Lecture # 1: Introduction

Note Title 8/26/2004

I. Introduction
II. Admin
III. why something new?
IV. The equivalence principle
V. Accelerated Frames

I. Intro & Admin

welcome to Phys 231A! I'm Don Marolf, & I've been of prof. of Grav physics for about 15 yrs.... Though I have not Taught This particular course very often before & I consider it still "under Development."

This is The first quarter of a 3-quarter sequence of GR courses. Let me Start w/ a brief outline to orient you to The year:

231A: I) Intro to GR, Fundamentals: Equivalence Principle
Accelerated Frames in S.R.

II) Math Tools: Tensors, metrics, Covariant Derivatives, Curvature

III) Physics Tools: Geodesics, Field Theories on curved space, how to Think about curved spacetimes

IV) Dynamics: Einstein's Equations & The Einstein-Hilbert Action

V) New physics: corrections to Newton (Light Bending, Time delay, perihelion precession.)

VI) A little cosmology

231 B (me, again): I) Linearized Gravity
II) Energy & conserved Quantities in G.R.
III) Black Holes

231 C (Gary Horowitz): "Special Topics"
I) Singularity Thms
II) BH's in String Theory

I'd like to start off today by going through a bit of admin & then explaining "why" GR is so different from E&M (despite the fact that Newton's Law of Gravity is so similar to Coulomb's Law).

Please note: what I will not do either today or later is to give a formal review ~~of~~ special relativity (though I will discuss some aspects that most of you will not have seen before.) While some review is definitely needed (both to refresh what may be long-dormant skills & to fill in potential holes in your background), I find it much better to do this via Homework.

For this reason, I am giving out HW #0 today, "Due" next Tuesday @ noon.

Comments: i) This may not be entirely review for some of you. That's ok. Ask your fellow students, read your textbooks, come to my office hours later today & Monday, etc.

Also: Feel free to ask for an extension if you really need it. (And similarly for later assignments, though I would prefer to keep these to a minimum.)

BTW, what is your background?

How about a quick round of introductions?

II. Admin

Textbooks: Carroll & Wald

I'll assign reading in both - most of you will prefer Carroll, but Wald has a bit more in it. We'll transition over more & more to Wald in 231 B & C.

HW: Due on Tuesdays @ noon. Place in 231A HW box on Broida ground floor. (Should exist by Tuesday.) (working together on hard problems is encouraged.) We will have a grader.

My OH (6131 Broida): M: 10-11, F: 2-3

Do these work for (almost) everyone?

Grading: 50% HW, 50% take-home final

(Details on syllabus & web)

Also note: My lecture notes will be posted on the web site (Hopefully at least the day before class.) Feel free to print them out & bring them with you to class to save writing.

Any questions?

III. Why Something new

As you all probably know already, G.R. was a very new kind of physical theory when it was introduced. It made spacetime dynamical instead of relying on a fixed background metric & introduced all sorts of complicated non-linearities (e.g., that lead to black holes). It is natural to ask "why did this happen?"

After all, Newton's law of gravity

$$F = -\frac{mMG}{r^2}$$

is almost identical to Coulomb's law

$$F = \frac{qQk}{r^2}.$$

They only differ by a (-) sign!

Yet Maxwell's equations are fairly simple (linear, non-dynamical Minkowski metric).

So, when Einstein went to look for a relativistic theory of gravity, why not just copy down Maxwell's Equations & change some signs?

Answer: The above sign turns out to be critical!

To see this, let's look at the action for the Maxwell field coupled to a point particle.
[Does everyone know this? If not, read ch. 1 of Carroll <u>carefully</u>.]

