# Phys 250 Quantum Optics, Final

Instructor: **Prof. D. Bouwmeester**
Office: 4123 Broida Hall
Email: bouwmeester@physics.ucsb.edu

*Due Tuesday, December 8th at 5 PM in the Homework Box.*

Graded finals can be picked up Thursday, December 10th from 3-5 PM in Dirk's office.

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1. Consider a single mode field in a non-linear medium represented by the Hamiltonian:

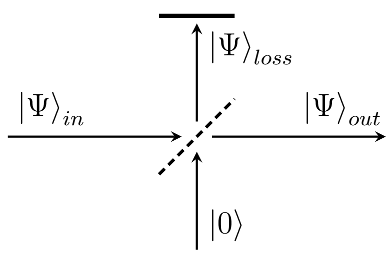
$$\hat{H} = \hbar\left[\omega \hat{a}^\dagger \hat{a} + \lambda\left(\hat{a}^\dagger \hat{a}\right)^2\right]$$

   (a) What is the time evolution of an initial coherent state, $|\alpha\rangle$, under the action of this Hamiltonian? (3 points)

   (b) Apart from the normal rotation of a coherent state, represented by the $e^{-i\omega t}$ phase shift, is this state periodic? If so, what is the period? (If you like, let $\omega \to 0$; this is equivalent to working in the interaction picture.) (3 points)

   (c) Express the state at a time $t = \frac{\pi}{2\lambda}$ as a sum of two coherent states. What is this state, and is it classical? Hint: first consider states of even and odd $n$ separately. (4 points)

2. In real quantum optics experiments, it is important to consider the effects of photon loss due to imperfect mirrors, detectors, etc. This can be modeled by inserting a weakly reflecting beam splitter, where in the end we trace over the state $|\Psi\rangle_{loss}$ in the beam spitter output which corresponds to the loss channel.

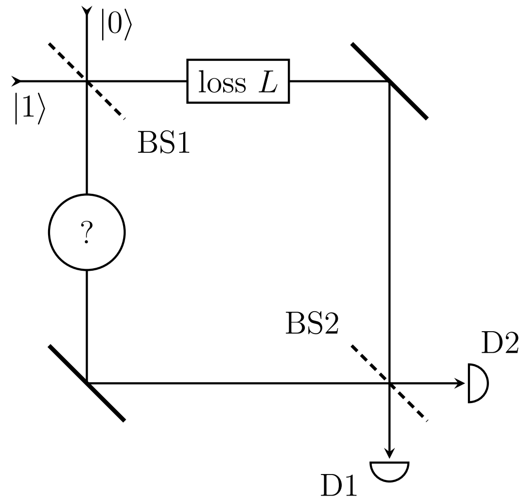



   The loss "beam splitter" is represented in the usual way:

$$\hat{a}_{out} = \cos\left(\theta_L/2\right)\hat{a}_{in} + i\sin\left(\theta_L/2\right)\hat{a}_0$$
$$\hat{a}_{loss} = i\sin\left(\theta_L/2\right)\hat{a}_{in} + \cos\left(\theta_L/2\right)\hat{a}_0,$$

   where $\theta_L$ is the loss angle, which is related to the loss probability by $L = \sin^2\left(\theta_L/2\right)$.

   Consider the interaction-free measurement discussed in section 6.4 of *Introductory Quantum Optics*, but where one of the interferometer arms is lossy:




Assume BS1 and BS2 are normal 50:50 beam splitters, and there is no relative phase shift between the arms ($\phi = 0$).

(a) What is the probability of a false object detection due to the lossy arm, or in other words what is the probability of finding a photon at D2 when no object is present? (4 points)

(b) What is the probability of detecting a photon at D2 when an object is present? (3 points)

(c) Consider a simpler loss model where we simply block the lossy arm completely in a fraction of the trials, with the fraction given by the loss probability $L$. In this case, what is the probability of finding a photon at D2 with and without the object in the non lossy arm? Is this the same as (a) and (b) above? (3 points)

3. Consider an effective Jaynes-Cummings model for a k-photon resonant interaction:

$$\hat{H}_{eff} = \hbar\lambda \left(\hat{a}^k \hat{\sigma}_+ + \hat{a}^{\dagger k} \hat{\sigma}_-\right),$$

where $k$ is a positive integer. (Ignore the field and atom Hamiltonians, $\hat{H}_A$ and $\hat{H}_F$, and consider only the interaction terms.) Note that the $k = 1, 2$ cases were covered in the lectures and homework.

