Physics 6C Summer 2006
Homework 4 Solutions by Michael Gary

All problems are from the 2nd edition of Walker. Numerical values are different for each student.

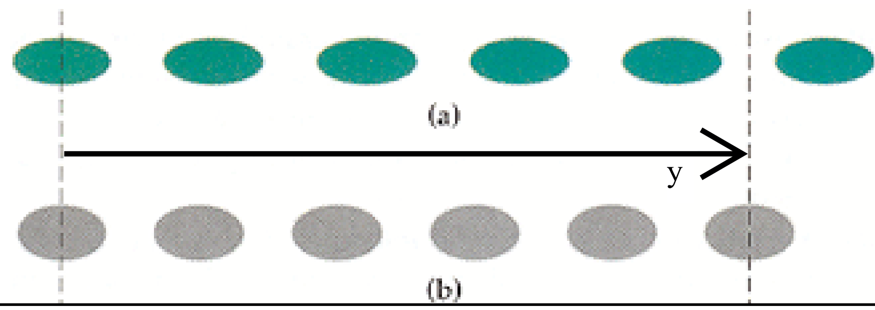
1) Chapter 28 Problem 13: Monochromatic light passes through two slits separated by distance of 0.0324 mm. If the angle to the third maximum above the central fringe is 3.51°, what is the wavelength of the light?

Solution:

$$\sin\theta = m\frac{\lambda}{d} = (3)\frac{\lambda}{d} = \frac{3\lambda}{d}$$

$$\lambda = \frac{1}{3}d\sin\theta = \frac{1}{3}(0.0324\times10^{-3}\text{ m})\sin 3.51° = \boxed{661\text{ nm}}$$

2) Chapter 28 Problem 21: The interference pattern shown in Figure 28–35 (a) is produced by green light with a wavelength of λ = 505 nm passing through two slits with a separation of 127μm. After passing through the slits, the light forms a pattern of bright and dark spots on a screen located 1.25 m from the slits. **(a)** What is the distance between the two vertical, dashed lines in Figure 28–35 (a)? **(b)** If it is desired to produce a more tightly packed interference pattern, like the one shown in Figure 28–35 (b), how could you do this by changing either the frequency of the light and/or the separation between the slits?



(a) HINT: the green spots are evenly spaced, and we can take the central maximum to be any one of them.

Solution: (a) $\sin\theta = \frac{m\lambda}{d}$

Assume the left dashed line is the central maximum. We will find the positions of the fourth and fifth maxima. The right dashed line is halfway between these points

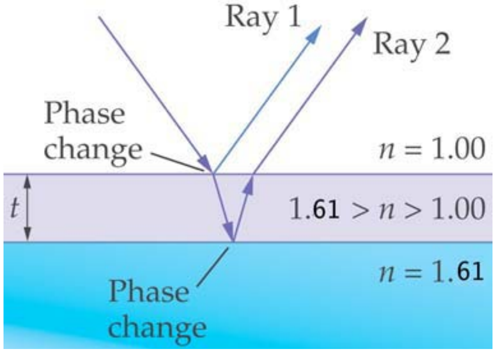
$$y_4 = L\tan\theta \approx L\sin\theta = L\frac{m\lambda}{d} = (1.25m)\frac{4(505\times10^{-9}m)}{127\times10^{-6}m} = 1.98cm$$

$$y_5 = L\tan\theta \approx L\sin\theta = L\frac{m\lambda}{d} = (1.25m)\frac{5(505\times10^{-9}m)}{127\times10^{-6}m} = 2.48cm$$

$$y = \frac{y_4 + y_5}{2} = 2.24cm$$

(b) The distance between the fringes is proportional to the wavelength, and so must be inversely proportional to the frequency. The distance between the fringes is also inversely proportional to the slit separation. A more tightly packed pattern would require a shorter wavelength and a higher frequency and/or an increased slit separation. Increase the frequency, increase the slit separation, or increase both simultaneously.

3) Chapter 28 Problem 33: A thin layer of magnesium fluoride $(n = 1.38)$ is used to coat a flint-glass lens $(n = 1.61)$. **(a)** What minimum thickness should the magnesium fluoride film have if the reflection of 595 nm light is to be suppressed? Assume that the light is incident at right angles to the film. **(b)** If it is desired to suppress the reflection of light with a higher frequency, should the coating of magnesium fluoride be made thinner or thicker?



(All rays move vertically; horizontal bending is only shown to distinguish rays.)

