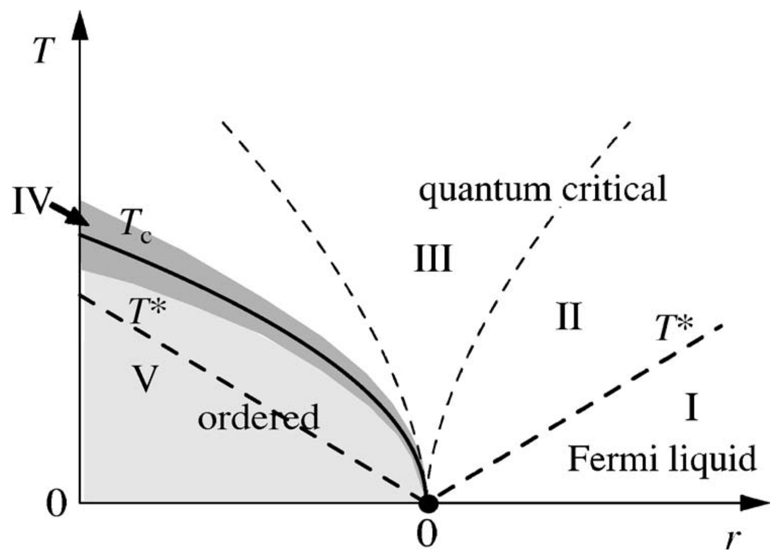
Phase boundary

- What determines the shape of the phase boundary?
  - Physics: thermal fluctuations suppress order



Phase boundary

- Fluctuation correction to *location* of critical point

$$S = \int \frac{d^d\mathbf{k}d\omega_n}{(2\pi)^{d+1}} \left\{ (k^2 + \frac{|\omega_n|}{k^a} + r)|\Phi_{\mathbf{k},\omega_n}|^2 \right\} + u \int d^d\mathbf{x}d\tau \, |\Phi_{\mathbf{r},\tau}|^4$$

- "Mean-field"-like approximation (technically self-energy correction)

$$u\Phi^4_{\mathbf{x},\tau} \to 6u \left\langle (\Phi_{\mathbf{x},\tau})^2 \right\rangle (\Phi_{\mathbf{x},\tau})^2$$

$$\boxed{r_{\text{eff}} = r + 6u \left\langle (\Phi_{\mathbf{x},\tau})^2 \right\rangle}$$

shifts critical point to r<0

The shift

- Fourier (introduce "cutoff" ε)

$$\langle \Phi^2_{\mathbf{x},\tau} \rangle = \frac{1}{\beta} \sum_{\omega_n} \int_0^\Lambda \frac{d^d\mathbf{k}}{(2\pi)^d} \frac{1}{k^2 + |\omega_n| + \epsilon\, \omega_n^2}$$

- We want to extract the small temperature behavior of this. Poisson formula:

$$\frac{1}{\beta} \sum_{\omega_n} = \frac{2\pi}{\beta} \int \frac{d\omega_n}{2\pi} \sum_m \delta(\omega_n - 2\pi m/\beta) = \sum_m \int \frac{d\omega_n}{2\pi} e^{im\beta\omega_n}$$

The shift

- We obtain

$$\langle \Phi^2_{\mathbf{x},\tau} \rangle = \int_0^\Lambda \frac{d^d\mathbf{k}}{(2\pi)^d} \int \frac{d\omega_n}{2\pi} \sum_{m=-\infty}^{\infty} \frac{e^{im\beta\omega_n}}{k^2 + |\omega_n| + \epsilon\,\omega_n^2}$$

- Separate m=0 (T=0) term:

$$\langle \Phi^2_{\mathbf{x},\tau} \rangle = I_0 + 2 \sum_{m=1}^{\infty} \underbrace{\int_0^\Lambda \frac{d^d\mathbf{k}}{(2\pi)^d} \int \frac{d\omega_n}{2\pi} \frac{\cos(m\beta\omega_n)}{k^2 + |\omega_n| + \epsilon\,\omega_n^2}}_{I_m}$$