(a) What are the dressed states for this system? (3 points)

(b) If the atom is in an initial excited state, what is the atomic inversion as a function of time for some arbitrary photon state described by $C_n$? (2 points)

(c) Obtain an *approximate* expression for the inversion revival and collapse times for a coherent light field in the limit $\bar{n} \gg k$. (You do **not** need to do the full expansion along the lines of 4.130-136 in *Introductory Quantum Optics*.) (2 points)

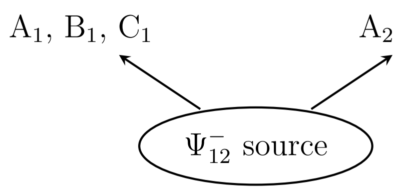
(d) By ignoring the $\hat{H}_A$ and $\hat{H}_F$ terms (4.96-97 in the textbook) we are effectively considering a system that is "on-resonance". What does "on-resonance" mean for the k-photon case? Why can't we ignore these terms when considering the system off-resonance? (3 points)

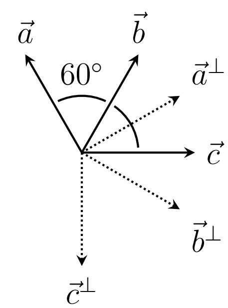
4. **Bell's inequalities and Greenberger-Horne-Zeilinger (GHZ) States**

(a) Explain how type II non-collinear down conversion can be used to generate the Bell state, $\Psi_{12}^{+} = \frac{1}{\sqrt{2}}\{|H\rangle_1 |V\rangle_2 + |V\rangle_1 |H\rangle_2\}$, where $|H\rangle_1$ corresponds to a horizontal polarized photon in mode 1, etc. (2 points)

(b) Given a birefringent element in mode 1 (or in mode 2) that provides a relative phase between the V and H polarization we can turn $\Psi_{12}^{+}$ into $\Psi_{12}^{-} = \frac{1}{\sqrt{2}}\{|H\rangle_1 |V\rangle_2 - |V\rangle_1 |H\rangle_2\}$. Show that $\Psi_{12}^{-}$ has anti-correlations, that is if the photon in mode 1 is H (or V) then the photon in mode 2 is V (or H), in any polarization basis. Does this also hold for $\Psi_{12}^{+}$? (2 points)

(c) In order to construct an explicit experimental test of Bell's inequalities we considered in the lectures a $\Psi_{12}^{-}$ source with the possibility of performing three different polarization measurements A, B, and C on each of the two modes. Measurement A projects a photon onto the $(\vec{a}, \vec{a}^{\perp})$ basis and measurements B and C on a 60° and 120° rotated basis with respect to A.




Give the arguments based on A,B,C measurements on photons in mode 1 and 2 that lead to an inconsistency between local realistic (hidden variable) predictions and quantum mechanical predictions. Why do we refer to this contradiction as a "statistical" contradiction? (2 points)

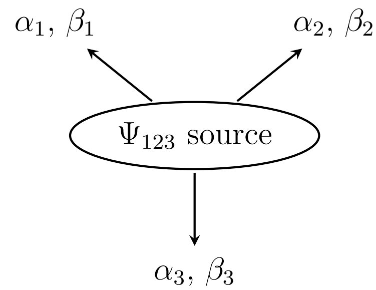
(d) We now consider a three-photon entangled state of the form:

$$\Psi_{123} = \frac{1}{\sqrt{2}}\{|H\rangle_1 |H\rangle_2 |V\rangle_3 - |V\rangle_1 |V\rangle_2 |H\rangle_3\},$$

where one photon in each of three modes is coming from a source. Consider two different detection bases in which each photon could be measured: the $\alpha$ basis (45° rotated linear polarization) and the $\beta$ basis (circular polarization).

$$\alpha - basis \begin{cases} |H'\rangle = \frac{1}{\sqrt{2}}(|V\rangle + |H\rangle) \\ |V'\rangle = \frac{1}{\sqrt{2}}(|V\rangle - |H\rangle) \end{cases}$$

$$\beta - basis \begin{cases} |L\rangle = \frac{1}{\sqrt{2}}(|H\rangle - i\,|V\rangle) \\ |R\rangle = \frac{1}{\sqrt{2}}(|H\rangle + i\,|V\rangle) \end{cases}$$




Rewrite $\Psi_{123}$ in the $\beta_1\beta_2\alpha_3$ detection basis. This should give 4 terms each corresponding to a possible $\beta_1\beta_2\alpha_3$ outcome with equally probabilities of 0.25. Do the same for $\beta_1\alpha_2\beta_3$ and $\alpha_1\beta_2\beta_3$ measurements. (2 points)

(e) Based on the possible outcomes of the $\beta_1\beta_2\alpha_3$, $\beta_1\alpha_2\beta_3$ and $\alpha_1\beta_2\beta_3$ measurements give a local realistic argument that provides predictions for the outcome of $\alpha_1\alpha_2\alpha_3$. Hint: start with one of the possible measurement outcomes of a $\beta_1\beta_2\alpha_3$ measurement, find the (two) outcomes of a $\beta_1\alpha_2\beta_3$ measurement that are consistent with it (the $\beta_1$ outcome must be the same), and for each of the two case find the unique $\alpha_1\beta_2\beta_3$ outcome that is consistent. Considering all combinations you should end up with 4 possible $\alpha_1\alpha_2\alpha_3$ outcomes. (3 points)

