Physics CS 140 **Set # 7** Winter 2014

(For T February 25 5:00 PM)

Read Feynman "Quantum Mechanics and Path Integrals": Ch.3, 3-2 and 3-3, and 3-11. Ch. 4

Problem 1 *Eigenfunctions and energy levels for the SHO.*

The propagator for the SHO was obtained earlier in class:

$$K(x_b, T; x_a, 0) = \sqrt{\frac{m\omega}{2\pi i\hbar \sin\omega T}} \exp\left\{\frac{iM\omega}{2\hbar \sin\omega T}\left[\left(x_a^2 + x_b^2\right)\cos\omega T - 2x_b x_b\right]\right\}$$

Use the summation formula for Hermite polynomials (Mehler's Formula), see Morse & Feshbach, *Methods of Theoretical Physics*, Vol. I, p.781.

$$\frac{1}{\sqrt{1-a^2}} \exp\left\{-\frac{1}{2(1-a^2)}\left[(x^2 + x'^2)(1+a^2) - 4xx'a\right]\right\}$$
$$= \exp\left(-x^2/2 - x'^2/2\right) \sum_{n=0}^{\infty} \frac{a^n}{2^n n!} H_n(x) H_n(x')$$

and obtain the normalized wave functions and energy levels for the SHO

$$\psi_n(x) = \left(\frac{1}{2^n n! \sqrt{\pi}}\right)^{1/2} \lambda^{-1/2} e^{-x^2/2\lambda^2} H_n(x/\lambda)$$

$$E_n = \hbar\omega\left(n + \frac{1}{2}\right)$$

with

$$\lambda = \sqrt{\frac{\hbar}{M\omega}}$$

Note: In problems 2 and 3 below use the formula derived in class for the amplitude factor $A(t_b, t_a)$ for 1-D systems where the Lagrangian is $L = \frac{m}{2}\dot{x}^2 - \frac{m}{2}W(t)x^2$

$$A(t_b, t_a) = \int_o^o \delta[y(t)] \exp\left\{\frac{i}{\hbar} S[y(t)]\right\} = \sqrt{m/2\pi i\hbar f(t_a) f(t_a) \int_{t_a}^{t_b} \frac{dt}{f^2(t)}}$$

where f is an arbitrary function with the property:

$$\left[\frac{d^2}{dt^2} + W(t)\right] f(t) = 0\ , \quad f(t_a) \neq 0$$

Problem 2 (Feynman & Hibbs Problem 3-8)

For a harmonic oscillator the Lagrangian is

$$L = \frac{m}{2}\dot{x}^2 - \frac{m\omega^2}{2}x^2$$

Show that the resulting kernel is

$$K = \left(\frac{m\omega}{2\pi i\hbar \sin\omega T}\right)^{1/2} \exp\left\{\frac{im\omega}{2\sin\omega T}\left[\left(x_b^2 - x_a^2\right)\cos\omega T - 2x_b x_a\right]\right\}$$

Problem 3 (Feynman & Hibbs Problem 3-9)

Find the kernel for a particle in a constant external field f where the Lagrangian is

