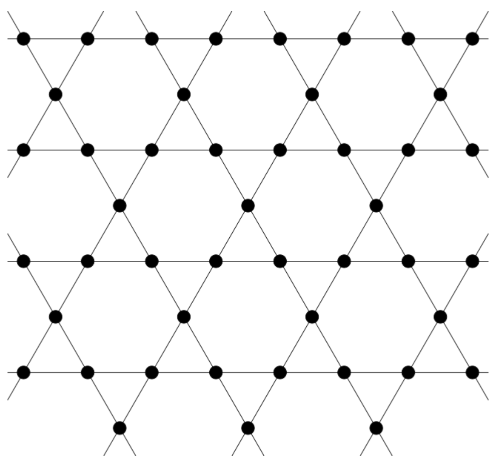
Physics 220: Problem Set 4
due May 17, 2012.

1. **Single tetrahedron:** As a crude approximation to spin ice, consider a single tetrahedron of four Ising spins $\sigma_i = \pm 1$, with Hamiltonian

$$H = J \sum_{i>j} \sigma_i \sigma_j, \tag{1}$$

with $J > 0$. Find the internal energy $U = \langle H \rangle$, the heat capacity $C = dU/dT$, and the entropy S as functions of T. Plot $C(T)$ and $S(T)$ versus $k_B T/J$.

2. **Kagomé ice:** If spin ice is subjected to a magnetic field along the (111) axis, it polarizes the 1/4th of the spins whose Ising axis is parallel to the field. This spin therefore points out of the "up" oriented tetrahedra, and into the "down" oriented tetrahedra. The remaining three spins per tetrahedra are organized into layers of *kagomé* lattice, as shown. Spins in each layer are



effectively decoupled, so we can consider one layer independently of the others. Considering the projection of the spins into the plane, the exchange interactions favor states in which two spins point in and one out within the "up" oriented triangles, and two spins point out and one in within the "down" triangles. Like in zero field, this gives rise to a zero temperature entropy. Estimate the entropy per spin using Pauli-like arguments.