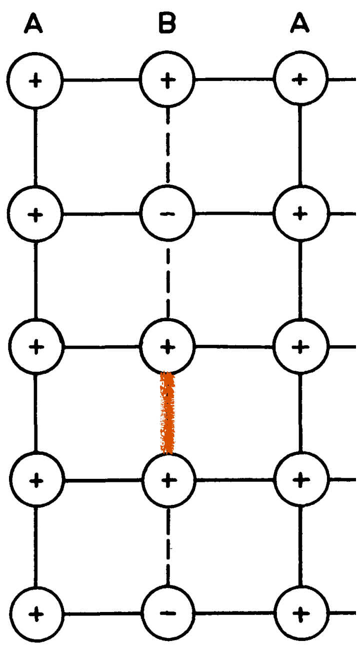
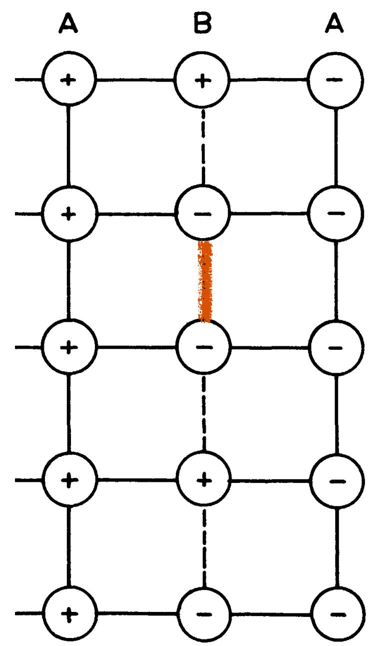
Very low T

- Two cases:

domain wall



a



b

energy $\Delta E = 2|J_{BB}| - 2J_{AB}$

$\Delta E = 2|J_{BB}|$

note: factor of 2 difference from Villain paper

B partition function

- We can place the domain wall in N'/2 places

$$Z_B \approx Z_{B0}\left(1+\frac{N'}{2}e^{-\beta\Delta E}\right) \approx Z_{B0}e^{\frac{N'}{2}e^{-\beta\Delta E}}$$

- This prefers ferromagnetic ordering

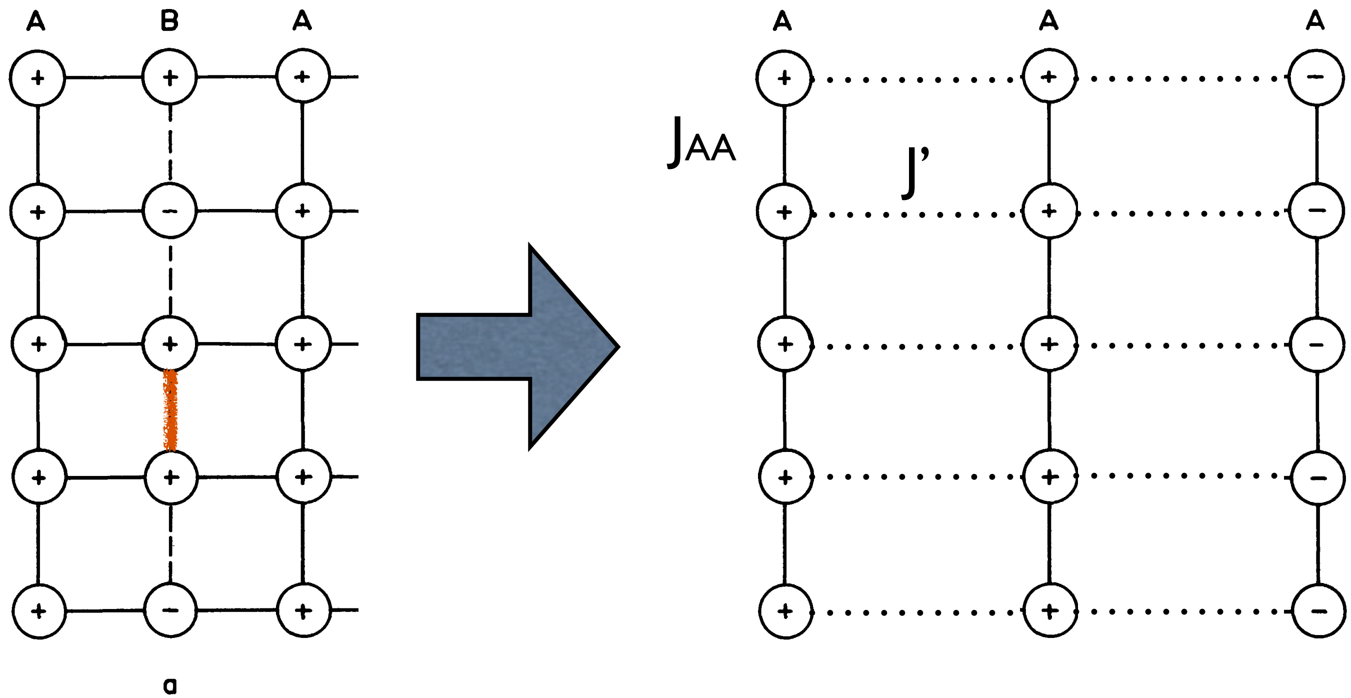
$$\frac{P(++)}{P(+-)} \approx e^{\frac{N'}{2}e^{-2\beta(|J_{BB}|-J_{AB})}}$$

