Physics CS 140 **Set # 3** Winter 2014
(For T January 28, 5:00 PM)

Read Feynman "Quantum Mechanics and Path Integrals": Ch.2. and Ch. 3 pp. 44-46, Sect. 3-4. Ch.5, Sect. 5-1, pp. 96-102

Problem 1 (Feynman Problem 3-2).
Show by substitution that the free-particle kernel $K_0(b,a)$ satisfies the differential equation

$$\frac{\partial K_0(b,a)}{\partial t_b} = -\frac{i}{\hbar}\left[-\frac{\hbar^2}{2m}\frac{\partial^2 K_0(b,a)}{\partial x_b^2}\right]$$

whenever t_b is greater than t_a .

Problem 2
a) Show that a representation for the Dirac δ-function is given by

$$\delta(x) = \lim_{\lambda\to\infty}\left(\frac{\lambda}{i\pi}\right)^{1/2} e^{i\lambda x^2}$$

where $\lambda > 0$

b) Use the result from part (a) and show that for the kernel of a free particle

$$K_0(x_b, t_a; x_a, t_a) = \delta(x_b - x_a)$$

Problem 3
Consider the normalized Gaussian wave-packet at $t = 0$ describing a free electron of mass m

$$\psi_0(x) = \sqrt[4]{\frac{2}{\pi l^2}}e^{-x^2/l^2}$$

a) Apply to this wave function the Feynman propagator with $\hbar = 1$ and show

$$\psi(x,t) = \left(\frac{2}{\pi}\right)^{1/4}\frac{1}{l(t)}\sqrt{l\left(1-\frac{2it}{ml^2}\right)}e^{-\left(1-2it/ml^2\right)\frac{x^2}{l^2(t)}}$$

where,

$$l(t) = \sqrt{l^2 + \frac{4t^2}{m^2 l^2}}$$

b) Show that the probability density for finding the electron at x at time t is

$$P(x) = \sqrt{\frac{2}{\pi}}\frac{1}{l(t)}e^{-2x^2/\,l^2(t)}$$

Describe the motion and shape of the wavepacked as it evolves in time.

An alternative method to obtain the free particle kernel :-)

Problem 4 (Feynman Problem 3-7).

The free particle kernel was shown to be of the form

$$K(b,a) = F(t_a,t_b)\exp\left\{\frac{im(x_b-x_a)^2}{2\hbar(t_b-t_a)}\right\}$$

Further information about this function F can be obtained from the property expressed by $K(b,a) = \int_{-\infty}^{\infty} K(b,c)K(c,a)dx_c$. First notice that the expression above for the free kernel imply that $F(t_b-t_a)$ can be written as $F(t)$,where t is the time interval t_b-t_a. By using this form for F in the expression above for the kernel and substituting into $K(b,a) = \int_{-\infty}^{\infty} K(b,c)K(c,a)dx_c$, express $F(t+s)$ in terms of $F(t)$ and $F(s)$, where $t = t_c - t_a$ and $s = t_a - t_c$. Show that if F is written as

$$F(t) = \left(\frac{m}{2\pi i\hbar t}\right)^{1/2} f(t)$$

the new function $f(t)$ must satisfy

$$f(t+s) = f(t)f(s)$$

This means that $f(t)$ must be of the form

$$f(t) = e^{\alpha t}$$

where a may be complex, that is, $a = \alpha + i\beta$. It is difficult to obtain more information about the function $f(t)$ from the principles we have laid down *(in Feynman's book)*. However, the especial choice of the normalizing factor A defined in Eq.(2.21): $A = \left(\frac{2\pi i\hbar\epsilon}{m}\right)^{1/2}$ implies that $f(\epsilon) = 1$ to first order in ϵ. This corresponds to setting α in $f(t) = e^{\alpha t}$ above equal to 0. The resulting value of $F(t)$ is in agreement with

$$K(b,a) = \left(\frac{m}{2\pi i\hbar(t_b-t_a)}\right)^{1/2} \exp\left\{\frac{im(x_b-x_a)^2}{2\hbar(t_b-t_a)}\right\}$$