(f) Now calculate the possible $\alpha_1\alpha_2\alpha_3$ outcomes by rewriting $\Psi_{123}$ directly in the $\alpha_1\alpha_2\alpha_3$ detection basis (this provides the prediction for such outcomes according to quantum mechanics). (2 points)

(g) Compare the local realistic predictions with the quantum mechanical predictions for $\alpha_1\alpha_2\alpha_3$ measurements and draw conclusions. In what sense is this result fundamentally different from the Bell-type conclusions in (c)? (2 points)

5. In the final lecture we discussed the general derivation of the "master equation." The book by Gerry and Knight does not give this derivation but introduces the master equation in a different way: read sections 8.1, 8.2 and 8.3. This method is usually referred to as the Monte Carlo Wave Function Method (MCWF), the single quantum trajectory method or the quantum jump approach. Study the attached article (that I wrote together with my colleagues when I was a grad student) on the topic of the MCWF method (some of the references are also posted in case you would like to consult them).

   Make a two to four page document in which you address the following issues:

   - What is the general setting for applying master equations?
   - What is the MCWF about?
   - Show that the master equation given in the article, Eqn. (8), is appropriate for describing spontaneous emission from a two-level atom (do this by providing explicit equations for the components of the reduced density matrix of the atom).
   - Show that Eqns. (13) and (16) are correct.
   - Summarize the results of the article (in as much detail as time permits).

   (10 points)

# Neoclassical radiation theory as an integral part of the Monte Carlo wave-function method

D. Bouwmeester, R. J. C. Spreeuw,* G. Nienhuis, and J. P. Woerdman
*Huygens Laboratory, University of Leiden, P.O. Box 9504, 2300 RA Leiden, The Netherlands*
(Received 16 December 1993)

We analyze the so-called Monte Carlo wave-function (MCWF) method from a conceptual point of view. This method has been recently introduced as a technique to simulate dissipative processes in quantum optics. For the case where dissipation consists of spontaneous emission, we find that the coherent decay part of the MCWF method is identical to neoclassical radiation theory. This unexpected reappearance of the neoclassical theory suggests the identification of the MCWF coherent decay with classical radiation reaction. It leads to an alternative interpretation of the MCWF method instead of the usual one which is based upon quantum measurement theory. We give a derivation of the MCWF method, illustrated by simple Feynman diagrams, in which the appearance of radiation reaction is shown to be a natural feature of the coherent decay part.

PACS number(s): 42.50.−p, 03.65.Sq

## I. INTRODUCTION

In this article we make a connection between two descriptions of spontaneous decay of a two-level atom. The first description is provided by the neoclassical radiation theory, put forth by Crisp and Jaynes [1], and Stroud and Jaynes [2]. Here the "spontaneous" decay of a single atom is described as the result of classical radiation reaction on an oscillating atomic dipole. Although the theory is conceptually elegant, it has been rejected for its disagreement with experimental data. It can be seen as one of the last attempts, in the spirit of Schrödinger [3] and Fermi [4], to get around the probabilistic interpretation of quantum mechanics. The key point of this article is to show that the neoclassical radiation theory makes an unexpected come back as an integral part of the Monte Carlo wave-function (MCWF) method.

This method, which has been recently developed [5–12] for simulating dissipative processes in quantum optics, provides the second description of spontaneous decay. The MCWF method is based on the evolution of the atomic wave function while a continuous measurement of the state of the radiation field is performed. To every realization of a continuous measurement there corresponds a specific evolution of the atomic wave function, a so-called single quantum trajectory. The essence of the MCWF method is that averaging over single quantum trajectories leads to the correct solution of the master equation for the reduced density matrix. The method, introduced for its computational advantages, replaces a cumbersome calculation of the density matrix of a quantum system interacting with a large reservoir. If we restrict ourselves to the example of dissipation in a two-level atom due to coupling to the vacuum radiation field the MCWF evolution consists of two parts. The first part is a stochastic process which brings the atomic population to its ground state accompanied by the emission of an observable photon. The process is stochastic in the sense that there is a prescribed probability for it to occur. The second part is a coherent evolution leading to a continuous decay of the excited-state population to the ground state without the emission of an observable photon.