$$L = \frac{m}{2}\dot{x}^2 + fx$$

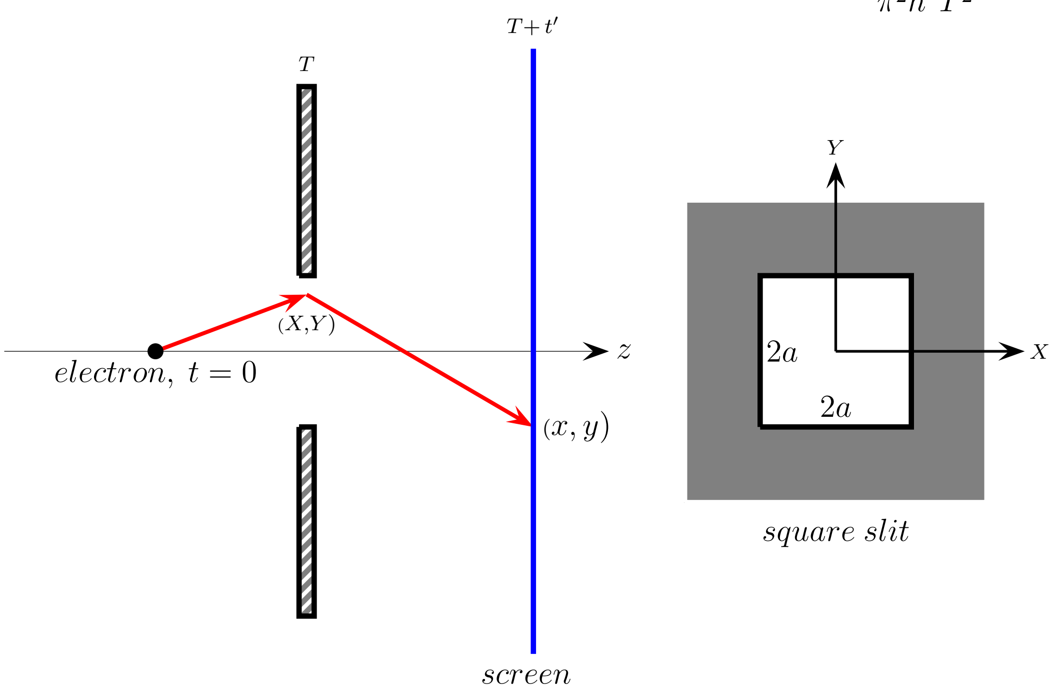
The result is

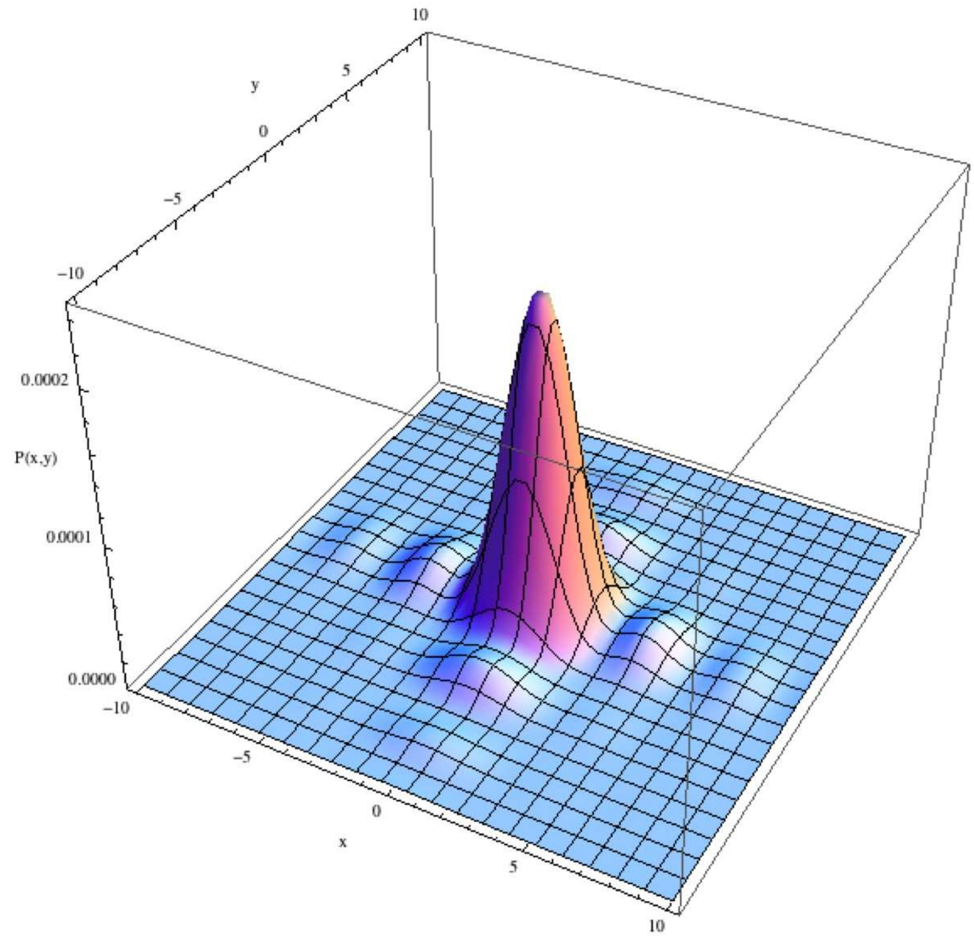
$$K = \left(\frac{m}{2\pi i\hbar T}\right)^{1/2} \exp\left\{\frac{i}{\hbar}\left[\frac{m(x_a - x_b)^2}{2T} + \frac{fT\left(x_b + x_a 2\right)}{2} - \frac{f^2T^3}{24m}\right]\right\}$$

Problem 4

An electron starts at the origin at time $t = 0$ and passes though a square slit of side $2a$ at the generic point (X, Y) at time T and it reaches the screen at the generic point (x, y) at time $T + t$. See figure below. The $z-$axis is perpendicular to the plane of the slit and the screen and passes through the center of the square slit. Use 2-D propagators in the variables (X, Y) and (x, y) and show that the probability per unit surface to go though the slit is equal to the probability to arrive at the slit

$$P(prob.\ to\ arrive\ at\ slit) = P(prob.\ to\ go\ through\ slit) = \frac{a^2m^2}{\pi^2\hbar^2T^2}$$





Note: If you plot the probability distribution $P(x, y)$ you should get something analogous to diffraction by a square slit for the probability per unit area for the electron to arrive at the screen.