

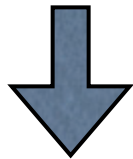
# Power counting

- Rescaling:  $k \rightarrow k/b$        $\omega_n \rightarrow \omega_n/b^z$

$$\Phi_{k,\omega_n} \rightarrow b^{d+z-d_\Phi} \Phi_{bk, b^z \omega_n}$$

$$S = \int \frac{d^d \mathbf{k} d\omega_n}{(2\pi)^{d+1}} \left\{ \left( k^2 + \frac{|\omega_n|}{k^a} + r \right) |\Phi_{\mathbf{k}, \omega_n}|^2 \right\} + u \int \frac{d^{3d} \mathbf{k}_i d^3 \omega_{n,i}}{(2\pi)^{3d+3}} \Phi_{k_1, \omega_{n1}} \Phi_{k_2, \omega_{n2}} \Phi_{k_3, \omega_{n3}} \Phi_{-k_1-k_2-k_3, -\omega_{n1}-\omega_{n2}-\omega_{n3}}$$

$$z = 2+a$$



$$-d - z - 2 + 2(d + z - d_\Phi) = 0$$

$$d + a - 2d_\Phi = 0$$

$$d_\Phi = \frac{d + a}{2}$$

# k versus x scaling

- Note: Fourier transform

$$\Phi_{\mathbf{k}, \omega_n} = \int d^d \mathbf{x} d\tau e^{-i\mathbf{k} \cdot \mathbf{x} - i\omega_n \tau} \Phi_{\mathbf{x}, \tau}$$

- Space-time scaling

$$\Phi_{\mathbf{x}, \tau} = b^{-d_\Phi} \Phi'_{\mathbf{x}/b, \tau/b^z}$$

- Hence

$$\Phi_{\mathbf{k}, \omega_n} = b^{-d_\Phi} \int d^d \mathbf{x} d\tau e^{-i\mathbf{k} \cdot \mathbf{x} - i\omega_n \tau} \Phi'_{\mathbf{x}/b, \tau/b^z}$$

note  
difference!

$$= b^{-d_\Phi + d + z} \int d^d \mathbf{x} d\tau e^{-ib\mathbf{k} \cdot \mathbf{x} - ib^z \omega_n \tau} \Phi'_{\mathbf{x}, \tau}$$

$$= b^{-d_\Phi + d + z} \Phi'_{b\mathbf{k}, b^z \omega_n}$$

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$$z = 2 + a$$

$$r \rightarrow b^2 r$$

$$d_\Phi = \frac{d+a}{2}$$

$$u \rightarrow b^{2-d-a} u = b^{4-d-z} u$$

u is “irrelevant” when  $d+a > 2$

# Aside: Classical Case

- Power counting  $x \rightarrow bx$   $\Phi_x \rightarrow b^{-d_\Phi} \Phi_{x/b}$

$$F = \int d^d x \left\{ (\nabla \Phi)^2 + r \Phi^2 + u \Phi^4 \right\}$$

- Gradient term  $d_\Phi = \frac{d-2}{2}$

- RG:

$$\begin{aligned} r &\rightarrow b^2 r \\ u &\rightarrow b^{4-d} u \end{aligned}$$

$u$  is irrelevant for  $d > 4$

# Upper critical dimension

- It turns out that for  $d > 4$ , one gets *mean field behavior*. We call  $d_{u.c.} = 4$  the *upper critical dimension*
- This coincides with - and is a consequence of - the fact that  $u$  is irrelevant, i.e. that the Gaussian fixed point is stable.
- Below the u.c.d., critical exponents are non-MF like

# Classical scaling for $d > 4$

- Correlation length:  $\nu = 1/2$

$$\xi = b g(r b^2, u b^{4-d}) = |r|^{-1/2} g(\pm 1, u |r|^{(d-4)/2})$$

- Free energy

$$f = b^{-d} \mathcal{F}(r b^2, u b^{4-d}) = |r|^{d/2} \mathcal{F}(\pm 1, u |r|^{(4-d)/2})$$

- $r > 0$ :  $f \sim |r|^{d/2}$

- $r < 0$ :  $u$  is necessary for stability

$$f \sim |r|^{d/2} [u |r|^{(4-d)/2}]^{-1} \sim r^2 / u \quad \alpha = 0$$

*u is a dangerously irrelevant operator*