Physics CS 140 **Set # 1** Winter 2014

(For T January 14, 5:00PM)

Read Feynman "Quantum Mechanics and Path Integrals": Ch.2 sections 2-1, 2-2, 2-3. Reading Ch.1 Sects 1-1, 1-2

Problem 1.

Consider a particle of mass m moving in an time-independent potential $V(x)$.
Show that if $x_{cl}(t)$is the classical path, then the action evaluated at the classical path is

$$S[x_{cl}(t)] = -(t_b - t_a)E_{cl} + \int_{x_a}^{x_b} dx \sqrt{2m(E_{cl} - V(x))} \quad ,$$

where E_{cl} is the total energy of the particle. Assume the velocity to be positive for all t.

From Feynman Ch. 2:

Problem 2-2.

For a harmonic oscillator $L = (m/2)\left(\dot{x}^2 - \omega^2 x^2\right)$. With T equal to $t_b - t_a$.

a) Show that on the classical path we can write the action as

$$S = \frac{m}{2}[x(t_b)\dot{x}(t_b) - x(t_a)\dot{x}(t_a)]$$

b) Show that the action evaluated at the classical path is

$$S_{cl} = \frac{m\omega}{2\sin\omega T}\left[\left(x_b^2 + x_a^2\right)\cos\omega T - 2x_b x_a\right]$$

Problem 2-3.

a) Find S_{cl} for a particle under a constant force f, that is, $L = (m/2)\dot{x}^2 + fx$.

b) Verify that the action satisfies the Hamilton-Jacobi equations:

$$H\left(x_a, -\frac{\partial S}{\partial x_a}, t_a\right) - \frac{\partial S}{\partial t_a} = 0$$

$$H\left(x_b, \frac{\partial S}{\partial x_b}, t_b\right) + \frac{\partial S}{\partial t_b} = 0$$

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Problem 2-4.

Classically, the momentum is defined as $p = \frac{\partial L}{\partial \dot{x}}$
Show that the momentum at a final point is

$$\left(\frac{\partial L}{\partial \dot{x}}\right)_{x=x_b} = +\frac{\partial S_{cl}}{\partial x_b}$$

while the momentum at an initial point is

$$\left(\frac{\partial L}{\partial \dot{x}}\right)_{x=x_b} = -\frac{\partial S_{cl}}{\partial x_a}$$

Hint: Consider the effect on Eq. (2.6) in Feynman's textbook of a change in the end points.

Problem 2-5.

Classically the energy is defined as $E = \dot{x}p - L$
Show that the energy at a final point is

$$\dot{x}_b \left(\frac{\partial L}{\partial \dot{x}}\right)_{x=x_b} - L(x_b) = -\frac{\partial S_{cl}}{\partial t_b}$$

while the energy at an initial point is

$$+\frac{\partial S_{cl}}{\partial t_a}$$

Hint: A change in the time of an end point requires a change in the path, since all paths must be classical paths.