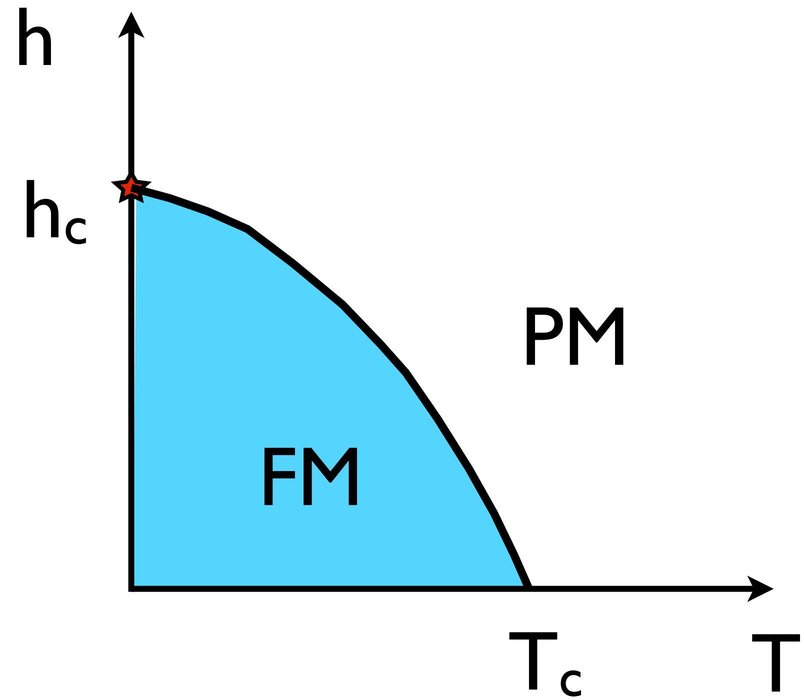
Quantum phase transitions in metals

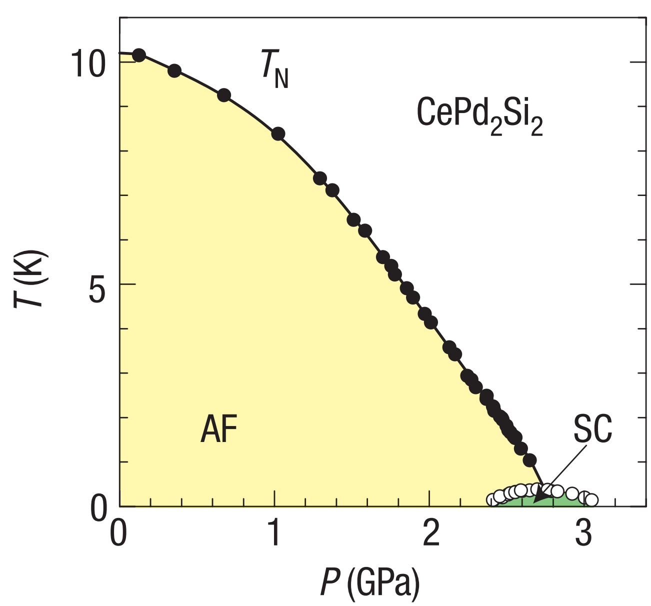
- Some quantum phase transitions are very similar to classical ones
  - recall TFIM



Actually the same field theory describes classical and quantum transitions in the Ising model