Check sign

$$A_\mu = (A_0, A_i) = (\Phi, A_i)$$

$$E_i = \partial_i \Phi - \partial_0 A_i \qquad B_i = \epsilon^{ijk} \partial_j A_k$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$S_{E\&M + \text{charged particle}} = -\frac{1}{4} \int d^4x \, F_{\mu\nu} F^{\mu\nu} + \int A_\mu J^\mu - m \int d\tau$$

ϵ

Do I need to explain any of the above notation?
(Raise index? Einstein summation? $d\tau^2 = -ds^2$?)

$$\Rightarrow \partial_\mu F^{\mu\nu} = -J^\nu \; (1) \; \& \quad \epsilon^{\alpha\beta\gamma\delta} \partial_\beta F_{\gamma\delta} = 0 \; (2)$$

& Lorenz Force law. (3)

(You'll check this on HW # 1).

Of course, this makes like charges repel.
What sign should be changed to make like charges attract?

Note that changing the sign of $A_\mu J^\mu$ merely sends $J \to -J$ or $q \to -q$ (i.e., switches + & − charges). This sign appears twice [in (1) & (3)] so the physical effects of changing this sign cancel.

Instead, we must change either the relative sign btwn F^2 & $A \cdot J$ or btwn $A \cdot J$ & $\int d\tau$, but not both!

E.g., one might guess the following action for gravity:

$$S_{grav} \underset{(guess)}{} = \oplus \frac{1}{4} \int F_{\mu\nu} F^{\mu\nu} d^4x + \int A_\mu J^\mu - m \int d\tau$$

This gives an attractive force!

However, recall that $E = H = \int d^3x \, (\Pi^\mu \dot{A}_\mu - \mathcal{L})$
w/
$$\Pi^\mu = \frac{\partial \mathcal{L}}{\partial \dot{A}_\mu}$$

What we have done is to change $\mathcal{L} \to -\mathcal{L}$ in the free Maxwell theory, so also $\Pi^\mu \to -\Pi^\mu$ & $H \to -H$.

So we have changed the sign of the energy! In particular, the propagating "photon" (graviton) now carries negative energy (while the point particle $-m\int d\tau$ still carries positive energy).

Such theories are unstable as radiation now acts to enhance any disturbance (e.g., sinusoidal oscillation of charge) rather than to damp it out. Einstein (being a solid believer in the beauty & stability of physics) would have dismissed such a theory out of hand.

Any such theory of gravity is also now in clear contradiction to observations (e.g., the Hulse-Taylor pulsar = Nobel prize in early 80's) which demonstrated that gravitational radiation emission removes energy from a system (& thus the graviton carries positive energy). More on this next quarter. [It is possible that negative energy gravitons as above were already observationally ruled out by solar system observations in Einstein's time, but I'm not sure.]

Note: Same effect if keep original sign for F^2 & change particle to $\to + m \int d\tau$. [Just $S_{(guess)} \to -S_{guess}$ & $J \to -J$]

So, what should one do? The simplest idea is to go off & construct a new theory of gravity ---- based on scalar fields. This turns out to naturally give an attractive force!!

Q: why? A: For the vector field A_μ, the Coulomb term comes from A_0. This means that an extra (−) sign sneaks into the calculation from the metric ($\eta_{00} = -1$) as compared w/ the scalar calculation. In particular, to find the electric field sourced by a charge density, we write

$$\partial_i \left(\partial^i A^0 - \partial^0 A^i \right) = -J^0 \quad \& \text{ solve for } A^0.$$

But the effect on another bit of charge comes via $A_\mu J^\mu = A_0 J^0 + A_i J^i = -A^0 J^0 + A^i J^i$

This last minus sign does not appear for the scalar field, which thus gives an attractive force for signs s.t. both radiation & the source particles have positive energy. (You'll verify this on HW #3.)

So why didn't Einstein like this theory? It failed to satisfy the Equivalence principle, to which we turn in a moment.

