

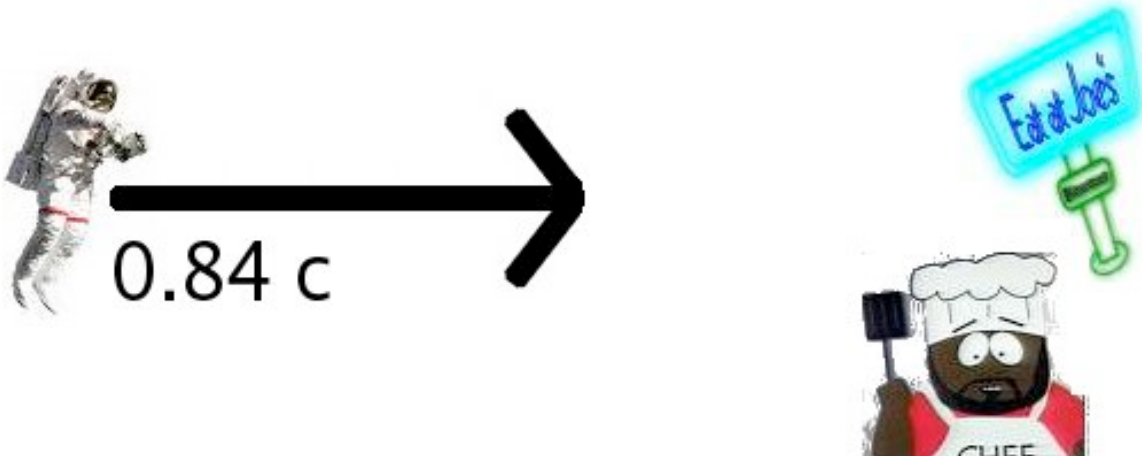
Physics 6C Summer 2006
Homework 5 Solutions

All problems are from the 2nd edition of Walker. Numerical values are different for each student.

1) Chapter 29 Conceptual Question 12: According to the theory of relativity, the maximum speed for any particle with mass is the speed of light. Is there a similar limit (due to motion) on the particle's momentum and energy, or can the momentum and/or energy be infinite?

Solution: The particle's momentum can be infinite. The rest energy of a particle does not depend on its speed, so it is constant, and hence not infinite. The kinetic energy of a particle can be infinite. Since the total energy is the sum of the rest energy and the kinetic energy, and the kinetic energy can be infinite, the total energy can also be infinite.

2) Chapter 29 Problem 2: A neon sign in front of a cafe flashes on and off once every 5.0 s, as measured by the head cook. How much time elapses between flashes of the sign as measured by an astronaut in a spaceship moving toward Earth at a speed of $0.84c$?

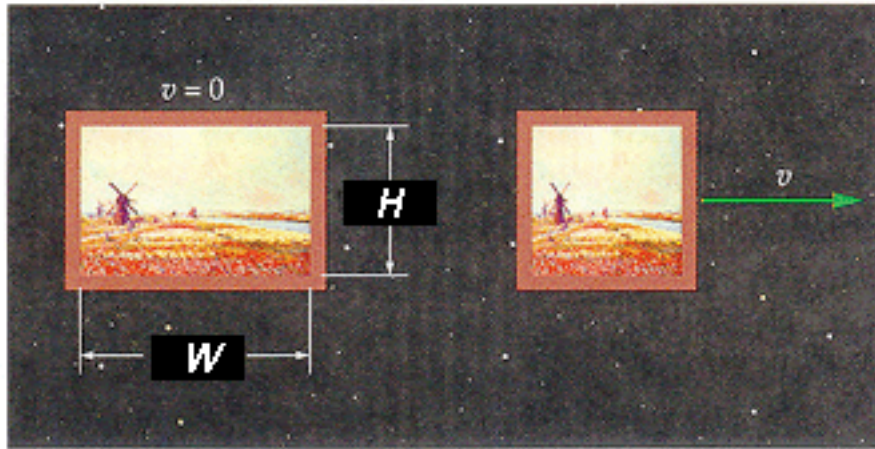


Solution: Since the head cook is standing still in the rest frame of the sign, the head cook measures the proper time elapsed between successive flashes of the sign, 5.0 s. The astronaut in the spaceship is moving towards earth, so in the astronaut's rest frame, the sign is moving towards the astronaut. Therefore, the time between flashes in the astronaut's frame is dilated. The dilated time measured by the astronaut is

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{5.0 \text{ s}}{\sqrt{1 - \left(\frac{0.84c}{c}\right)^2}} = \boxed{9.2 \text{ s}}.$$

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3) Chapter 29 Problem 21: A rectangular painting is $W=124$ cm wide and $H=80.5$ cm high, as indicated in **Figure 29-28**. At what speed, v , must the painting move parallel to its width if it is to appear to be square?