But some are not

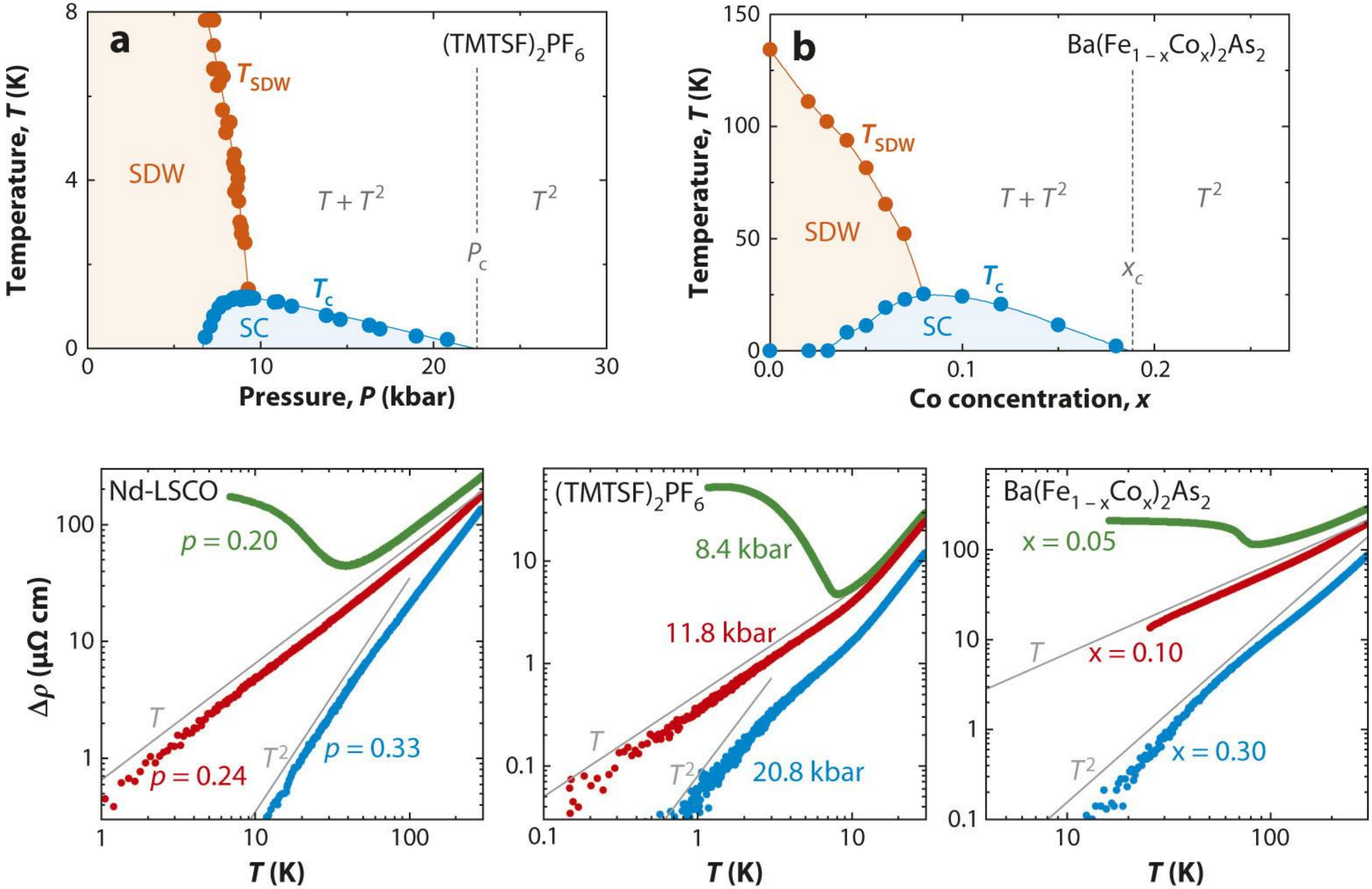
- Especially: quantum phase transitions in metals



"heavy fermion"

- Often superconducting state "covers" QCP
- Critical exponents non-classical
- Anomalous metallic behavior

Linear resistivity



Why does metal make a difference?

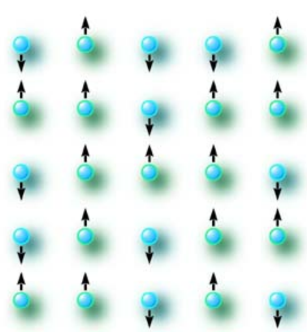
- These phase transitions are nominally similar to those in insulators

$$\langle \vec{S}(\mathbf{r}) \rangle = \vec{\Phi} e^{i\mathbf{Q}\cdot\mathbf{r}} + \mathrm{c.c.}$$

- Might expect a Landau theory in Φ to apply
- But...usual assumption is that the only contributions to the critical behavior come from the ordering fluctuations, as *only these persist to long distances* (up to ξ)
- In a metal, there are other long-distance fluctuations and correlations which are due to low energy quasiparticles

Connection of quantum and classical stat. mech.

- In classical stat. mech., the partition function is a sum/integral over degrees of freedom in *d* dimensions



$$Z = \sum_{\{\sigma_{\mathbf{r}}\}} e^{-\beta \sum_{\mathbf{r}} \mathcal{H}_{\mathbf{r}}}$$

$$\sim \int [d\Phi(\mathbf{r})] e^{-\beta \int d^d\mathbf{r}\, \mathcal{H}[\Phi(\mathbf{r}), \nabla\Phi(\mathbf{r})]}$$

Connection of quantum and classical stat. mech.