Notice though, that from above we conclude (or guess) that the attractive/repulsive nature of a force (given Lorentz invariance) is determined by the number of indices the potential carries.

$$\text{odd }\#\ (\text{e.g. } A_\mu) \Rightarrow \text{repulsive}$$

$$\text{even }\#\ (\text{e.g. } \phi) \Rightarrow \text{attractive}$$

So, next simplest attractive force is a Tensor field $h_{\mu\nu}$.

IV. The Equivalence Principle

I'm sure you recall the basic idea of the equivalence principal from Newtonian physics:

$$F = m_I\, a \,, \qquad F = m_G\, g$$

$$\& \quad m_I = m_G \quad \Rightarrow \quad a = g$$

So that all objects "fall at the same rate" in a Newtonian grav. field.

This is often called the "weak" equivalence principle.

To Einstein, this suggested a much stronger idea. Perhaps all physics experienced in a gravitational field was equivalent to that found in an accelerated reference frame! (Strong equiv. principle)

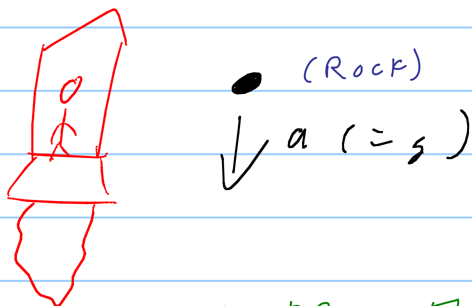
Note: Despite the intrinsic ambiguity of the Newtonian theory, this principle clearly states that light falls in a gravitational field ⇒ not compatible w/ a scalar theory. Sun

But this suggests a radical change of perspective. The easiest way to deal w/ accelerations is usually to describe them from inertial reference frames. I.e., to "get rid" of the fictitious inertial forces by a change of reference frame. The analogous idea in the gravitational context is to "get rid" of the gravitational forces by passing to a freely falling reference frame.

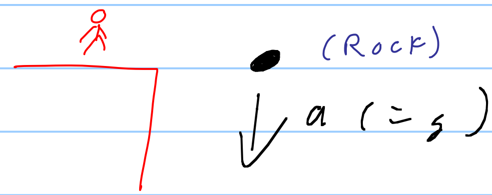
The SEP says that it is these freely falling frames that act like inertial frames in special relativity (and so where we understand proper time, maxwell's eqn's, & so forth), despite the fact that Newton would have called these accelerating frames!

I.e., Einstein says "Free fallers are 'inertial' & we (sitting on the Earth's surface) are accelerating upward!" ⇒ Gravity is a "Fictitious" Force!

This is a brilliant idea! ... But, why didn't Newton think of it? Because, at least at first glance, it is also obviously wrong. Let's look at the details, Comparing an accelerating rocket in flat space to an observer on (say) the surface of the earth



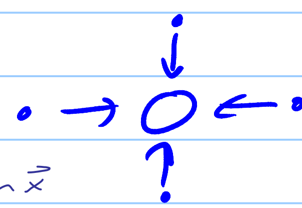
vs.



Note the differences!

The most obvious is that the rocket requires an engine! This is related to the fact that the grav field $\vec{g}(z)$ is a particular function of z (determined just by S.R.) which does not vanish anywhere! (Rocks "fall" @ any z.)

On the other hand, the earth requires no engine. This is related to the fact that $\vec{g} = 0$ at the center of the earth (so that the c.m. is in free fall). This is possible because the dependence of $\vec{g}$ on $\vec{x}$ is dictated not by kinematics, but by the source mass density $\rho(\vec{x})$ through Poisson's eqn.



$$\text{I.e.} \quad \vec{g}_{\text{Rocket}}(\vec{x}) \neq \vec{g}_{\text{Earth}}(\vec{x}),$$

so all physics cannot be the same.

In particular, as viewed in a freely falling frame, in the rocket case there is no relative accel. btwn two freely falling rocks, but there is in the case of the earth because $\vec{\nabla} \vec{g} \neq 0$!

Einstein's great insight was really that this issue could be fixed, & also that it should be fixed!

