Gauge/gravity; solutions for assignment 2

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Problem 1 a
We try \( \phi = e^{ikx} z^\alpha \psi(z) \)

\[
\nabla^2 \phi - m^2 \phi = z^{d+1} \partial_z (z^{1-d} \partial_z z^\alpha \psi) - ((kz)^2 + m^2) z^\alpha \psi \\
= z^{d+1} \partial_z (\alpha z^\alpha \psi + z^{\alpha-d+1} \psi') - ((kz)^2 + m^2) z^\alpha \psi \\
= z^\alpha (\alpha (\alpha - d) \psi + \alpha z \psi' + (\alpha - d + 1) z \psi' + z^2 \psi'') - ((kz)^2 + m^2) z^\alpha \psi
\]

so by setting \( \alpha = (d - 1)/2 \) we kill the linear term and get

\[
- \psi'' + \left[ k^2 + z^{-2} \left( m^2 + \frac{d^2 - 1}{4} \right) \right] \psi = \omega^2 \psi(z)
\]

Problem 1 b
Integrating over a constant \( x^0 \) surface and taking the normalization \( \phi = e^{ikx} z^\alpha \psi(z)/\sqrt{V} \) with \( V \) the volume of a timeslice of the boundary,

\[
(\phi, \phi)_{KG} = -i \int d^{d-1}x \sqrt{-\gamma} n^\mu \bar{\phi} \gamma_\mu \phi = 2\omega \int dz z^{d-1} \bar{\phi} \phi = 2\omega \int dz \psi^* \psi = 2\omega (\psi, \psi)_{\text{Schr}}
\]

where \( n^\mu = (\partial_0)^\mu / |\partial_0| = z \delta_0^n \).

Problem 1 c
Set \( k = 0 \) and take negative energy \( -\omega^2 \) so that \( \omega \) remains real and positive, also let \( \delta^2 = m_{BF}^2 - m^2 = -d^2/4 - m^2 \) then the S-equation becomes

\[
-z^2 \psi'' + \left( z^2 \omega^2 - \delta^2 - \frac{1}{4} \right) \psi = 0
\]

One solution to this is \( \sqrt{z} H_\delta(i\omega z) \) with \( H \) a Hankel function of the first kind. This has Schrödinger norm

\[
\int_0^\infty dz z |H_{\delta}(i\omega z)|^2
\]

A NIntegrate with \( \delta = \omega = 1 \) shows that this is finite: we have found an integrable negative energy solution below the BF bound. Of course that doesn’t show we don’t have such modes above the BF bound. McGreevy works this out in great detail.