- In quantum stat. mech., the partition function is a trace

$$
\begin{aligned}
Z &= \mathrm{Tr}\,\left[e^{-\beta H}\right] \\
&= \sum_{\{\sigma^z_{\mathbf{r}}\}} \langle \sigma^z_{\mathbf{r}_1}\sigma^z_{\mathbf{r}_2}\cdots|e^{-\beta H}|\sigma^z_{\mathbf{r}_1}\sigma^z_{\mathbf{r}_2}\cdots\rangle
\end{aligned}
$$

- There is nothing local about the matrix elements of exp[-βH]

Connection of quantum and classical stat. mech.

- Trotter formula

$$Z = \mathrm{Tr}\left[e^{-\beta H}\right]$$

$$= \mathrm{Tr}\left[\underbrace{e^{-\delta\tau H} e^{-\delta\tau H} \cdots e^{-\delta\tau H}}_{\text{N factors}}\right] \qquad \delta\tau = \beta/N$$

$$= \sum_{\{\sigma^z_{\mathbf{r},\tau}\}} \langle\{\sigma^z_{\mathbf{r},\beta}\}|e^{-\delta\tau H}|\{\sigma^z_{\mathbf{r},\beta-\delta\tau}\}\rangle\langle\{\sigma^z_{\mathbf{r},\beta-\delta\tau}\}|e^{-\delta\tau H} \cdots \langle\{\sigma^z_{\mathbf{r},\delta\tau}\}|e^{-\delta\tau H}|\{\sigma^z_{\mathbf{r},0}\}\rangle$$

Connection of quantum and classical stat. mech.

- Trotter formula

$$Z = \mathrm{Tr}\left[e^{-\beta H}\right] = \sum_{\{\sigma^z_{\mathbf{r},\tau}\}} e^{-\sum_{\mathbf{r},\tau}\mathcal{L}_{\mathbf{r},\tau}}$$

$$\sim \int [d\Phi(\mathbf{r},\tau)] e^{-\int d^d\mathbf{r}\, d\tau\, \mathcal{L}[\Phi(\mathbf{r},\tau),\partial_\mu\Phi(\mathbf{r},\tau)]}$$

$$= \int [d\Phi(\mathbf{r},\tau)] e^{-S[\Phi(\mathbf{r},\tau)]}$$

"Euclidean action"

- So one expects there to be a relation between the *d* dimensional quantum problem and a classical-like problem in *d* space and *one* "time-like" direction

Degrees of freedom

- But...in a metal we do not just have spins
  - really the trace must include the states of the electrons

$$H = \frac{1}{2}\sum_{\mathbf{r},\mathbf{r}'} J_{\mathbf{r},\mathbf{r}'}\vec{S}_{\mathbf{r}}\cdot\vec{S}_{\mathbf{r}'} + \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} c^{\dagger}_{\mathbf{k},\alpha} c_{\mathbf{k},\alpha}$$

$$+J_K \sum_{\mathbf{r},\mathbf{k},\mathbf{k}'} e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}}\vec{S}_{\mathbf{r}}\cdot c^{\dagger}_{\mathbf{k},\alpha}\frac{\vec{\sigma}_{\alpha\beta}}{2}c_{\mathbf{k}',\beta}$$

  - Trace includes S_r and c_k

Degrees of freedom

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$$H = \frac{1}{2}\sum_{\mathbf{r},\mathbf{r}'} J_{\mathbf{r},\mathbf{r}'}\vec{S}_{\mathbf{r}}\cdot\vec{S}_{\mathbf{r}'} + \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} c^{\dagger}_{\mathbf{k},\alpha} c_{\mathbf{k},\alpha}$$

$$+J_K \sum_{\mathbf{r}} \vec{S}_{\mathbf{r}}\cdot\vec{s}_{\mathbf{r}}$$

- Trace includes S_r and c_k - so does the action

Path integral

- Formally

$$Z = \int [d\Phi][dc\,dc^\dagger]e^{-S[\Phi,c,c^\dagger]}$$

- We can *try* to reduce this to a *d+1*-dimensional "classical" problem by integrating out c, $c^\dagger$
- How feasible is this?