Solution: Only directions parallel to the observed direction of motion contract, so the height H will not change. Therefore, we want the length contracted W to be equal to H so that the painting appears to be square.

$$H = W \sqrt{1 - \frac{v^2}{c^2}}$$

$$\frac{H^2}{W^2} = 1 - \frac{v^2}{c^2}$$

$$v = c \sqrt{1 - \frac{H^2}{W^2}} = c \sqrt{1 - \left(\frac{80.5 \text{ cm}}{124 \text{ cm}} \right)^2} = 0.761c$$

4) Chapter 29 Problem 52: When a certain capacitor is charged, its mass increases by 8.3×10^{-16} kg (this extra mass is not the mass of the charge that it has accumulated). How much energy is stored in the capacitor?

Solution: $E = \Delta mc^2 = (8.3 \times 10^{-16} \text{ kg}) \left(3.00 \times 10^8 \frac{\text{m}}{\text{s}} \right)^2 = \boxed{75 \text{ J}}$

5) Chapter 29 Problem 64: Find the radius to which the Sun must be compressed for it to become a black hole.

Solution: $R = \frac{2GM}{c^2} = \frac{2 \left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2} \right) (2.00 \times 10^{30} \text{ kg})}{\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}} \right)^2} = \boxed{2.96 \text{ km}}$ (See equation 29-10).

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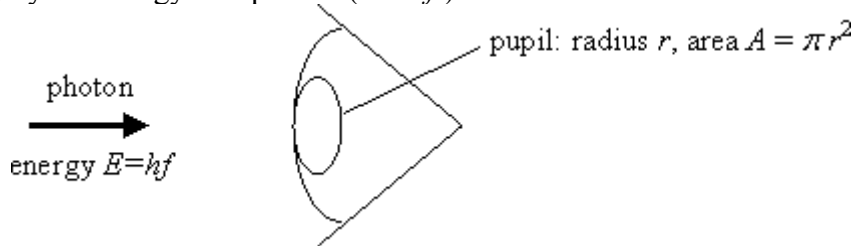
6) Chapter 30 Conceptual Question 14: Suppose you perform an experiment on the photoelectric effect, using light with a frequency high enough to eject electrons. If the intensity of the light is increased while the frequency is held constant, describe whether the following quantities increase, decrease, or stay the same: **(a)** The maximum kinetic energy of an ejected electron; **(b)** the minimum de Broglie wavelength of an electron; **(c)** the number of electrons ejected per second; **(d)** the electric current in the phototube.

Solution: Note that increasing the intensity while holding the frequency constant simply means that more photons – each with the original energy – strike the metal surface per second.

(a) Stays the same. **(b)** Stays the same. **(c)** Increases. **(d)** Increases.

7) Chapter 30 Problem 30: Owls have large, sensitive eyes for good night vision. Typically, the pupil of an owl's eye can have a diameter of 8.5 mm (as compared with a maximum diameter of about 7.0 mm for humans). In addition, an owl's eye is about 100 times more sensitive to light of low intensity than a human eye, allowing owls to detect light with an intensity as small as $5.0 \times 10^{-13} \text{ W/m}^2$. Find the minimum number of photons per second an owl can detect, assuming a frequency of $7.0 \times 10^{14} \text{ Hz}$ for the light.

Solution: The intensity that enters the pupil is: $I = \text{energy per second per unit area}$. The energy per second that enters the pupil is: $(\text{intensity})(\text{area of pupil}) = IA = I(\pi r^2)$. To find the number of photons per second that enter the pupil, we must divide the energy per second (IA) by the energy of a photon ($E = hf$)



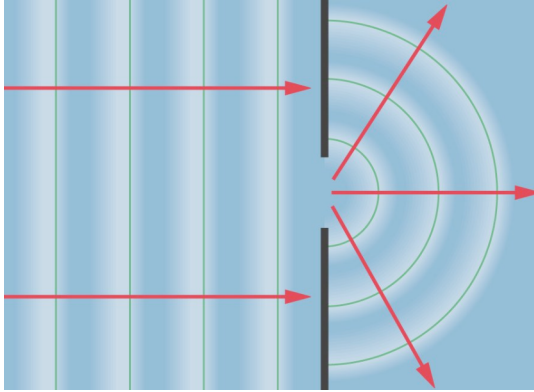
$$\frac{IA}{E} = \frac{I\pi r^2}{hf} = \frac{\left(5.0 \times 10^{-13} \frac{\text{W}}{\text{m}^2}\right) \pi \left(\frac{0.0085 \text{ m}}{2}\right)^2}{(6.63 \times 10^{-34} \text{ J}\cdot\text{s})(7.0 \times 10^{14} \text{ Hz})} = \boxed{61 \text{ photons/s}}$$

