PHY231A Final Exam – Due at noon on Friday 12/09/11
Bring your completed exam to Don Marolf in Broida 6131 or to Jessica Wirts (or Debbie Ceder) in Broida 5014

Name: ______________________________________

Time & date started: ____________ Time & date stopped work: ____________

Instructions:

1) Write the tiime you first looked at this exam at the top of your Solution. You must stop working on your exam 12 hours later. Also write down the time you stopped working. The exam may then be turned in at your leisure (before the due date above).

2) You may use your notes from class and both textbooks (Wald and Carroll). However, do not consult any other texts or any other person until you have turned in your exam.

3) I believe the problems below are clearly stated. However, if a problem seems unclear, please just write down what you think the problem is asking for and proceed. Asking me for clarification is not recommended, as this process could consume valuable time that you could have spent working the exam.

Problems:

1) [30points] In class, we considered the weak field limit for a static gravitational field sourced by a point mass. Our calculation in class was for $\Lambda = 0$, where Λ is the cosmological constant.

   a) Repeat the above-mentioned calculation for $\Lambda \neq 0$ and find the effective Newtonian potential Φ. Sketch a (qualiative) graph of your $\Phi(r)$ vs. r for both signs of Λ. Treat both the mass m of the point particle and the cosmological constant Lambda as being "small" so that you may neglect terms of order m^2, $m\Lambda$, or Λ^2.

      the case where Λ is small (compared to any other scales) and *positive*. Does the motion of geodesics differ qualitatively from the $\Lambda = 0$ case? You may consider the non-relativistic limit of the geodesic motion, $v \ll c$.

   b) Consider the case where Λ is small (compared to any other scales) and *negative*. Does the motion of geodesics differ qualitatively from the $\Lambda = 0$ case? You may consider the non-relativistic limit of the geodesic motion, $v \ll c$.

2) [30 points] Consider the metric

$$ds^2 = -(1 - r^2/\ell^2)dt^2 + \frac{dr^2}{1 - r^2/\ell^2} + r^2 d\Omega^2. \tag{1}$$

This metric is another representation of de Sitter space, which we have discussed several times (remember the inflating universe?). It is a solution to the Einstein equations with $T_{ab} = 0$ and $\Lambda = 3/\ell^2$ (in four dimensions).

a) Show by explicit calculation that ∂_t satisfies Killing's equation.

b) Calculate the worldline of a general timelike radial geodesic. Radial geodesics are those with $\frac{d\theta}{d\tau} = 0 = \frac{d\phi}{d\tau}$. The easiest way to proceed is to use energy conservation for geodesics and the fact that $U^a U_a = -1$.

c) Does this behavior agree qualitatively with your results from problem 1?

3) [40 points] Consider the metric

$$ds^2 = -(1 + r^2/\ell^2)dt^2 + \frac{dr^2}{1 + r^2/\ell^2} + r^2 d\Omega^2. \tag{2}$$

This spacetime is called *anti*-de Sitter space. It is a solution to the Einstein equations with $T_{ab} = 0$ and $\Lambda = -3/\ell^2$ (in four dimensions).

a) Show by explicit calculation that ∂_t satisfies Killing's equation.

b) Calculate the worldline of a general timelike radial geodesic; in particular, find $r(\tau)$ where τ is the proper time. Radial geodesics are those with $\frac{d\theta}{d\tau} = 0 = \frac{d\phi}{d\tau}$. The easiest way to proceed is to use energy conservation for geodesics and the fact that $U^a U_a = -1$.

c) Does the above behavior agree qualitatively with your results from problem 1?

d) Calculate the worldline of a general *null* radial geodesic. Show that $r \to \infty$ at a finite value of t, but that this occurs at an infinite value of any affine parameter along the geodesic.