- Effectively this is like a FM exchange

$$2\beta J' = \frac{1}{2}e^{-2\beta(|J_{BB}|-J_{AB})}$$

Order?

- Effective rectangular lattice



J' is shrinking fast at low T

but correlation length of A chains is growing very fast

- Orders if $J'\xi_A \sim k_B T$

Order?

- Estimate
  - 1d Ising $\xi_A \sim e^{2\beta J_{AA}}$
  - Entropy $2\beta J' = \frac{1}{2} e^{-2\beta(|J_{BB}| - J_{AB})}$
- Together

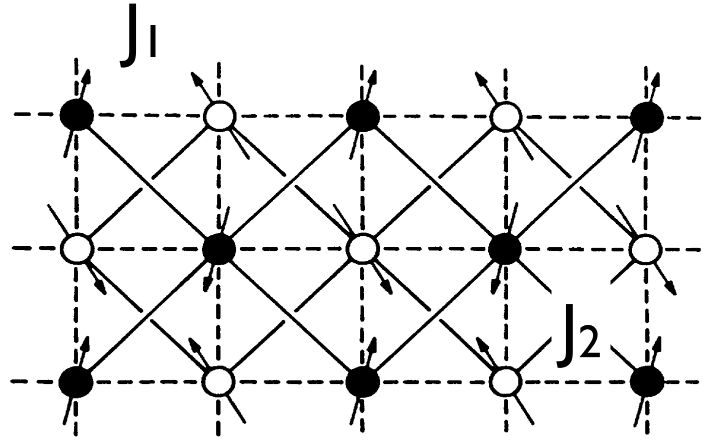
$$\beta J' \xi_A \sim e^{-2\beta(|J_{BB}| - J_{AB}|)} e^{2\beta J_{AA}}$$

$$\gg 1 \qquad\qquad J_{AA} > |J_{BB}| - J_{AB}$$

Thus the A spins are ferromagnetically ordered!

Continuous Spins

- Actual strictly Ising systems are rather rare in magnets, but similar phenomena can occur for continuous spins
- Example: frustrated square lattice "XY" AF - spins are unit vectors in the plane