8) Chapter 30 Problem 48: An X-ray photon scatters through 180° from (i) an electron or (ii) a helium atom. In which case is the change in wavelength $\Delta\lambda$ of the X-ray greater, and why?

Solution: Since $\Delta\lambda \propto \frac{1}{m}$ and $m_e < m_{\text{He}}$, the change in wavelength of the X-ray is greater for the electron.

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9) Chapter 30 Problem 58: Diffraction effects become significant when the width of an aperture is comparable to the wavelength of the waves being diffracted. **(a)** At what speed will the de Broglie wavelength of a 65-kg student be equal to the 0.76-m width of a doorway? **(b)** At this speed, how long will it take the student to travel a distance of 1.0 mm? (For comparison, the age of the universe is approximately 4×10^{17} s. The correct answer to **(b)** explains why the diffraction in this problem cannot be observed.)



Solution:

$$\begin{aligned} \text{(a)} \lambda &= \frac{h}{p} \\ &= \frac{h}{mv} \\ v &= \frac{h}{\lambda m} \\ &= \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(0.76 \text{ m})(65 \text{ kg})} \\ &= \boxed{1.3 \times 10^{-35} \text{ m/s}} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad x &= vt \\ t &= \frac{x}{v} \\ &= \frac{x\lambda m}{h} \\ &= \frac{(0.0010 \text{ m})(0.76 \text{ m})(65 \text{ kg})}{6.63 \times 10^{-34} \text{ J} \cdot \text{s}} \\ &= \boxed{7.5 \times 10^{31} \text{ s}} \end{aligned}$$

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10) Chapter 30 Problem 62: The position of a 0.20 kg air track cart is determined to within an uncertainty of 2.2 mm. What is the minimum uncertainty of the cart's speed as a result of the position measurement?

Solution:

$$\Delta p_y \Delta y \geq \frac{h}{2\pi}$$

$$m \Delta v_y \Delta y \geq \frac{h}{2\pi}$$

$$\Delta v_y \geq \frac{h}{2\pi m \Delta y}$$

$$\Delta v_y \geq \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{2\pi (0.20 \text{ kg})(0.0022 \text{ m})}$$

$$\Delta v_y \geq \boxed{2.4 \times 10^{-31} \text{ m/s}}$$

11) Chapter 30 Problem 65: An excited state of a particular atom has a mean lifetime of $0.60 \times 10^{-9} \text{ s}$, which we may take as the uncertainty Δt . What is the minimum uncertainty in any measurement of the energy of this state?

Solution:

$$\Delta E \Delta t \geq \frac{h}{2\pi}$$

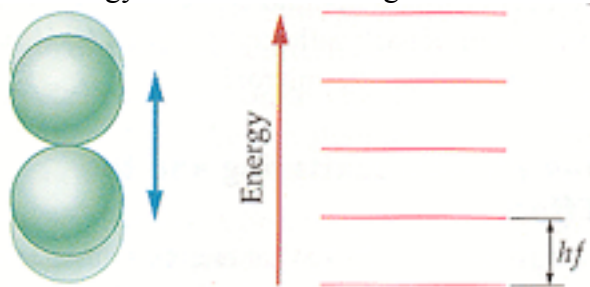
$$\Delta E \geq \frac{h}{2\pi \Delta t}$$

$$\Delta E \geq \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{2\pi (0.60 \times 10^{-9} \text{ s})}$$

$$\Delta E_{\min} = \boxed{1.8 \times 10^{-25} \text{ J}}$$

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12) Chapter 30 Problem 9: An oxygen molecule (O_2) vibrates with an energy identical to that of a single particle of mass $m = 1.340 \times 10^{-26} \text{ kg}$ attached to a spring with a force constant of $k = 1215 \text{ N/m}$. The energy levels of the system are uniformly spaced, as indicated in Figure 30-20, with a separation given by hf . **(a)** What is the vibration frequency of this molecule? **(b)** How much energy must be added to the molecule to excite it from one energy level to the next higher level?