Solution: (a) Solution using the book's method:

Find the difference in phase changes of the light reflected from each interface. Note that λ is the wavelength in vacuum.

<u>air-magnesium fluoride interface</u>

$$\ell_{\text{eff, 1}} = \frac{1}{2}\lambda$$

$$\frac{\ell_{\text{eff, 1}}}{\lambda} = \frac{1}{2}$$

<u>magnesium fluoride-glass interface</u>

$$\ell_{\text{eff, 2}} = 2t + \frac{1}{2}\lambda_n$$

$$\frac{\ell_{\text{eff, 2}}}{\lambda_n} = \frac{2t}{\lambda_n} + \frac{1}{2}$$

$$\frac{\ell_{\text{eff, 2}}}{\lambda} = \frac{2nt}{\lambda} + \frac{1}{2}$$

difference in phase changes $= \dfrac{2nt}{\lambda} + \dfrac{1}{2} - \dfrac{1}{2} = \dfrac{2nt}{\lambda}$

For destructive interference: $\dfrac{2nt}{\lambda} - \dfrac{1}{2} = m$ where $m = 0, 1, 2, \ldots$.

Determine the minimum film thickness ($m = 0$).

$$\frac{2nt}{\lambda} = \frac{1}{2}$$

$$t = \frac{\lambda}{4n} = \frac{595\text{ nm}}{4(1.38)} = \boxed{108\text{ nm}}$$

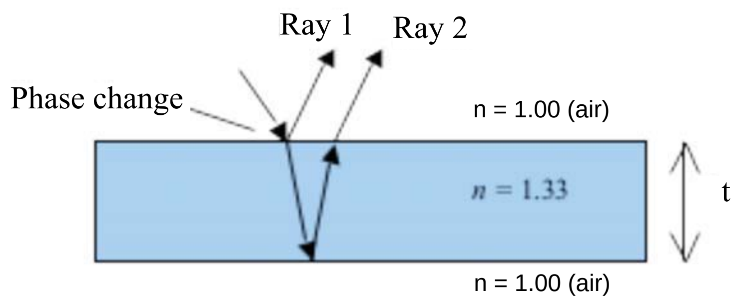
See next page for alternate solution.

Solution using the method from lecture:
There is a phase change of $\frac{1}{2}\lambda_n$ at each of the reflections in the figure (see the phase change rule in Figure 28-8) so this does not introduce any phase difference between rays 1 and 2. Thus the path difference ($2t$) alone must account for all of the desired phase difference. To get destructive interference, we want $2t = (m + \frac{1}{2})\lambda_n$ for some integer m so the smallest thickness t has $m = 0$:

$$2t = (0 + \tfrac{1}{2})\lambda_n$$
$$t = \frac{1}{4}\lambda_n = \frac{1}{4}\frac{\lambda}{n} = \frac{595\text{ nm}}{4(1.38)} = 108\text{ nm}$$

(b) Higher frequency corresponds to lower wavelength. So, since $t \propto \lambda$, the coating should be made $\boxed{\text{thinner}}$.

4) Chapter 28 Problem 34: White light is incident normally on a thin soap film $(n = 1.33)$ suspended in air. **(a)** What are the two minimum thicknesses that will constructively reflect yellow $(\lambda = 590\text{nm})$ light? **(b)** What are the two minimum thicknesses that will *destructively* reflect yellow $(\lambda = 590\text{nm})$ light?



Due to reflection, there is a phase difference of $\frac{1}{2}\lambda_n$ between ray 1 and ray 2 (see the phase change rule in Figure 28-8). Again, all rays move vertically; horizontal bending is only shown to distinguish them.

Solution using the book's method: Again, λ is the wavelength in vacuum.

(a) $\dfrac{2nt}{\lambda} - \dfrac{1}{2} = m$ (equation 28-11)

$$\frac{2nt}{\lambda} = m + \frac{1}{2}$$

$$t = \frac{\left(m + \frac{1}{2}\right)\lambda}{2n}$$

$$= \frac{\left(m + \frac{1}{2}\right)(590\text{ nm})}{2(1.33)}$$

m	0	1
t (nm)	110	330

The two minimum thicknesses are $\boxed{110\text{ nm and }330\text{ nm}}$.

(b) $\dfrac{2nt}{\lambda} = m$ (equation 28-10)

$$t = \frac{m\lambda}{2n}$$

$$= \frac{m(590\text{ nm})}{2(1.33)}$$

m	1	2
t (nm)	220	440

The two minimum thicknesses are $\boxed{220\text{ nm and }440\text{ nm}}$.

