sim e^{E_m/k_B T}$$

- Rapid rise below 2K due to Coulomb!

Experiment/Theory

- Theory including Coulomb interactions (Monte Carlo):



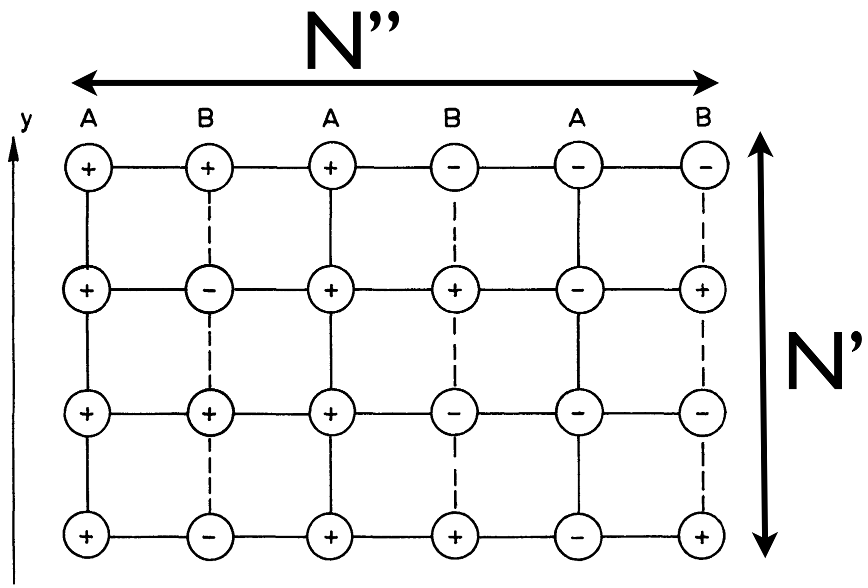
- Rise is due to *binding* of monopole-antimonopole pairs

Order by Disorder

- In spin ice, the ground state degeneracy seems to prevent an ordered phase forming
- Actually, this is not so obvious at low but non-zero temperature
- In fact, many models with ground state degeneracy *break* that degeneracy at $T>0$ due to fluctuations
  - "Order by disorder", due to J. Villain
  - Idea: *free* energy of states is generally different once fluctuations are included

J. Villain *et al*, J. Physique **41**, 1263 (1980).

Domino Model



$$H = -\frac{1}{2}\sum_{ij} J_{ij}\sigma_i\sigma_j$$

J_{AA}, J_{AB} ferromagnetic

J_{BB} antiferromagnetic

$$0 < J_{AB} < |J_{BB}| < J_{AA}$$

- Ground states are FM A chains and AF B chains, with $2^{N''}$ degeneracy

Order

- However, one can show that the model has a phase transition (by exact solution)
- Evidently it is ordered at low T despite the degeneracy - this is due to fluctuations.
- Let's understand this in some simple limits

Very low T

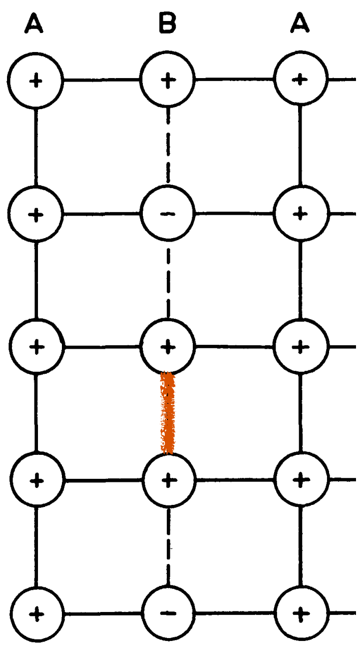
- $k_B T << J_{AA}, |J_{BB}|, J_{AB}$: only rare excitations within each chain
  - Ask: is there any preference for successive A chains to be aligned vs anti-aligned?
  - Do this by "integrating out" B chain between each pair of A chains

