Physics CS 140 **Set # 2** Spring 2014
(For T January 21, 5:00 PM)

Read Feynman "Quantum Mechanics and Path Integrals": Ch.1. and Ch. 7 Sect. 7-2 pp. 170 and 171

Problem 1.
Show the following results for functional derivatives:

a) $$\frac{\delta Q(t')}{\delta Q(t)} = \delta(t' - t)$$

b) $$F[Q] = \int_{t_a}^{t_b} Q(t') f(t') dt',$$

$$\frac{\delta F[Q]}{\delta Q(t)} = f(t)$$

c) $$F[Q] = \exp\left[\int_{t_a}^{t_b} Q(t') f(t') dt'\right] ,$$

$$\frac{\delta F[Q]}{\delta Q(t)} = f(t) \exp\left[\int_{t_a}^{t_b} Q(t') f(t')\, dt'\right]$$

Problem 2 (Feynman Problem 7-1).

If $S[x(t)] = \int_{t_a}^{t_b} L(\dot{x}, x, t) dt$, show that, for any s inside the range t_a to t_b,

$$\frac{\delta S}{\delta x(s)} = -\frac{d}{ds}\left(\frac{\partial L}{\partial \dot{x}}\right) + \frac{\partial L}{\partial x}$$

where the partial derivatives are evaluated at $t = s$.

Problem 3.
Consider a particle of mass m constrained to move along a 1-D axis under the action of a given force $F(t)$. The Lagrangian is $L = \frac{m}{2}\dot{q}^2 + qF(t)$. The action is defined as the functional

$$S[q(t), F(t)] = \int_{t_1}^{t_2} dt L(t, q, \dot{q}, F)$$

The motion is such that $q(t_1) = q_1$ and $q(t_2) = q_2$.

a) Show the general result

$$\frac{\delta S[q,F]}{\delta F(t)} = q(t)$$

b) Verify that the solution for this motion with the given end point conditions above is given by the expression

$$q_{cl}(t) = \frac{(t_2-t)q_1 + (t-t_1)q_2}{t_2-t_1} + \int_{t_1}^{t_2} dt' G(t,t') \frac{F(t')}{m} \quad ,$$

where $G(t,t')$ is the Green's function for this problem and is defined by

$$\frac{d^2}{dt^2} G(t,t') = \delta(t-t') \; ,$$

with $\quad G(t_1,t') = G(t_2,t') = 0 \;\; , \qquad t_1 < t' < t_2$

c) The classical action is:

$$S_{cl} = \frac{m}{2}\frac{(q_2-q_1)^2}{t_2-t_1} + \int_{t_1}^{t_2} dt \frac{(t_2-t)q_1 + (t-t_1)q_2}{t_2-t_1} F(t) + \frac{1}{2m}\int_{t_1}^{t_2} dt \int_{t_1}^{t_2} dt' F(t) G(t,t') F(t')$$

(You don't need to evaluate this result for now)

(i) Obtain $q_{cl}(t)$ by using the general result obtained in part (a) above

(ii) Evaluate the second functional derivative

$$\frac{\delta^2 S_{cl}}{\delta F(t) \delta F(t')}$$