Analyzing the integral

- Rotate contour $\omega_m = i\,y$

$$I_m = 2\mathrm{Re}\int_0^\Lambda \frac{d^d\mathbf{k}}{(2\pi)^d}\int_0^\infty \frac{d\omega_n}{2\pi}\frac{e^{im\beta\omega_n}}{k^2+\omega_n+\epsilon\,\omega_n^2}$$

$$= 2\mathrm{Re}\int_0^\Lambda \frac{d^d\mathbf{k}}{(2\pi)^d}\int_0^\infty \frac{dy}{2\pi}\frac{i\,e^{-m\beta y}}{k^2+iy-\epsilon\,y^2}$$

$$= 2\int_0^\Lambda \frac{d^d\mathbf{k}}{(2\pi)^d}\int_0^\infty \frac{dy}{2\pi}\frac{ye^{-m\beta y}}{y^2+(k^2-\epsilon\,y^2)^2}$$

Analyzing the integral

- Rescale: y = T u, k = $T^{1/2}$ q

$$I_m = 2\int_0^{\Lambda} \frac{d^3\mathbf{k}}{(2\pi)^3} \int_0^{\infty} \frac{dy}{2\pi} \frac{y e^{-m\beta y}}{y^2 + (k^2 - \epsilon\, y^2)^2}$$

$$= 2T^{3/2} \int_0^{\frac{\Lambda}{\sqrt{T}}} \frac{d^3\mathbf{q}}{(2\pi)^3} \int_0^{\infty} \frac{du}{2\pi} \frac{u e^{-mu}}{u^2 + (q^2 - \epsilon\, T^2 u^2)^2}$$

$$\approx 2T^{3/2} \int_0^{\infty} \frac{d^3\mathbf{q}}{(2\pi)^3} \int_0^{\infty} \frac{du}{2\pi} \frac{u e^{-mu}}{u^2 + q^4}$$

$$= c_m T^{3/2}$$

So finally...

- We obtain $\left\langle \Phi^2_{\mathbf{x},\tau} \right\rangle = I_0 + cT^{3/2}$
- Which implies

$$r_{\text{eff}} = r + 6u \left\langle \left(\Phi_{\mathbf{x},\tau}\right)^2 \right\rangle$$