The "Einstein equivalence principle" is as follows:

Any gravitational field is locally indistinguishable from having an accelerating reference frame in Minkowski space. Differences appear only in 2nd derivatives of fundamental quantities. (potentials)

Idea: This is still enough to think of gravity as a fictitious force!

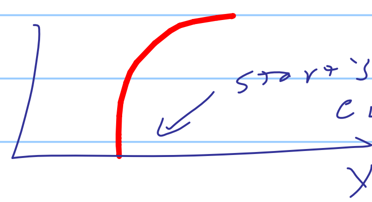
Now, why did he think this would work?

Answer: He found an analogy in non-Euclidean geometry (which had "recently" been developed by Riemann).

Let's be precise abt the problem:

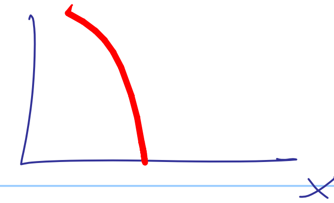
suppose start at relative rest

How does the rock's motion look in the observer's reference frame?

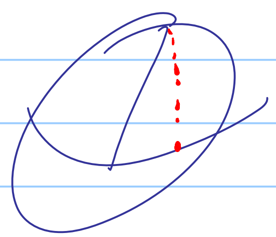


starts orthog, but curves

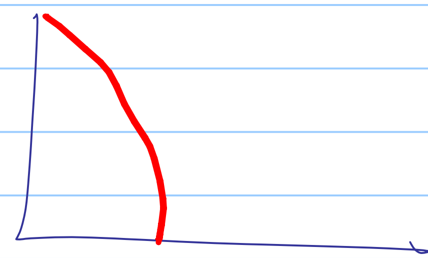
or,



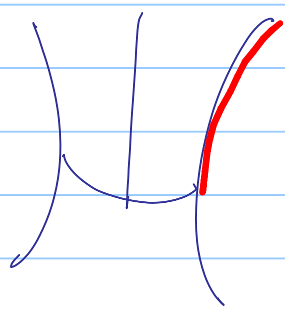
Let's compare this with what happens in studying the geometry of curved surfaces. E.g., consider a sphere. The "natural" path for an observer to follow is a great circle. (this is a geodesic, the path obtained by attempting to follow the straightest possible path, e.g. by placing one foot directly in front of the other at each step.)

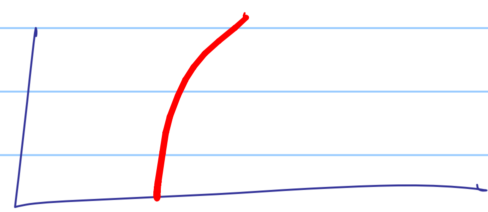


$\Rightarrow$



Also consider a hyperboloid:





This looks just like what we found in our study of the earth's gravitational field!

I.e., perhaps relative accelerations btwn freely falling observers can be understood as being due to spacetime curvature?

$$\boxed{\text{Gravity} = \text{Spacetime curvature?}}$$

this idea left Einstein w/ the rather serious task of learning the (then still novel) mathematics of general curved surfaces (i.e., differential geometry).

This is also the task that we now face (since Diff geom. not a pre-req. for this course.

This took Einstein quite awhile (years to get it right!), but was essential for a proper formulation of the theory. So it is also worth our time to get it right $\Rightarrow$ we have a lot of math to go through before being able to extract the desired physics, but worth our while to do it right the first time.

This extra math is probably what led to the early reputation of G.R. as "hard." [In 1919 NYT quoted Eddington as saying the theory was understood by no more than "12 wise men."] But it's not really so hard, & you'll all be experts soon!