Apart from the computational advantages, the MCWF method is interesting from the point of view of quantum measurement theory [10]. The specific measurement of the state of the radiation field determines the MCWF evolution. Even if no photon is detected, i.e. during the coherent decay part of the method, the evolution of the atomic state vector is influenced by the measurement. This aspect of the MCWF method is the subject of this article. We will show, for the case of spontaneous decay of an ideal two-level atom and detection of the radiation field by an ideal photodetector, that the coherent decay part of the MCWF evolution is identical to the neoclassical result for spontaneous emission. For the identity to hold it is essential that the wave function remains normalized throughout its evolution. This requirement is not necessary from a computational point of view, and is therefore omitted in some versions of the MCWF method [10]. The stochastic character of the first part of the MCWF method reflects the indeterministic features inherent in a quantum mechanical description of spontaneous decay.

Since the MCWF method is expressed in terms of pure-state evolutions, it should be derivable directly from QED without using the master equation. We give such a derivation for the case of spontaneous emission by including the action of a certain measurement performed on the radiation field. In the derivation we make use of simple Feynman diagrams. This treatment of spontaneous emission brings out the essential features of both

*Present address: National Institute of Standards and Technology, United States Department of Commerce, Phys A167, Gaithersburg, MD 20899.

the neoclassical theory and the MCWF method and illuminates their intimate link.

Throughout the article we assume that the atomic level shifts, although in principle inseparable from spontaneous decay, play no significant role in our treatment. In the MCWF treatment and in QED the level shifts are time independent and can therefore simply be included in the atomic part of the Hamiltonian, which makes them unimportant for the description of spontaneous decay. In the neoclassical treatment, however, the level shifts are time dependent, which makes it difficult to include them in a simple way in the theory. Knowing that the time-dependent level shifts are generally small compared to the level separation we assume that we may ignore them in our treatment of spontaneous emission. In Sec. VI we briefly discuss the complications that arise when one attempts to treat the level shifts and spontaneous decay on equal footing.

Our paper is organized as follows. In Sec. II the neoclassical radiation theory is briefly reviewed. In Sec. III the MCWF method is presented and in Sec. IV its relation with the neoclassical radiation theory is shown. A derivation of the MCWF method based on radiation reaction is given in Sec. V. We discuss the results and draw some conclusions in Sec. VI.

## II. NEOCLASSICAL THEORY

We first address the two-level atom from the perspective of the neoclassical radiation theory [1,2]. If the atom is in a coherent superposition of the excited state and the ground state it is treated as a classical oscillating dipole. The atomic dipole-moment operator in quantum mechanics is replaced by its expectation value,

$$\boldsymbol{\mu}(t) = e \int d^3r \;\; \boldsymbol{r} |\psi(\boldsymbol{r},t)|^2, \tag{1}$$

where $e$ is the electron charge. The corresponding (Schrödinger) interpretation of the wave function $\psi(\boldsymbol{r},t)$ is that of a charge density instead of a probability distribution. The mechanism responsible for spontaneous decay is taken to be classical radiation reaction [13]. It is a dissipative process for which no fluctuating radiation field is required, hence the coherences between the two atomic levels are conserved throughout the atomic evolution.

Formally, the evolution of the two-level atom is described by the motion of a Bloch vector, $\boldsymbol{R}(t) \equiv (x(t), y(t), z(t))$. Its components are expressed in terms of the complex amplitudes $\alpha_g(t)$ and $\alpha_e(t)$ of the ground state and the excited state, respectively:

$$x(t) \equiv \alpha_g(t)\alpha_e^*(t) + \alpha_g^*(t)\alpha_e(t) = 2\,\mathrm{Re}\,\rho_{ge}(t), \tag{2}$$

$$y(t) \equiv -i\left[\alpha_g(t)\alpha_e^*(t) - \alpha_g^*(t)\alpha_e(t)\right] = 2\,\mathrm{Im}\,\rho_{ge}(t), \tag{3}$$

$$z(t) \equiv |\alpha_e(t)|^2 - |\alpha_g(t)|^2 = \rho_{ee}(t) - \rho_{gg}(t), \tag{4}$$

where $\rho(t)$ represents the density matrix of a pure state. In the absence of external fields, and neglecting phase shifts related to the neoclassical Lamb shift, the neoclassical theory results in the following evolution equations [2]:

$$\frac{d}{dt}x(t) = \beta x(t) z(t)\,, \tag{5}$$

$$\frac{d}{dt}y(t) = \beta y(t) z(t)\,, \tag{6}$$

$$\frac{d}{dt}z(t) = -\beta[1 - z^2(t)]\,, \tag{7}$$

where $\beta \equiv 2\mu^2\omega^3/3\hbar c^3$ is the coupling coefficient due to radiation reaction. This set of nonlinear equations describes "spontaneous" decay of a two-level atom in a neoclassical way; its solution gives a tanh decay of the inversion $z(t)$. The spectral line shape, defined as the squared amplitude of the Fourier transform of the dipole moment, $x(t) + iy(t)$, is given by a sech$^2$. These results are at variance with experiments and QED calculations, which yield an exponential decay of the atomic inversion and a Lorentzian line shape. Note that the norm of the Bloch vector is conserved throughout the evolution described by Eqs. (5)–(7). This is in contrast to the usual quantum mechanical evolution of the Bloch vector for spontaneous decay where all three of the vector components are decaying.

