Physics CS 140 **Set # 4** Spring 2014

(For T February 4, 5:00 PM)

Read Feynman ”Quantum Mechanics and Path Integrals”: Ch.3, Sects. 3-1 through 3-5.

Problem 1 (Feynman Problem 3-5).
In Set 3 you obtained that the free particle kernel satisfies the equation

$$\frac{\partial K_0(b,a)}{\partial t_b} = -\frac{i}{\hbar}\left[-\frac{\hbar^2}{2m}\frac{\partial^2 K_0(b,a)}{\partial x_b^2}\right]$$

Use this result and the equation

$$\psi(x_b, t_b) = \int_{-\infty}^{\infty} K(x_b, t_b; x_c, t_c)\psi(x_c, t_c)dx_c$$

to show that the wavefunction of a free particle satisfies the equation

$$\frac{\partial \psi}{\partial t} = -\frac{i}{\hbar}\left[-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2}\right]$$

which is the Schröndinger equation for a free particle.

Problem 2 (The Gaussian slit. See Fig.1 next page)
Consider the amplitude for a particle that starts at $x = 0$ at $t = 0$, passes through a slit of width $2b$, at time T, and at time t' later reaches a point on a screen a distance x' from the center of the slit. That is, eqn.(3.19) in the text, which includes the sum over all paths passing through the slit that connect the origin to the point x' on the screen. All the distances are measured on the x-axis.

$$\psi(x') = \int_{-b}^{b} K(X + x', T + t'; X + y, T)K(X + y, T; 0, 0)dy$$

where K is the free particle propagator and X is the position of the center of the slit.
Next introduce a Gaussian slit in the integrand above via the factor $G(y) = e^{-y^2/2b^2}$, with y being the distance from the center of the slit to a point in the slit.

a) Obtain the expression for the amplitude

$$\psi(x') = \sqrt{\frac{m}{2\pi i\hbar}}\left(T + t' + it'T\frac{\hbar}{mb^2}\right)^{-1/2} \times \exp\left\{\frac{im}{2\hbar}\left(\frac{x'^2}{t'} + V^2 T\right) + \frac{(m^2/2\hbar^2 t'^2)(x' - Vt')^2}{(im/\hbar)(1/t' + 1/T) - 1/b^2}\right\}$$

where $V = X/T$ is the classical velocity to get from the origin of the x-axis to the center of the slit.

b) Show that the probability for the particle to reach x' on the screen is

$$P(x') = \frac{m}{2\pi\hbar}\frac{b}{T\Delta x}\exp\left\{-\frac{(x'-Vt')^2}{(\Delta x)^2}\right\}$$

where we have used the substitution

$$(\Delta x)^2 = b^2\left(1+\frac{t'}{T}\right)^2 + \frac{\hbar^2 t'^2}{m^2 b^2}$$

Interpret this result.

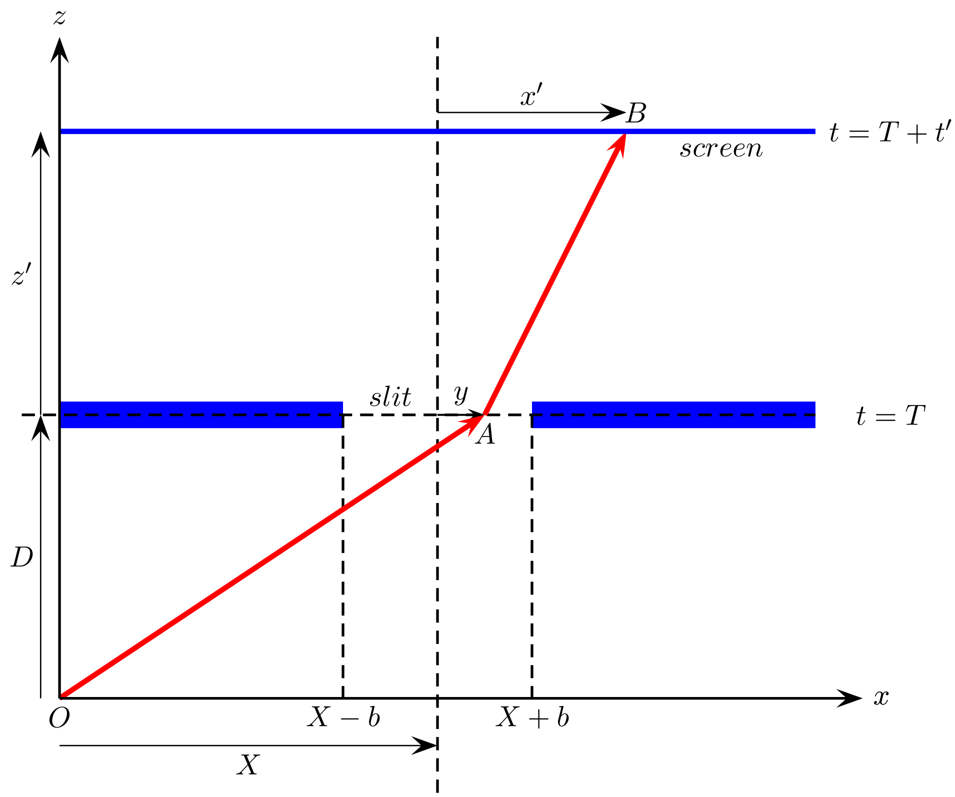

Fig.1

Problem 3

Consider the infinite wall

$$V(x) = \begin{cases} 0\,, & x > 0 \\ \infty\,, & x < 0 \end{cases}$$

A particle has a wave function at $t = 0$ given by

$$\psi(x,0) = e^{ip_0 x/\hbar}\delta(x - x_0)$$

with $x_0 > 0$.

Apply to $\psi(x,0)$ the propagator derived in lecture 7 and obtain the following form for the wave-function at $t > 0$

$$\psi(x,t) = A(x,t;\lambda,V)e^{i\lambda(x_0+Vt)^2}\sin(2\lambda x x_0)$$

Find the constant λ, the velocity V, and the factor $A(x,t;\lambda,V)$. These are the only variables and constants, including x_0, that should appear in $\psi(x,t)$.

Problem 4

A particle of mass m is in an infinite square well of width L

$$V(x) = \begin{cases} 0 & 0 < x < L \\ \infty & x < 0, \;\; x > L \end{cases}$$

The particle can go from x_i to x_f inside the well either directly or bouncing off one wall and the other wall, once, twice, etc. Show that the sum over all paths for this case yields the kernel

$$K(x_f,t_f;x_i,t_i) = \sum_{n=-\infty}^{\infty}\sqrt{\frac{m}{2\pi i\hbar(t_f-t_i)}}\left\{\exp\left[\frac{i}{\hbar}\frac{m}{2}\frac{(2nL+x_f-x_i)^2}{t_f-t_i}\right] - \exp\left[\frac{i}{\hbar}\frac{m}{2}\frac{(2nL-x_f-x_i)^2}{t_f-t_i}\right]\right\}$$