Physics CS 140 **Set # 6** Winter 2014

(For T February 18, 5:00 PM)

Read Feynman "Quantum Mechanics and Path Integrals": Ch.3, Sects. 3-1 through 3-5 with special attention to 3-2 and 3-3. Ch. 4, Sects. 4-1 and 4-2

Problem 1 *Electron going through two slits.*

Consider the geometry shown in the figure below for a two-slit interference with an electron. The electron starts from the origin at $t = 0$, passes either one of the very narrow slits at time T, and then hits the screen at time $T + t'$. The distance from the origin to the slits is D. The distance from the slits to the screen is Z. The electron hits the screen at a distance x' from the z-axis.

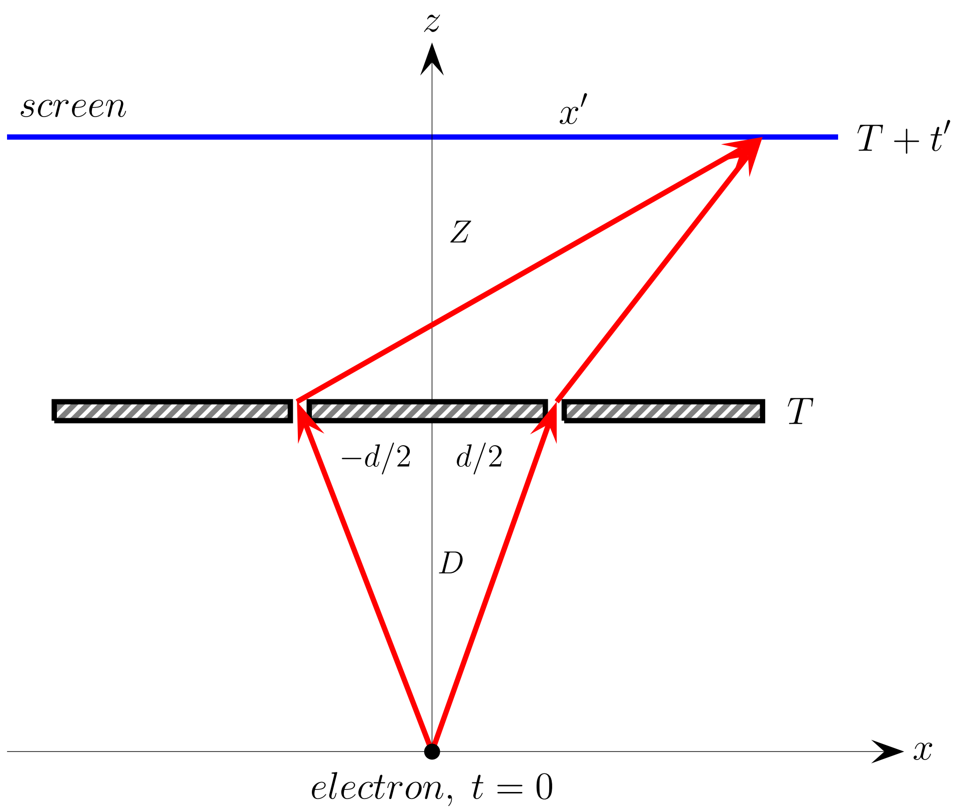

Fig.1. The two-slit experiment with electrons

Show that the relative probability distribution for locating the electron a distance x' on the screen is

$$P(x') = \frac{m^4}{(2\pi)^4 \hbar^4 T^2} \frac{2}{t'^2} \cos^2\left(\frac{md}{2\hbar t'} x'\right)$$

Sketch the probability distribution at times $t_1 < t_2 < t_3$.

Problem 2

A particle of mass m is trapped in an infinite square well and hits the walls and bounces off the walls and moves freely inside the well. The potential is

$$V(x) = \begin{cases} 0 & 0 < x < L \\ \infty & x < 0\,,\ x > L \end{cases}$$

a) Construct the propagator using the eigenfunctions and eigenvalues of H:

$$H\phi_n(x) = E_n\phi_n(x)$$

Compare your answer with the result of problem 3 in set #5.

b) The particle is at $x = L/2$ at $t = 0$. Find the wave function at times $t > 0$. Interprete the result.

Problem 3

Consider a particle of mass m hitting and bouncing off an infinite barrier and otherwise moving freely in the region $x > 0$:

$$V(x) = \begin{cases} 0 & 0 < x \\ \infty & x < 0 \end{cases}$$

Consider the set of wave functions parametrized by the wavenumber k

$$\phi_E(x) = \begin{cases} \phi_k(x) = \sqrt{\dfrac{2m}{\pi k\hbar^2}}\sin kx & x > 0 \\ 0 \quad x < 0 \end{cases}$$

where $E = k^2\hbar^2/2m$

a) Show that they satisfy the energy eigenvalue equation $H\phi_E(x) = E\phi_E(x)$ with the correct boundary condition.

b) Show that they satisfy the normalization

$$\int_{-\infty}^{\infty} \phi_E^*(x)\phi_{E'}(x)dx = \delta(E - E')$$

c) Construct the propagator using the the sum over products of eigenfunctions. Check that your answer agrees with the result obtained in the lecture when the sum over paths was done:

$$K(x,t;x',0) = \sqrt{\frac{m}{2\pi i\hbar t}}\left[e^{i\frac{m}{2\hbar t}(x-x')^2} - e^{i\frac{m}{2\hbar t}(x+x')^2}\right]$$

d) At $t = 0$ the particle is at the point $x = x_o > 0$. Find $\psi(x,t)$ and interpret the result.