## III. MONTE CARLO WAVE-FUNCTION METHOD

We now address the two-level atom from the perspective of the MCWF method. This method aims at solving the master equation for the reduced atomic density matrix $\rho_A$ [5–12]. We briefly review its essence, following Dalibard, Castin, and Mølmer [5], in order to expose the connection with the neoclassical theory.

The master equation for $\rho_A$ is given by

$$\frac{d}{dt}\rho_A(t) = \Gamma S^- \rho_A(t) S^+ - i[H\rho_A(t) - \rho_A(t)H^\dagger]\,, \tag{8}$$

$$H = H_0 - i\frac{\Gamma}{2}S^+S^-\,, \tag{9}$$

where $S^-$ and $S^+$ are the atomic lowering and raising operators, and † indicates the Hermitian conjugate. The spontaneous decay rate is represented by $\Gamma$, and we have set $\hbar = 1$. $H_0$ contains the atomic part of the Hamiltonian, including the frequency shifts related to the QED Lamb shift. Interaction of the atom with external electromagnetic fields can also be included in $H_0$. Since our interest is in the evolution due to spontaneous decay we set $H_0$ equal to zero. Now suppose that at time $t$ the state vector $|\Psi(t)\rangle$ of the coupled atom-field system is given by

$$|\Psi(t)\rangle = |\psi(t)\rangle \otimes |0\rangle\,, \tag{10}$$

where the atomic-state vector $|\psi(t)\rangle$ is a superposition of the ground state $|g\rangle$ and the excited state $|e\rangle$,

$$|\psi(t)\rangle = \alpha_g(t)|g\rangle + \alpha_e(t)|e\rangle\,, \tag{11}$$

and the quantized radiation field is in its ground state $|0\rangle$. To analyze the evolution of $|\Psi(t)\rangle$ the changes during a small time interval $[t, t+dt]$ are considered. The time interval is chosen to be much smaller than the spontaneous lifetime of the atom, $dt \ll \Gamma^{-1}$.

The first term on the right-hand side of Eq. (8) is interpreted as the possibility of a quantum jump to the ground state. The probability $dp$ for a jump to occur and thus for an observable photon to appear in the time interval $[t, t + dt]$ is given by

$$dp = \Gamma|\alpha_e(t)|^2 dt \ll 1 . \tag{12}$$

This part of the MCWF method we will call the stochastical part.

If no observable photon has been emitted in the time interval $[t, t + dt]$ the atomic state $|\psi(t)\rangle$ must have evolved into $|\psi(t + dt)\rangle$ governed by the second term on the right-hand side of Eq. (8). This term contains the non-Hermitian Hamiltonian $H = -\frac{1}{2}i\Gamma S^+ S^-$. The effect of this Hamiltonian operating for a short time $dt$ on $|\psi(t)\rangle$ is

$$|\psi(t + dt)\rangle = \eta\alpha_g(t)|g\rangle + \eta\left(1 - \frac{\Gamma}{2}\, dt\right)\alpha_e(t)|e\rangle , \tag{13}$$

where an extra normalization factor $\eta$ has been introduced [see Eq. (16) below]. This normalization factor is not necessary from a computational point of view, but it is of crucial importance for our interpretation of the MCWF method, as we show in the next section. Note that Eq. (13) describes a coherent decay of the atomic inversion without the emission of an observable photon. This second part of the MCWF method we call the coherent decay part.

Since, at time $t + dt$, there are two distinguishable states of the radiation field, namely, the zero- and one-photon states, the reduced atomic density matrix has evolved from a pure state at time $t$ into a mixed state at time $t + dt$. The MCWF method, as presented by Dalibard, Castin, and Mølmer [5], consists of sequences of Gedanken measurements of the number of photons in the radiation field. In between these Gedanken measurements, which are separated in time by $dt$, the state vector $|\Psi(t)\rangle$ evolves as described above. A Gedanken measurement is numerically performed using a random number $\epsilon$, uniformly distributed between 0 and 1. This random number determines whether the Gedanken measurement yields a one-photon result ($\epsilon < dp$), or a zero-photon result ($\epsilon > dp$), which projects the state vector $|\Psi(t)\rangle$ onto $|g\rangle \otimes |0\rangle$ or onto $|\psi(t+dt)\rangle \otimes |0\rangle$, respectively. Note that a one-photon result means that the atom has emitted a photon which propagates away, and does not act back on the future evolution of the atom. Therefore the Gedanken measurement always leaves $|\Psi(t)\rangle$ in the form of Eq. (10). This justifies a repetition of the evolution described above. If $dt$ becomes infinitesimal each sequence describes a pure-state evolution of the atomic system. It has been demonstrated that averaging over these pure-state evolutions is equivalent to the standard density-matrix treatment of dissipative processes in quantum optics [5–12].