$J_2 > J_1/2$

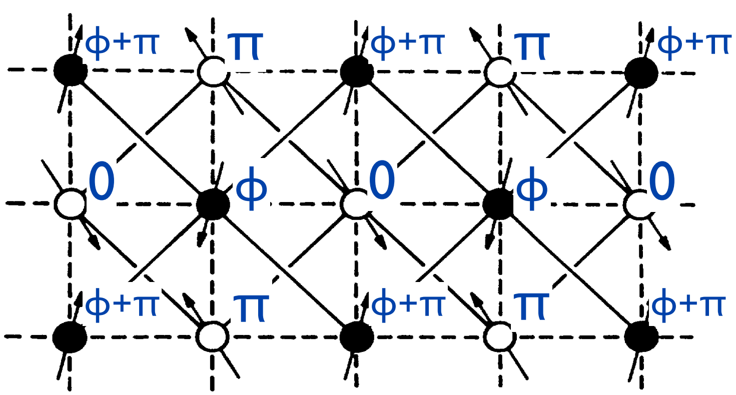
C. Henley, 1989

Thermal fluctuations

- Consider expansion around an arbitrary ground state

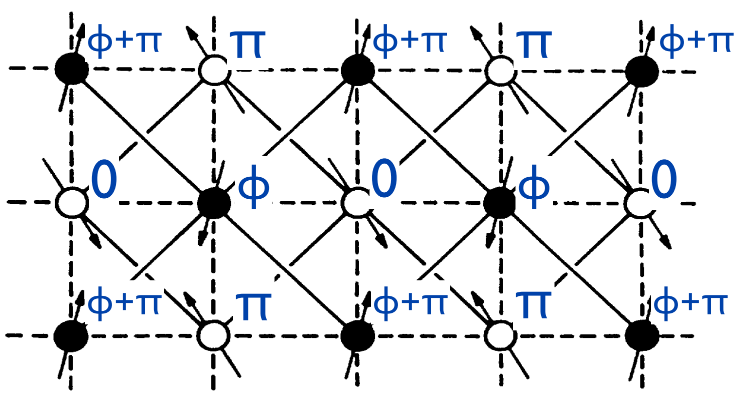
$$H = -\frac{1}{2}\sum_{ij} J_{ij}\cos(\theta_i - \theta_j)$$

$$\approx E_0 + \frac{1}{4}\sum_{ij} J_{ij}\cos(\theta_{ij}^{(0)})(\delta\theta_i - \delta\theta_j)^2$$



Thermal fluctuations

- Consider expansion around an arbitrary ground state

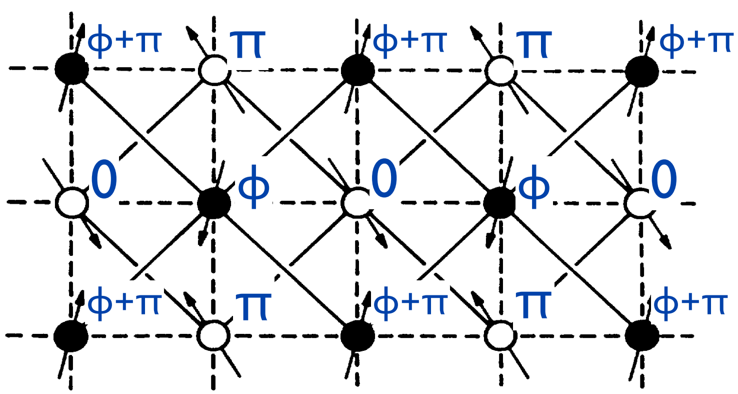


$$H \approx \frac{J_1}{2} \sum_{xy} \cos\phi \left[(\delta\theta_{xy} - \delta\theta_{x+1,y})^2 - (\delta\theta_{xy} - \delta\theta_{x,y+1})^2 \right]$$

$$- \frac{J_2}{2} \sum_{xy} \left[(\delta\theta_{xy} - \delta\theta_{x+1,y+1})^2 + (\delta\theta_{xy} - \delta\theta_{x+1,y-1})^2 \right]$$

Thermal fluctuations

- Consider expansion around an arbitrary ground state



$$\delta\theta_{xy} = \frac{1}{\sqrt{N}} \sum_k e^{i\mathbf{k}\cdot\mathbf{r}} \delta\theta_{\mathbf{k}}$$

$$H \approx \frac{J_1}{2} \sum_{\mathbf{k}} 2\cos\phi(\cos k_y - \cos k_x)|\delta\theta_{\mathbf{k}}|^2$$

$$-\frac{J_2}{2} \sum_{\mathbf{k}} \left[4 - 2\cos(k_x + k_y) - 2\cos(k_x - k_y)\right] |\delta\theta_{\mathbf{k}}|^2$$

I covered up to and including this page!!!

