Phys 250 Quantum Optics, Homework #2, Solutions

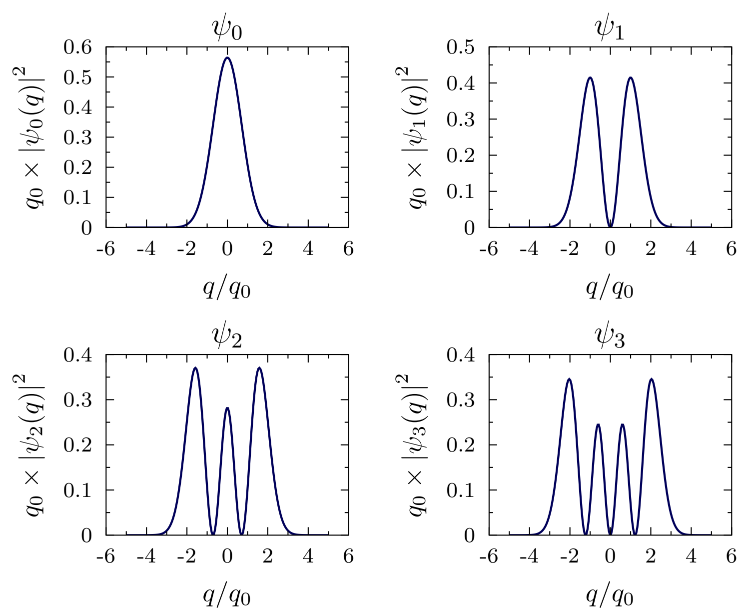
2. Study the Harmonic Oscillator (section 2.2) and make sure you understand it in detail. Derive an explicit expression of the wave function, $\psi(q)$, of the ground state and the first three excited states and plot $|\psi(q)|^2$ for each.

From (2.15):

$$\psi_n(q) = i^n \left(\sqrt{\pi} q_0 2^n n!\right)^{-1/2} \left(\xi - \frac{\partial}{\partial \xi}\right)^n e^{-\xi^2/2}$$

Where $q_0 = \sqrt{\frac{\hbar}{m\omega}}$ and $\xi = q/q_0$. So...

$$\begin{aligned}
\psi_0(q) &= \left(\sqrt{\pi} q_0\right)^{-1/2} e^{-\xi^2/2} \\
\psi_1(q) &= i \left(2\sqrt{\pi} q_0\right)^{-1/2} 2\xi e^{-\xi^2/2} \\
\psi_2(q) &= - \left(8\sqrt{\pi} q_0\right)^{-1/2} (4\xi^2 - 2) e^{-\xi^2/2} \\
\psi_3(q) &= - i \left(48\sqrt{\pi} q_0\right)^{-1/2} (8\xi^3 - 12\xi) e^{-\xi^2/2}
\end{aligned}$$



4. Work out the expectation value of the position, $\langle q \rangle$, for an energy eigenstate of a harmonic oscillator, $|n\rangle$, as well as the states $\frac{1}{\sqrt{2}}(|n\rangle + e^{i\phi}|n+1\rangle)$ and $\frac{1}{\sqrt{2}}(|n\rangle + e^{i\phi}|n+2\rangle)$.

From (2.6):

$$q = i\frac{q_0}{\sqrt{2}}(a - a^\dagger)$$

So...

$$
\begin{aligned}
\langle n|\, q\, |n\rangle =& \frac{q_0}{\sqrt{2}} \langle n| \left(\sqrt{n}\, |n-1\rangle - \sqrt{n+1}\, |n+1\rangle \right) \\
=& 0
\end{aligned}
$$

And...

$$
\begin{aligned}
|\psi_1\rangle =& \sqrt{\frac{1}{2}} \left(|n\rangle + e^{+i\phi} |n+1\rangle \right) \\
\langle \psi_1|\, q\, |\psi_1\rangle =& \frac{iq_0}{2\sqrt{2}} \left(\langle n| + e^{-i\phi} \langle n+1| \right) \times \\
& \left(\sqrt{n}\, |n-1\rangle - \sqrt{n+1}\, |n+1\rangle + e^{+i\phi}\sqrt{n+1}\, |n\rangle - e^{+i\phi}\sqrt{n+2}\, |n+2\rangle \right) \\
=& \frac{iq_0}{2} \sqrt{\frac{n+1}{2}} \left(e^{+i\phi} - e^{-i\phi} \right) \\
=& -q_0 \sqrt{\frac{n+1}{2}} \sin\phi
\end{aligned}
$$

And...

$$
\begin{aligned}
|\psi_2\rangle =& \sqrt{\frac{1}{2}} \left(|n\rangle + e^{+i\phi} |n+2\rangle \right) \\
\langle \psi_2|\, q\, |\psi_2\rangle =& \frac{iq_0}{2\sqrt{2}} \left(\langle n| + e^{-i\phi} \langle n+2| \right) \times \\
& \left(\sqrt{n}\, |n-1\rangle - \sqrt{n+1}\, |n+1\rangle + e^{+i\phi}\sqrt{n+2}\, |n+1\rangle - e^{+i\phi}\sqrt{n+3}\, |n+3\rangle \right) \\
=& 0
\end{aligned}
$$