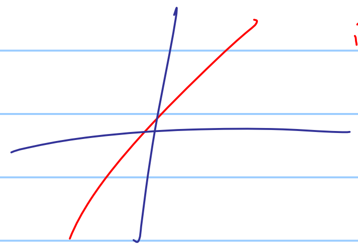
Before jumping into this, however, we also see that it is worth understanding the physics of accelerated frames (in S.R.) as well as possible. I'll leave much of this for you to explore via HW, but I do want to cover some essential points in class.

V. Accelerated Frames

Let's consider 1+1 Minkowski space

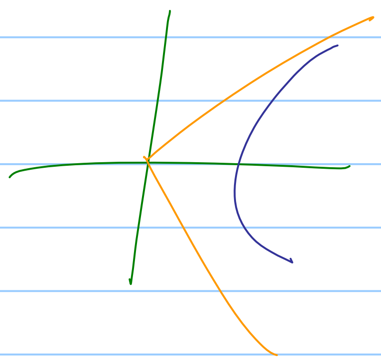
$$ds^2 = -dt^2 + dx^2$$

There are many classes of interesting worldlines in this system. One such class are the <u>inertial</u> worldlines. Note that any inertial world line is invariant under some translation symmetry which acts as a time translation along the world line.



This guarrantees that physics as described by such observers has a time-trans symmetry; i.e., that their experiences are <u>uniform</u> along their worldline.

What about worldlines invariant under boost symmetries?



Hyperbolas:

$$x = \ell \cosh(\lambda)$$

$$t = \ell \sinh(\lambda)$$

$$\Rightarrow x^2 - t^2 = \ell^2$$

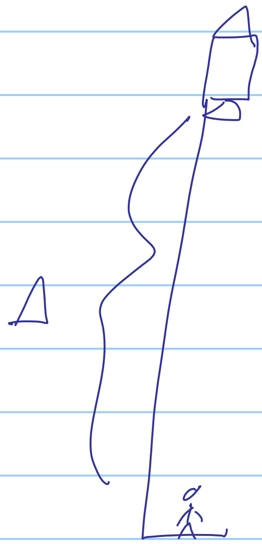
The experiences of such observers must <u>also</u> be uniform. But they are cleary not inertial, so their <u>proper</u> <u>acceleration</u> is non-zero. But A as expressed in each instantaneous co-moving reference frame it must be independent of the proper time. In particular, the magnitude $|A|$ is constant. As you saw in your HW, $|A| = 1/\ell$.

You also computed

$$d\tau = \sqrt{dt^2 - dx^2} = \ell\, d\lambda \left(\cosh^2 - \sinh^2\right)^{1/2}$$

$$\Rightarrow \lambda = \tau/\ell$$

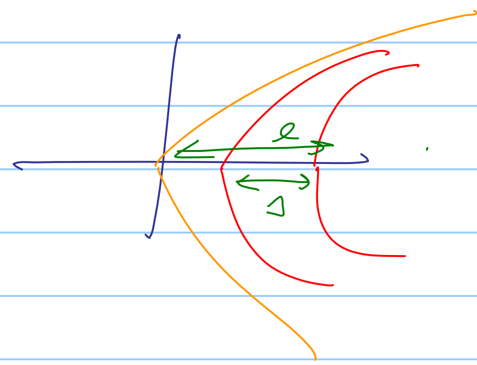
So, suppose we lower a platform a proper distance Δ below our rocket on a rope



The entire situation remains uniform in time, so this must preserve the symmetry. Thus the platform moves along another hyperbol w/ <u>larger</u> proper acceleration.

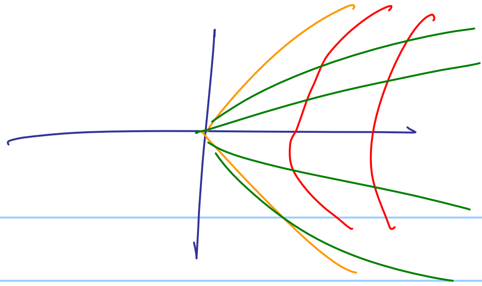
$$|A_{platform}| = \frac{1}{\ell - \Delta}$$

where the simplest way to see which hyperbola is the right world line is to compute the separation btwn them at $t=0$ when both are at rest (so proper distance $= \Delta x$; i.e., inst. co-moving frame is x,t as drawn).



