1. Tensors
   a. Show that the linear vector spaces $V$ and its dual $V^*$ have the same dimension and hence are isomorphic.
   b. Prove that the tensor operation of contraction is independent of the choice of basis for the vector space.

2. Addition of velocities
   a. A cart rolls across a table with speed $v$ with respect to the table; on top of the cart is another cart, rolling with speed $\tilde{v}$ with respect to the first cart, and in the same direction. What is the speed of the second cart with respect to the table?
   b. Assume that $\tilde{v} = v$. On the second cart is a third cart moving with speed $v$ with respect to the second, and so on up to $n$ carts. What is the speed of the $n^{th}$ cart with respect to the table? What is the speed as $\lim_{n \to \infty}$?

3. Faster than light travel?
   a. One often hears the statement that “nothing” can travel faster than light. Consider a light house with a beam of light rotating at constant frequency $\omega$. Suppose there is a screen at some large distance $R$ away. Can the beam of light move across the screen faster than $c$? If so, how is this consistent with special relativity.
   b. Imagine that a spaceship, Enterprise, has been built that can travel faster than light. The Enterprise travels from Earth to Regulus $\alpha$ Leo, taking a mere 30 hours (as measured by Earth clocks) to cover the 26 parsec distance. According to the crew on a space-tug traveling in the same direction, they agree the trip took 30 hours, but to them it happened in reverse order! The Enterprise left Regulus and travelled to the Earth! How fast was the space-tug traveling with respect to the Earth. This is an example of how causal relationships get confused when things travel faster than the speed of light.

4. Accelerated observers
   At the LHC, a proton moves in a circular orbit of radius $r$ and period $T$ as measured in the lab frame. How much proper time elapses during one revolution as seen by the proton?