## IV. THE INTIMATE LINK

We now proceed to demonstrate that the coherent decay part of the MCWF method is identical to the neoclassical theory of spontaneous decay presented by Eqs. (5)–(7). During the coherent decay part of the MCWF method the ground-state and excited-state amplitudes are given by

$$\alpha_g(t + dt) = \eta\alpha_g(t) , \tag{14}$$

$$\alpha_e(t + dt) = \eta\left(1 - \frac{\Gamma}{2}\, dt\right)\alpha_e(t). \tag{15}$$

From the requirement that $|\alpha_g(t+dt)|^2 + |\alpha_e(t+dt)|^2 = 1$ it follows that

$$\eta \approx \left[1 - \Gamma\, dt\, |\alpha_e(t)|^2\right]^{-\frac{1}{2}} , \tag{16}$$

where it is assumed that $dt \ll \Gamma^{-1}$. Using Eqs. (2)–(4) one finds for the three Bloch-vector components

$$\begin{aligned} x(t + dt) &= \eta^2\left(1 - \frac{\Gamma}{2}dt\right)x(t) \\ &\approx \left(1 + \frac{\Gamma}{2}dt\, z(t)\right)x(t) , \end{aligned} \tag{17}$$

$$\begin{aligned} y(t + dt) &= \eta^2\left(1 - \frac{\Gamma}{2}dt\right)y(t) \\ &\approx \left(1 + \frac{\Gamma}{2}dt\, z(t)\right)y(t) , \end{aligned} \tag{18}$$

$$\begin{aligned} z(t + dt) &= \eta^2\left\{(1 - \Gamma dt)|\alpha_e(t)|^2 - |\alpha_g(t)|^2\right\} \\ &\approx z(t) - [1 - z(t)]\Gamma dt|\alpha_e(t)|^2 \\ &= z(t) - \frac{\Gamma}{2}[1 - z^2(t)]dt. \end{aligned} \tag{19}$$

This set of equations is identical to the neoclassical equations of motion, Eqs. (5)–(7), if we set $\Gamma \equiv 2\beta$ . Note that the normalization leads to nonlinear equations of motion which is an important feature of the neoclassical theory.

We emphasize that in the case of spontaneous decay, the neoclassical description with the inclusion of a stochastic jump to the ground state is identical to a single quantum trajectory as used in the MCWF method. More specifically, the addition of quantum jumps to the ground state at a rate proportional to $|\alpha_e(t)|^2$ transforms, after averaging over many trajectories, the neoclassical tanh decay and $\text{sech}^2$ line shape into the QED exponential decay and Lorentzian line shape. Since the probability of jumps is proportional to $|\alpha_e(t)|^2$, the role of the jumps becomes less important for atomic systems with small inversion. This corresponds to the well known result that the neoclassical radiation theory approaches QED in the limit of small atomic inversion. In this limit the relation $\Gamma = 2\beta$ is also confirmed [2].

## V. INTERPRETATION OF THE COHERENT DECAY

The evolution part in the MCWF method which leads to a coherent decay of the excited state to the ground state without the emission of a photon is usually interpreted within the context of quantum measurement theory [5,8]. The zero detection result of the number of emitted photons provides information on the atomic-state vector which modifies the state vector. Our observation that the jump-free evolution in the MCWF method can be seen as neoclassical in nature provides an alternative interpretation in terms of classical radiation reaction. In this section we show how the appearance of the neoclassical radiation theory in the MCWF method can be understood.

The master equation, Eq. (8), is generally derived from a QED calculation of the coupled atom-field system by taking the trace over the radiation field. By taking this trace information is lost and therefore classical statistics is introduced. Hence the master equation describes a mixed-state evolution. In contrast, in a full QED treatment a pure state always remains a pure state. The MCWF method unravels a mixed-state evolution into an ensemble of pure-state evolutions. The unraveling depends in general on the specific measurement performed on the radiation field. This unraveling procedure is like going back to the full QED treatment of the problem but with the influences of a specific measurement device included. We will elaborate on this point of view and show that the equivalence of the neoclassical theory with the evolution between jumps is not sheer coincidence but could be a principle of general validity.

The starting point for our considerations is the standard perturbation treatment of QED. The interaction Hamiltonian for the electron of a two-level atom and the radiation field is given by

$$H_{\rm int} = -\frac{e}{2m}[\mathbf{p}\cdot\mathbf{A}(\mathbf{x},t) + \mathbf{A}(\mathbf{x},t)\cdot\mathbf{p}] + \frac{e^2}{2m}\mathbf{A}(\mathbf{x},t)\cdot\mathbf{A}(\mathbf{x},t)\,, \qquad (20)$$

where $\mathbf{p}$ is the momentum operator for the atomic electron, and $\mathbf{A}(\mathbf{x},t)$ is the operator for the vector potential. The electron charge $e$ is the coupling constant and $m$ is the electron mass. It is always possible to make a decomposition of the radiation field $\mathbf{A}(\mathbf{x},t)$ in a complete set of mode functions. The choice of which decomposition to use is arbitrary. However, if a specific measurement is performed on the radiation field it becomes advantageous to use a decomposition corresponding to the specific measurement.