Note then that
$$\tau_{platform} = (\ell - \Delta)\lambda$$
$$= \frac{(\ell - \Delta)}{\ell}\tau_{rocket} \quad (\star)$$

Here we compare proper times along lines of constant λ (which is the only thing consistent w/ the symmetry). One can also see that this is the right thing to do (in the ICM RF) by acting w/ the



Symmetry on the $t=0$ line.

Using the equivalence principle, (A) implies that there must also be a gravitational redshift -- that clocks higher up in a gravitational field run faster. But of course (A) does not give the correct dependence on the proper length Δ of the rope in a general gravitational field. However, it does tell us what the right rule is! (As I will now explain.)

Consider an arbitrary time-independent (stationary) gravitational field in which the proper acceleration of freely falling observers is $\vec{g}(\vec{x})$ (where I can think of these as vectors in space due to the lack of time-dependence). Suppose that we have chosen some curve $\vec{x}(\lambda)$ in space and that we have synchronized a set of static clocks along this curve at some $t=0$. By time-translation invariance, for $t \neq 0$ the proper times must be $\tau(t,\lambda) = f(\lambda)\, t$ for some $f(\lambda)$. The relative rate at which two nearby clocks run should be given by the flat space result.

For small Δ, $$\frac{\tau_{platform}}{\tau_{rocket}} = \frac{\ell - \Delta}{\ell} = 1 - \frac{\Delta}{\ell}$$

For $\lambda = -\Delta$ = proper distance "above" origin in grav field

$$\Rightarrow \frac{\tau(\lambda)}{\tau(0)} = 1 + \lambda |A|$$

$$\Rightarrow \frac{\tau'(0)}{\tau(0)} = -\underbrace{A_a \, dx^a}_{\text{inner product}} \Big|_{\lambda=0}$$

But we can use the equivalence principle to map <u>any</u> static observer in our spacetime to some such rocket, so we must have

$$-A_a dx^a = \frac{d\tau(\vec{x})}{\tau(\vec{x})} = d \ln \tau(\vec{x}) \quad \text{for all } \vec{x}.$$

So, if we normalize our definition of t by choosing $f(0) = 1$, the Equivalence principle tells us that

$$f(\vec{x}) = \exp\left(-\int_0^{\vec{x}} A_a dx^a \right)$$

for all points $\vec{x}$ on our curve.

This should then be an <u>exact</u> result. The Einstein Equiv principle treats proper distances & proper times as "fundamental quantities" & gives exact results for first order changes (i.e., for the derivative $\frac{d\tau}{d\lambda}$). Discrepancies between the grav field & the accel frame do arise at 2nd order, but only because $\vec{g}(\lambda)$ is not in general the one given by a uniformly accelerating frame in flat space.

In addition to being an example of how to use the equivalence principle, this tells us something about the form that G.R. must take. We see that describing a static gravitational field requires a <u>metric</u> at least as complicated as

$$ds^2 = -f(\vec{x})\, dt^2 + d\vec{x}^2$$

But the gravitational field may depend on time, & by Lorentz invariance we expect to need factors in front of the dx^2, dy^2, & dz^2 terms as well. And considering a few more examples (like rotating reference frames, where clocks cannot all be synchronized!) suggests that we will need off-diagonal terms as well.

So then a general gravitational field should be described by a general metric

$$ds^2 = g_{ab}\, dx^a dx^b$$

which is indeed a 2-index tensor as we suggested earlier. (So we expect to find an attractive force once we write down an appropriate action!) This is also consistent w/ the analogy w/ differential geometry (where the metric plays a primary role).