Thermal Fluctuations

- Collecting terms

$$\delta H \approx \frac{1}{2} \sum_{\mathbf{k}} A_{\mathbf{k}}(\phi) |\delta\theta_{\mathbf{k}}|^2$$

$$A_{\mathbf{k}}(\phi) = 4J_2(1 - \cos k_x \cos k_y) - 2J_1 \cos\phi(\cos k_x - \cos k_y)$$

- Gaussian integral

$$Z \approx e^{-\beta E_0} \int [\prod_{\mathbf{k}} d\delta\theta_{\mathbf{k}}] e^{-\delta H} \sim e^{-\beta E_0} \prod_{\mathbf{k}} \frac{1}{\sqrt{A_{\mathbf{k}}}}$$

Entropy

- Free energy

$$F = -k_B T \ln Z \approx E_0 + \frac{k_B T}{2} \sum_{\mathbf{k}} \ln A_{\mathbf{k}}$$

$$\equiv E_0 - T S_0$$

$$S_0 = -N \frac{k_B}{2} \int \frac{d^2\mathbf{k}}{(2\pi)^2} \ln A_{\mathbf{k}}$$

more entropy if A_k is smaller

$$\ln A_{\mathbf{k}} = \ln[4J_2(1 - \cos k_x \cos k_y)] + \ln[1 - \frac{J_1 \cos\phi}{2J_2} \frac{\cos k_x - \cos k_y}{1 - \cos k_x \cos k_y}]$$

indep. of ϕ

Entropy

- Up to a constant

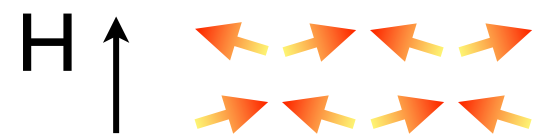
$$S_0(\phi) = \text{const} - \frac{Nk_B}{2}\int \frac{d^2\mathbf{k}}{(2\pi)^2} \ln\left[1 - X\frac{\cos k_x - \cos k_y}{1 - \cos k_x \cos k_y}\right]$$

$$X = \frac{J_1 \cos\phi}{2J_2}$$

- This is an *increasing* function of |X|, so minimized when ϕ=0 or π: *collinear state*
  - See this, e.g. by expanding in X using ln (1-ε) = -ε -ε^2+...

Collinear states

- Why collinear states?
- Think about each sublattice as an antiferromagnet in a fluctuating field due to the other sublattice
  - An antiferromagnet likes to “flop” normal to an applied field



  - The fluctuating field from A sublattice on the B spins is normal to the A spins

Collinear states

- So...the normal to A spins should be normal to B spins, i.e. A and B should be collinear!
- It has been suggested (Henley) that this is rather general.

Quantum Fluctuations

- At T=0, we can imagine quantum zero point motions of the spins plays the role of thermal fluctuations
- Simple idea: quantize the normal mode frequencies corresponding to the modes $\delta\theta_k$:

$$\hbar\omega_{\mathbf{k}} = \sqrt{A_{\mathbf{k}}/m}$$

- This corresponds to the semi-classical "1/*S*" or spin-wave expansion

Zero point energy

- Harmonic oscillators

$$E_{0-\mathrm{pt}} = \sum_{\mathbf{k}} \frac{\hbar\omega_{\mathbf{k}}}{2} \sim \frac{1}{\sqrt{2m}} \sum_{\mathbf{k}} \sqrt{A_{\mathbf{k}}}$$

- The zero point energy is again minimized if A_k is smaller -
  - one can check that this is again $\phi=0,\pi$

Seeing ObD

- In models, this is a generic phenomena: small fluctuations break “accidental” degeneracies
- But...many other perturbations also remove the accidental degeneracies
  - e.g. explicit small J’ interaction
  - How can you ever really know - in an experiment - if order is due to disorder or just some interaction you missed?
- Lucile will tell you Thursday!