To calculate the probability to go from a certain initial state of the atom-field system to a certain final state one should calculate the complex probability amplitudes of all the different paths that lead to the final state. The different paths, which can be represented by Feynman diagrams, act as different channels in an interferometer. The total transition probability, including interference, is obtained by squaring the sum of the probability amplitudes.

To make the connection with the MCWF method the result of a specific measuring device acting upon the radiation field must be included in the QED description. If the radiation field is constantly monitored an atom-field state containing one photon is distinguishable from an atom-field state containing zero photons. Therefore the coherence between these states is lost due to the measurement. One can order the QED diagrams in groups leading to the distinguishable measurement results. The group of diagrams that describe a single-photon measurement will map onto the stochastical part of the MCWF method. The group of diagrams that describe the zero-photon measurement is based on radiation reaction and will map onto the coherent decay part of the MCWF method. The proper atomic-state vector in each one of these two cases occurs after normalization.

As an illustration we describe a two-level atom coupled to the radiation field which is constantly monitored by an ideal photodetector. The atomic-state vector at time $t$ is represented by Eq. (11). For the initial state we require that $\alpha_e(t) \neq 0$ and that the radiation field is in its vacuum state.

Up to second order in $H_{\rm int}$ [see Eq. (20)], only the element shown in Fig. 1 leads to photodetection. This diagram comes from the $[\mathbf{p}\cdot\mathbf{A}(\mathbf{x},t) + \mathbf{A}(\mathbf{x},t)\cdot\mathbf{p}]$ term in $H_{\rm int}$ [see Eq. (20)]. Standard time-dependent perturbation theory leads to a rate of increase of the number of photons in the radiation field and to a corresponding rate of increase of the atomic ground-state population,

$$\frac{d}{dt}|\alpha_g(t)|^2 = \Gamma|\alpha_e(t)|^2. \qquad (21)$$

The probability $dp$ for the detection of a photon in the time interval $[t, t+dt]$, $dt \ll 1/\Gamma$, is given by

$$dp = \Gamma|\alpha_e(t)|^2 dt, \qquad (22)$$

and the normalized atomic state after the detection is the ground state, $|\alpha_g(t+dt)|^2 = 1$. The mapping to the stochastical part of the MCWF method is evident.

The group of diagrams resulting in zero-photon detection contains, to second order in $H_{\rm int}$, three elements as shown in Fig. 2. The first diagram represents the free evolution of the atomic system. The second diagram comes from the $\mathbf{A}(\mathbf{x},t)\cdot\mathbf{A}(\mathbf{x},t)$ term in $H_{\rm int}$ [see Eq. (20)]. This diagram can be omitted since it is independent of the atomic parameters and therefore leads to an overall shift of the total energy which does not influence the atomic transition properties. The third diagram represents the spontaneous emission of a photon followed by the absorption of the same photon. This diagram is second order in $H_{\rm int}$ and can be interpreted as radiation reaction. The contribution of the diagrams shown in Figs. 2(a,c) to the atomic-state evolution is a complex-valued correction $\Delta E_e$ to the excited-state energy [14]. Hence $\alpha_e(t)$ can be written as

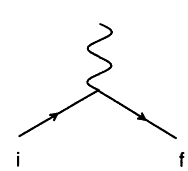



FIG. 1. One-photon detection diagram.

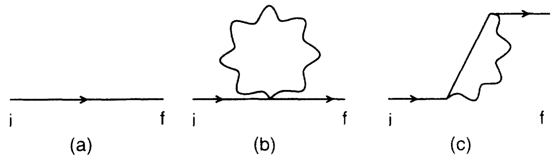



FIG. 2. Zero-photon detection diagrams.

