Physics CS 140 **Set # 10** Winter 2014
(For Th March 20 , 5:00 PM)

Read Feynman "Quantum Mechanics and Path Integrals": Ch 6, Sects 6-1 through 6-4. Read the class notes for the weeks of T March 4 thrhough Th March 13

Problem 1 (Feynman Problem 6-6)

Suppose the potential is that of a central force. Thus $V(\mathbf{r}) = V(r)$. Show that $v(\Delta\mathbf{p})$ can be written as

$$v(\Delta\mathbf{p}) = v(|\Delta\mathbf{p}|) = \frac{4\pi\hbar}{|\Delta\mathbf{p}|}\int_0^\infty \left(\sin\frac{|\Delta\mathbf{p}|\, r}{\hbar}\right) V(r)\, r dr$$

Suppose V is the Coulomb potntial $-Ze^2/r$. In this case the integral for $v(\Delta p)$ is oscillatory at the upper limit. But convergence of the integral can be artificially forced by introducing the factor $e^{-\epsilon r}$ and then taking the limit of the result as $\epsilon \to 0$. Following through this calculation, show that the cross section corresponds to the Rutherford cross section

$$\frac{\sigma_{\text{Ruth}}}{d\Omega} = \frac{4m^2Z^2e^4}{|\Delta\mathbf{p}|^4} = \frac{Z^2e^4}{16K^2\sin^4(\theta/2)}$$

where $e =$ charge on a proton

$K = mu^2/2$

$u = (r_a + r_b)/T$

$\Delta\mathbf{p} = \mathbf{p_a} - \mathbf{p_b}$

$\Delta p = 2p\sin(\theta/2) = 2mu\sin(\theta/2)$

$\theta =$ angle between the vectors $-\mathbf{x_a}$ and $\mathbf{x_b}$

Problem 2 (Feynman Problem 6-14)

Use the wavefunction approach to discuss the scattering of an electron from a sinusoidally oscillating field whose potential is given by

$$V(\mathbf{x}, t) = U(\mathbf{x})\cos\omega t$$

For all practical purposes let

$$x^2 = \frac{mR_{bc}^2}{2\hbar(t_b - t_c)}$$

and show the following result for the wavefunction $\psi(\mathbf{x_b}, t_b)$:

$$\psi(\mathbf{x_b}, t_b) = e^{(i/\hbar)(\mathbf{p_a}\cdot\mathbf{x_a} - E_a t_b)} - \frac{m}{4\pi\hbar^2}\Bigg[e^{(i/\hbar)(E_a - \hbar\omega)t_b}\int d^3x_c\, U(\mathbf{x}_c)e^{(i\hbar)\mathbf{p_a}\cdot\mathbf{x_c}}$$

$$\times\frac{1}{R_{bc}}e^{(i/\hbar)R_{bc}\sqrt{p^2 - 4m\hbar\omega}} + e^{(i/\hbar)(E_a + \hbar\omega)t_b}\int d^3x_c\, U(\mathbf{x}_c)e^{(i\hbar)\mathbf{p_a}\cdot\mathbf{x_c}}\frac{1}{R_{bc}}e^{(i/\hbar)R_{bc}\sqrt{p^2 + 4m\hbar\omega}}\Bigg]$$

Interprete this result when r_b is large.