Solution: (a) $\omega = \sqrt{\frac{k}{m}} = 2\pi f$

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \sqrt{\frac{1215 \frac{\text{N}}{\text{m}}}{1.340 \times 10^{-26} \text{ kg}}} = \boxed{4.792 \times 10^{13} \text{ Hz}}$$

(b) $E = hf = (6.63 \times 10^{-34} \text{ J} \cdot \text{s})(4.792 \times 10^{13} \text{ Hz}) = \boxed{3.18 \times 10^{-20} \text{ J}}$

13) Chapter 31 Problem 13: What is the minimum energy is required to ionize hydrogen when it is in the $n = 4$ state?

Solution: The photon energy is the difference between the initial and final energies, labeled by n and n_f , respectively. Since we are trying to just barely ionize the hydrogen atom, the electron will be just barely free, and thus have zero energy, which corresponds to the state $n_f = \infty$. It is generally a good idea to use absolute values for the photon energy, since the photon energy is always positive.

$$|E_n| = \frac{13.6 \text{ eV}}{n^2}$$

$$E_4 = \frac{13.6 \text{ eV}}{4^2}$$

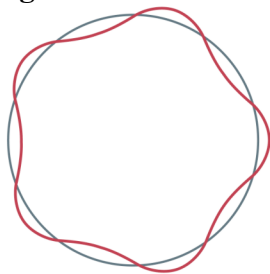
$$= \boxed{0.850 \text{ eV}}$$

14) Chapter 31 Problem 14: Find the energy of the photon required to excite a hydrogen atom from the $n = 2$ state to the $n = 5$ state.

Solution: $\Delta E = E_5 - E_2 = (-13.6 \text{ eV})\left(\frac{1}{5^2} - \frac{1}{2^2}\right) = \boxed{2.86 \text{ eV}}$

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15) Chapter 31 Problem 31: What is the radius of the hydrogen-atom Bohr orbit shown in Figure 31–30?



Solution: There are five wavelengths, which corresponds to the $n = 5$ state of hydrogen.

$$r_5 = 5^2 r_1 = 25(5.29 \times 10^{-11} \text{ m}) = \boxed{1.32 \times 10^{-9} \text{ m}}$$

16) Chapter 31 Problem 35: Give the value of the quantum number ℓ if one exists, for a hydrogen atom whose orbital angular momentum has a magnitude of (a) $\sqrt{6} \left(\frac{h}{2\pi} \right)$ (b)

$\sqrt{15} \left(\frac{h}{2\pi} \right)$ (c) $\sqrt{30} \left(\frac{h}{2\pi} \right)$ or (d) $\sqrt{36} \left(\frac{h}{2\pi} \right)$

Solution: $L = \sqrt{\ell(\ell+1)} (h / 2\pi)$

(a) $\sqrt{6} = \sqrt{\ell(\ell+1)}$ yields $\ell = \boxed{2}$

This can be seen explicitly by squaring both sides of the above equation, yielding the quadratic equation

$$6 = \ell(\ell+1)$$

$$\ell^2 + \ell - 6 = 0$$

$$\ell = \frac{-1 + \sqrt{1 - 4(-6)}}{2} = 2$$

Alternatively, we could plug in successive integers and see if any work. For example, $\ell=1$ gives $\sqrt{1(1+1)} = \sqrt{2}$, and we could continue to guess and check until we found an integer that works or got a number larger than $\sqrt{6}$.

(b) $\sqrt{15} = \sqrt{\ell(\ell+1)} \rightarrow \boxed{\text{no integer } \ell \text{ exists}}$

We can see this by plugging in integers and if there are two neighboring integers, one smaller than the answer, and one larger, then there is no solution. In this case, $\ell=3$ gives $\sqrt{3(3+1)} = \sqrt{12}$ while $\ell=4$ gives $\sqrt{4(4+1)} = \sqrt{20}$, so there is no solution.

(c) $\sqrt{30} = \sqrt{\ell(\ell+1)}$ yields $\ell = \boxed{5}$ by methods similar to (a)

(d) $\sqrt{36} = \sqrt{\ell(\ell+1)} \rightarrow \boxed{\text{no integer } \ell \text{ exists}}$

This is similar to (b).

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17) Chapter 31 Problem 40: Give the electronic configuration for the ground state of carbon (for other elements, see table 31-4).

Solution: Carbon has six protons ($Z = 6$) and six electrons. The first two electrons will fill up the 1s subshell, the next two electrons will fill the 2s subshell, and the remaining two electrons will go into the 2p subshell. $1s^2 2s^2 2p^2$