$$\alpha_e(t) = e^{-i\Delta E_e t/\hbar}. \tag{23}$$

The real part of $\Delta E_e$ leads after mass renormalization to small level shifts. These shifts were already included in $H_0$ [see Eq. (9)] and can therefore be omitted. The imaginary part of $\Delta E_e$ has a value of $-\Gamma/2$ and therefore leads to damping of the excited-state amplitude without the emission of an observable photon. Since the measurement device distinguishes this atomic-state evolution from other possible evolutions, the atomic-state vector must be normalized and be interpreted as the actual state of the atom. The evolution of $\alpha_g(t)$ and $\alpha_e(t)$ are then given by the same equations as Eqs. (14) and (15), which lead to a tanh decay of the atomic inversion and a sech$^2$ line shape. Note that the normalization significantly influences the evolution of the atom. If, for example, at time $t = 0$ the atom is prepared in a pure excited state the normalization prevents the atom from decaying. The probability $dp$ for the detection device to measure no photon in the time interval $[t, t+dt]$ is equal to the probability that the excited-state population remains unchanged,

$$dp = \left(1 - \frac{\Gamma}{2}|\alpha_e(t)|^2 dt\right)^2 \approx 1 - \Gamma|\alpha_e(t)|^2 dt, \tag{24}$$

which equals 1 minus the probability for the emission of an observable photon, as it should. This completes the mapping of the direct inclusion of a specific measurement in QED onto the MCWF method.

Note that the neoclassical radiation theory describes the evolution of the atomic state when the continuous measurement of the radiation field yields zero result. In this situation it is apparently not necessary to quantize the radiation field to determine the evolution of the atomic state.

We should mention two publications in which observations related to the above were presented. Mollow [15] already has introduced the concept of radiation reaction to explain the decay of the excited-state amplitude of a two-level atom. His work, published in 1975, contains preliminaries of the essential features of the MCWF method. More recently, Gardiner, Parkins, and Zoller [11] used the concept of the Stratonovich and Ito equations to generalize the work of Mollow and to show the connection with the MCWF method. In the Stratonovich and Ito equations, when used in quantum optics, radiation reaction is included in a mathematically convenient way. Both publications, however, deal with the QED description of radiation reaction and do not mention any relation to the neoclassical theory.

Our treatment of the coherent decay part of the MCWF method in terms of radiation reaction has been stimulated by an article of Dicke on "apparently" interaction-free measurements [16]. Dicke considers the position measurement of a particle in a plane using a narrow light beam which is normal to the plane. The presence or absence of fluorescence photons is used to measure whether the particle is located within or outside the illuminated spot. The absorption and subsequent emission of a photon from the light beam changes the particle wave function without the emission of a detectable fluorescence photon.

## VI. CONCLUSIONS

We have demonstrated, in the case of spontaneous emission, that the coherent decay part of the MCWF method can be described in a neoclassical way. In such a description the mechanism responsible for the decay of the atomic inversion is classical radiation reaction. The rate of decay is then proportional to the atomic dipole moment. The occurrence of radiation reaction is also present in a QED description of the zero-photon detection. In QED, radiation reaction is proportional to $|\alpha_e|^2$ because it starts with the emission of a photon. Without taking a measurement into account radiation reaction leads to a non-Hermitian contribution to the evolution of the atomic wave function. However, if zero-photon detection is taken into account it is required that the atomic wave function remains normalized. As a consequence the rate of decay due to radiation reaction becomes proportional to the atomic dipole moment thus providing the link with the neoclassical theory. We conclude that quantization of the radiation field is superfluous for the description of the atomic evolution as long as no photons are detected. A complementary statement is that the neoclassical theory provides predictions which are inconsistent with experimental data because the way in which radiation is detected is not taken into account.

It might be that the occurrence of a semiclassical description during zero-result measurements is a principle of general validity, since the corpuscular character of radiation is not revealed by a zero-result measurement. This has significant consequences since a nonperturbative approach of nonlinear semiclassical theories can have a rich dynamical structure. This structure can never be described by the standard perturbation approach of QED since there it is assumed that small changes of the parameters give small changes in the dynamics. Of course one must realize that one can only see a glimpse of the atomic dynamics by looking at the emitted photons.

The suggestion that all of the coherent dynamics of an atom is determined by semiclassical equations would be stronger if we were able to include the neoclassical Lamb shift in the coherent evolution of the atom. However, the QED Lamb shift is calculated using renormalization of the electron mass. This obscures a simple connection with the neoclassical theory where the atomic electron is not a localized quantity and therefore mass renormalization seems inappropriate. Our interpretation of the

MCWF method leads to a combination of the neoclassical theory and quantum jumps. The evolution of the atomic wave function is obtained by a Schrödinger-like point of view as long as the photodetector yields a zero recording. In addition there is the possibility of quantum jumps to the ground state, with a probability proportional to $|\alpha_e(t)|^2$, corresponding to the detection of a photon. The fact that quantum jumps are still important suggests that the notion of a localized electron is still a valid concept in our interpretation. It seems that mass renormalization should therefore be included in the neoclassical theory. A related remark can be found in the review article on semiclassical radiation theories by Milonni [3].

## ACKNOWLEDGMENTS

We are grateful to K.A.H. van Leeuwen for stimulating discussions. This work is part of the research program of the Foundation for Fundamental Research on Matter (FOM) and was made possible by the financial support from the Netherlands Organization for Scientific Research (NWO). R.J.C.S. acknowledges the Niels Stensen Foundation for financial support.

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