See next page for alternate solutions to (a) and (b).

Solution using the method from lecture: (a) For constructive interference, the total phase difference between rays 1 and 2 must be an integer multiple of λ_n so to offset the $\frac{1}{2}\lambda_n$ phase difference due to reflection, we want the path difference (2*t*) to be:

$$2t = (m + \tfrac{1}{2})\lambda_n$$
$$t = \frac{(m + \frac{1}{2})}{2}\lambda_n$$

So the two smallest thicknesses *t* have *m* = 0 and *m* = 1:

$$t = \frac{(0 + \frac{1}{2})}{2}\lambda_n = \frac{\lambda}{4n} = \frac{590\text{ nm}}{4(1.33)} = 110\text{ nm}$$
$$t = \frac{(1 + \frac{1}{2})}{2}\lambda_n = \frac{3\lambda}{4n} = \frac{3(590\text{ nm})}{2(1.33)} = 330\text{ nm}$$

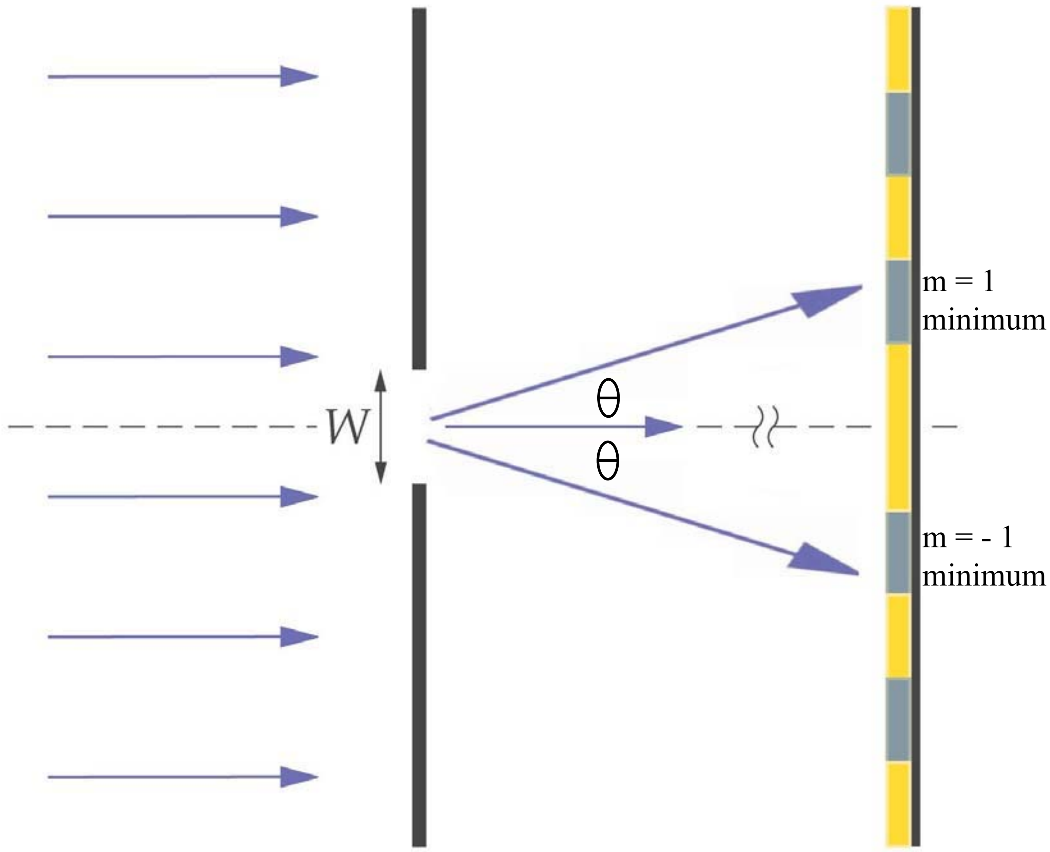
(b) For destructive interference, the total phase difference between rays 1 and 2 must be a half-integer multiple of λ_n. Since the phase difference due to reflection is already $\frac{1}{2}\lambda_n$, we want the path difference (2*t*) to be:

$$2t = m\lambda_n$$
$$t = \frac{m}{2}\lambda_n$$

So the two smallest (nonzero) thicknesses *t* have *m* = 1 and *m* = 2:

$$t = \frac{1}{2}\lambda_n = \frac{\lambda}{2n} = \frac{590\text{ nm}}{2(1.33)} = 220\text{ nm}$$
$$t = \frac{2}{2}\lambda_n = \frac{\lambda}{n} = \frac{590\text{ nm}}{1.33} = 440\text{ nm}$$

5) Chapter 28 Problem 38: Diffraction also occurs with sound waves. Consider 1100-Hz sound waves diffracted by a door that is 82 cm wide. What is the angle between the two first-order diffraction minima?



