1. **Thermal Equilibrium and Equipartition** A subject mass $M$ is embedded in an infinite homogeneous sea of field stars, with mass $m \ll M$ and isotropic distribution function $f(v)$.

(a) Using the Fokker-Planck equation and the diffusion coefficients (Binney & Tremaine equation 7.88), show that when the subject mass is in thermal equilibrium with the field stars, with a Maxwellian distribution function, its velocity dispersion is:

$$\sigma^2_M = \frac{m}{M} \int_0^\infty dv v^2 f(v) f(0)$$

(b) Show that when $f(v)$ is Maxwellian, this condition reduces to the requirement of energy equipartition between the subject mass and the field stars.

2. **Galactic Center** Suppose that the massive dark objects in the Galactic Center ($M = 2.6 \times 10^6 M_\odot$, $\rho > 10^{17} M_\odot pc^{-3}$) and NGC 4258 ($M = 3.6 \times 10^7 M_\odot$, $\rho > 4 \times 10^9 M_\odot pc^{-3}$) are not black holes, but dense star clusters consisting of say, $1.4 M_\odot$ neutron stars. By estimating the evaporation time scale of such a cluster in each case, comment if this is a viable alternative.

3. **Stellar Distribution around a Black Hole** Read §7.5.9b of Binney & Tremaine, which derives the steady-state distribution of stars around a central black hole, in the case where the relaxation time is shorter than the age of the system. Consider the following alternate derivation. Assume that the density near the hole is a power law, $n(r) \propto r^{-s}$. The mean-square velocity at any radius should be of order $\langle v^2 \rangle \approx GM_{BH}/r$, so the local relaxation time is $t_{\text{relax}} \approx \langle v^2 \rangle^{3/2}/(G^2 m^2 n) \propto r^{s-3/2}$. Relaxation among the $N(r)$ cusp stars interior to $r$ can lead to a flow of stars through the shell at radius $r$ that is of order $N(r)/t_{relax} \sim n(r)v^3/t_{relax} \sim r^{-2s+9/2}$. In a steady state, this flow must be independent of radius, so $s = 9/4$. This differs from the (correct) result $s = 7/4$ in Binney & Tremaine. What is wrong with the argument presented here?