**Physics CS 140** **Set # 8** Winter 2014

(For T March 4, 5:00 PM)

**Read Feynman "Quantum Mechanics and Path Integrals":** Ch. 3 Sects. 3-6 and 3-11. Read the class notes for the weeks of T February 18, and T February 25

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**Problem 1** (Feynman & Hibbs Problem 3-11)

Suppose the harmonic oscillator of Prob. 3-8 is driven by an external force $f(t)$. The Lagrangian is

$$L = \frac{m}{2}\dot{x}^2 - \frac{m\omega^2}{2}x^2 + f(t)x$$

The classical action was obtained in class:

$$S_{cl} = \frac{m\omega}{2\sin\omega(t_b - t_a)}[(x_a^2 + x_b^2)\cos\omega(t_b - t_a) - 2x_a x_b]$$

$$+\int_{t_a}^{t_b} dt \frac{x_b \sin\omega(t - t_a) - x_a \sin\omega(t - t_b)}{\sin\omega(t_b - t_a)} f(t) + \frac{1}{2m}\int_{t_a}^{t_b} dt \int_{t_a}^{t_b} dt' f(t) G(t,t') f(t')$$

where, $G(t,t')$ is the Green's function for the SHO satisfying the end point conditions $G(t_a, t') = G(t_b, t') = 0$

$$G(t,t') = \frac{\cos\omega(t_b - t_a - |t - t'|) - \cos\omega(t_a + t_b - t - t')}{2\omega\sin\omega(t_b - t_a)}$$

Show that the resulting kernel can be written, with $T = t_b - t_a$

$$K = \left(\frac{m\omega}{2\pi i\hbar\sin\omega T}\right)^{1/2} \exp\left\{\frac{im\omega}{2\hbar\sin\omega T}\left[(x_a^2 + x_b^2)\cos\omega T - 2x_a x_b\right.\right.$$
$$+\frac{2x_b}{m\omega}\int_{t_a}^{t_b} f(t)\sin\omega(t - t_a)dt + \frac{2x_a}{m\omega}\int_{t_a}^{t_b} f(t)\sin\omega(t_b - t)dt$$
$$\left.\left.-\frac{2}{m^2\omega^2}\int_{t_a}^{t_b}\int_{t_a}^{t} f(t)f(s)\sin\omega(t_b - t)\sin\omega(s - t_a)dsdt\right]\right\}$$

**Problem 2**

Suppose a free particle of mass $m$ is acted on by an external force $F(t)$. The Lagrangian is

$$L = \frac{m}{2}\dot{q}^2 + qF(t)$$

The Green's function for this problem is determined by the equation

$$\frac{d^2}{dt^2}G(t,t') = \delta(t-t')$$

with the end point conditions $G(t_1,t') = G(t_2,t') = 0$.

a) Derive the classical action for this system

$$S_{cl} = \frac{m}{2}\frac{(q_2-q_1)^2}{t_2-t_1} + \int_{t_1}^{t_2} dt \frac{(t_2-t)q_1 + (t-t_1)q_2}{t_2-t_1}F(t) + \frac{1}{2m}\int_{t_1}^{t_2} dt \int_{t_1}^{t_2} dt' F(t)G(t,t')F(t')$$

where thr Green's function is

$$G(t,t') = -\frac{1}{t_2-t_1}[(t_2-t)(t'-t_1)\theta(t-t') + (t_2-t')(t-t_1)\theta(t'-t)]$$

b) Obtain the Feynman propagator for this system.

**Problem 3**

Suppose that the Lagrangian of a particle of mass $m$ is

$$L = \frac{m}{2}\dot{x}^2 - 2\frac{mt}{T^2}x\dot{x} + \frac{m}{T^2}x^2 + xF(t) + A\cos\omega t$$

where $T = 2\pi/\omega$, and $f(t)$ is an external force. Show that the relative probability for the transition $(x_a,t_a)$ to $(x_b,t_b)$ wih $t_b > t_a$ is

$$P(b,a) = \frac{m}{\pi\hbar T\sinh[2(t_b-t_a)/T]}$$