Solution:

$$\sin\theta = m\frac{\lambda}{W}$$

$$\theta = \sin^{-1}\frac{m\lambda}{W}$$

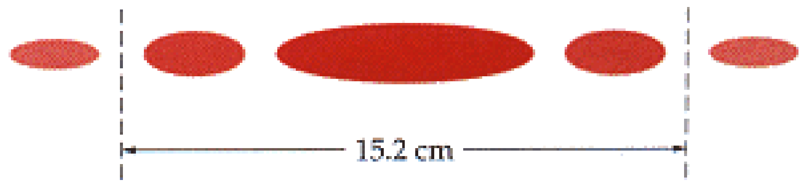
$$2\theta = 2\sin^{-1}\frac{m\lambda}{W}$$

$$= 2\sin^{-1}\frac{mv}{Wf}$$

$$= 2\sin^{-1}\frac{(1)\left(343\ \frac{\text{m}}{\text{s}}\right)}{(0.82\ \text{m})(1100\ \text{Hz})}$$

$$= \boxed{45^\circ}$$

6) Chapter 28 Problem 44: The diffraction pattern shown in **Figure 28–38** is produced by passing He–Ne laser light ($\lambda = 632.8$ nm) through a single slit and viewing the pattern on a screen 1.50 m behind the slit. **(a)** What is the width of the slit? **(b)** If monochromatic yellow light with a wavelength of 591 nm is used with this slit, will the distance indicated in Figure 28–38 be greater than or less than 15.2 cm? Explain.



Solution: (a) $\sin\theta = m\dfrac{\lambda}{W} = (2)\dfrac{\lambda}{W} = \dfrac{2\lambda}{W}$

$$W = \frac{2\lambda}{\sin\theta}$$

Find θ.

$$y = L\tan\theta$$

$$\theta = \tan^{-1}\frac{y}{L}$$

Substitute.

$$W = \frac{2\lambda}{\sin\left(\tan^{-1}\frac{y}{L}\right)} = \frac{2(632.8\times10^{-9}\text{ m})}{\sin\left(\tan^{-1}\frac{\frac{0.152\text{ m}}{2}}{1.50\text{ m}}\right)} = \boxed{25.0\ \mu\text{m}}$$

(b) Since the angle a wave diffracts is greater the larger the wavelength of the wave, the distance indicated will be $\boxed{\text{less than}}$ 15.2 cm (591 nm < 632.8 nm). If, instead, the wavelength λ > 632.8 nm, the distance indicated will be greater than 15.2 cm.

7) Chapter 28 Problem 50: The Hubble Space Telescope (HST) orbits Earth at an altitude of 613 km. It has an objective mirror that is 2.4 m in diameter. If the HST were to look down on Earth's surface (rather than up at the stars), what is the minimum separation of two objects that could be resolved using 550-nm light? [*Note*: The HST is used only for astronomical work, but a (classified) number of similar telescopes are in orbit for spy purposes.]

Solution:

$$\begin{aligned} y &= L\tan\theta_{\min} \\ &= L\tan\left(1.22\frac{\lambda}{D}\right) \\ &= (613\times10^{3}\text{ m})\tan\frac{1.22(550\times10^{-9}\text{ m})}{2.4\text{ m}} \\ &= \boxed{17\text{ cm}} \end{aligned}$$

8) Chapter 28 Problem 62: White light strikes a diffraction grating (760 lines/mm) at normal incidence. What is the longest wavelength that forms a second-order maximum?

Solution: We want the second order maximum to just barely appear on the screen. As long as the location of the second order maximum is less than 90°, it will appear on the screen at some location. However, if the second order maximum is at 90°, it will no longer appear on the screen. Therefore, to find the longest possible wavelength, we should solve for what wavelength makes the second order maximum appear at 90°.

$$\sin\theta = m\frac{\lambda}{d} = mN\lambda$$

$$\lambda = \frac{\sin\theta}{mN} = \frac{\sin 90°}{2(760\ \text{mm}^{-1})\left(\frac{1000\ \text{mm}}{1\ \text{m}}\right)} = \boxed{660\ \text{nm}}$$