$$= r_{\text{eff}}(T=0) + c\,u\,T^{3/2}$$

- So the critical point occurs when

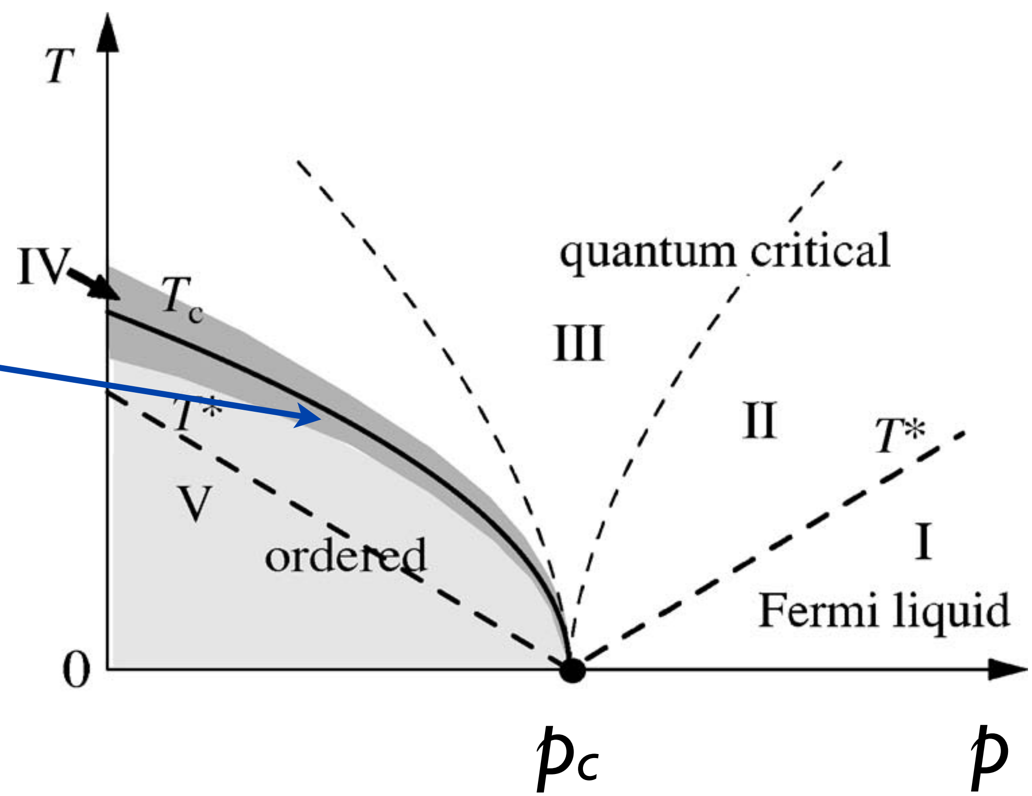
$$r_{\text{eff}}(T) = 0$$

$$\boxed{T_c = \left(\frac{-r_{\text{eff}}(T=0)}{cu} \right)^{2/3}}$$

Phase boundary

- This gives the shape:

$$T_c \sim (p_c - p)^{2/3}$$



Resistivity

- This is very complicated, even in Hertz theory above the upper critical dimension!
  - but...in general power-law behavior is expected, and usually different from that in an normal metal, i.e. away from the QCP
  - In the simplest approximation, for d=3, z=2, one obtains $\rho \sim \rho_0 + A\,T^{3/2}$ See von Löhneysen *et al*, RMP **79**, 1015, sec. IIIF
  - c.f. in a usual Fermi liquid, at low temperature $\rho \sim \rho_0 + A\,T^2$

Resistivity

- Behavior in $CeNi_2Ge_2$ seems consistent with the "simple" theory, which is expected to apply when the material is not too clean

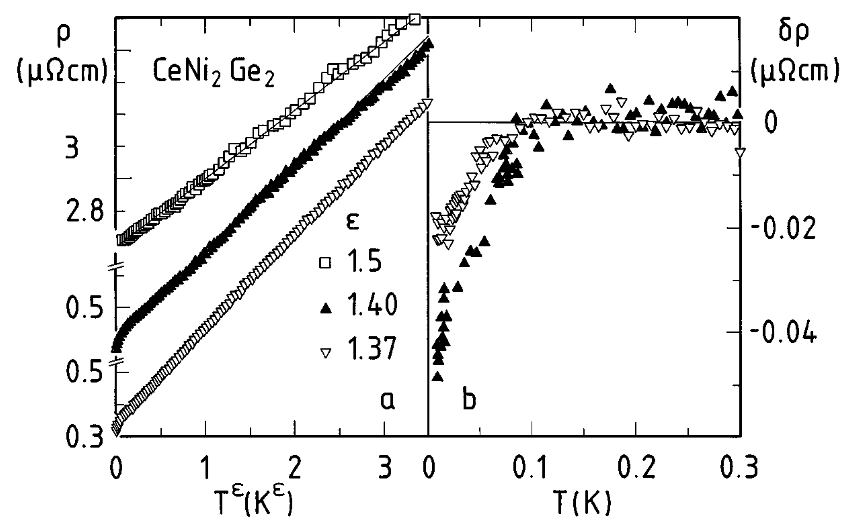


FIG. 2. Electrical resistivity as a function of temperature for three $CeNi_2Ge_2$ samples with $\rho_0 = 2.7\ \mu\Omega$ cm (□), $0.43\ \mu\Omega$ cm (▲), and $0.34\ \mu\Omega$ cm (▽) as ρ vs T^ε with differing exponents ε (a) and $\delta\rho = \rho - (\rho_0 + \beta T^\varepsilon)$ vs T (b).

When does it work?

- Not obvious: the assumption that integrating our electrons does nothing to higher order terms is questionable
- People have looked at these and it seems that it is OK when $Q \neq 0$ in $d=3$
- For $Q=0$ in $d=2,3$ and for $Q \neq 0$ in $d=2$ there are many singularities not captured by Hertz action
- In all these cases, one should try to study the QCP without integrating out fermions
  - This is much more complicated and still a matter of current research