$$P[\{\sigma_{i\in A}\}] = \frac{1}{Z} \sum_{\sigma_j \in B} e^{-\beta H}$$

Very low T

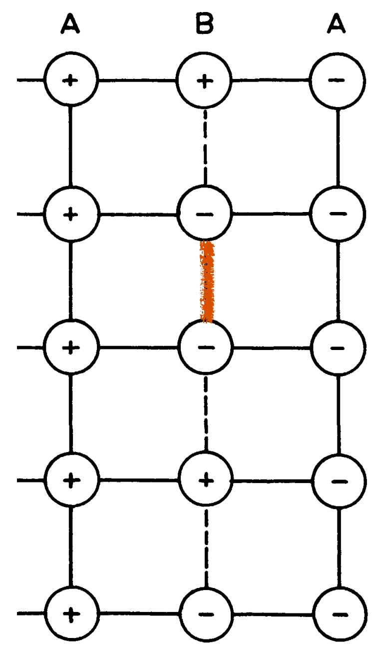
- Two cases:

domain wall



a

excitation lowers J_{AB} energy



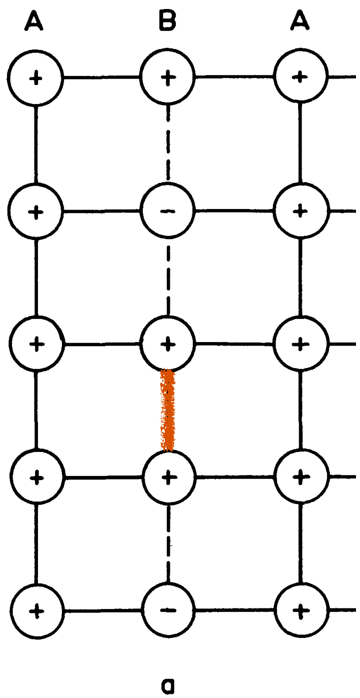
b

excitation does not lower J_{AB} energy

Very low T

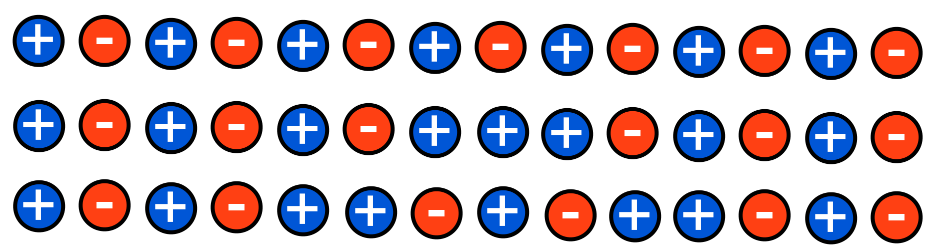
- Two cases:

domain wall



$$H_B = \sum_i \{ |J_{BB}|\, \sigma_i \sigma_{i+1} - 2 J_{AB}\, \sigma_i \}$$

$$\Delta E_B = 2|J_{BB}| - 2 J_{AB} M_{DW}$$

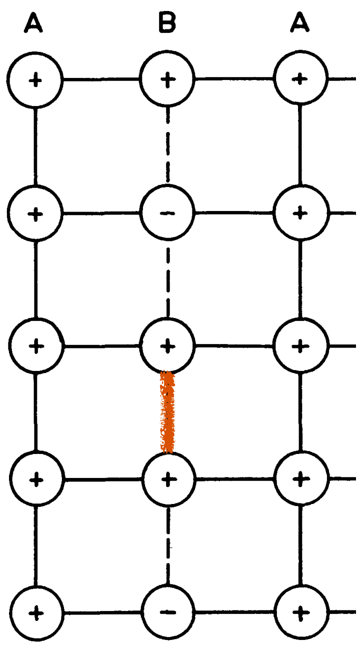


M=2=1+1

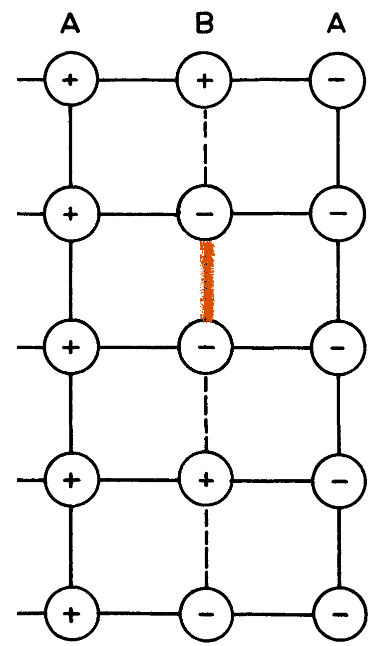
Very low T

- Two cases:

domain wall



a



b

energy $\Delta E = 2|J_{BB}| - 2J_{AB}$

$\Delta E = 2|J_{BB}|$

note: factor of 2 difference from Villain paper