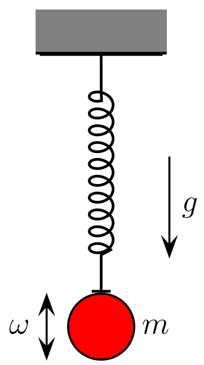
Physics CS 140 **Set # 9** Winter 2014
(For T March 11, 5:00 PM)

Read Feynman "Quantum Mechanics and Path Integrals": Read Feynman Ch 6, Sects 6-1 through 6-3. Read the class notes for the weeks of T February 25 through Th March 6

Problem 1

A particle of mass m is hanging vertically from a spring of natural frequency ω. The potential energy is



$$V(x) = V_0 - mgx + \frac{m}{2}\omega^2 x^2$$

where V_0 is a constant. Find the probability amplitude for the particle starting at x_0 and ending up at x at time T. That is,

$$\langle x, T | x_0, 0 \rangle$$

Problem 2

a) Evaluate the matrix element of the position operator $\hat{x}(t)$

$$\langle x_2, t_2 \,|\, \hat{x}(t) \,|\, x_1, t_1 \rangle$$

for a simple harmonic oscillator of mass m and frequency ω with

$$L = \frac{m}{2}[\dot{x}^2 - \omega^2 x^2]$$

b) Evaluate the matrix element of the time ordered product for the position operators $\hat{x}(t)$ and $\hat{x}(t')$

$$\langle x_2, t_2 \,|\, T(\hat{x}(t)\hat{x}(t')) \,|\, x_1, t_1 \rangle$$

for a free particle of mass m with

$$L = \frac{m}{2}\dot{x}^2$$

Problem 3

Consider a system such that at the initial time t_a the system is in a state described by the wave function $\psi(x_a, t_a)$. At a later time t_b the original state will develop into the state $\phi(x_b, t_b)$. The transition amplitude that this system is found to be in the specific state described by the wave function $\chi(x_b, t_b)$ at the time t_b is given by

$$\langle \chi\, t_b \,|\, \phi\, t_b \rangle$$

and the transition probability for this process is

$$|\langle \chi\, t_b \,|\, \phi\, t_b \rangle|^2$$

where $|\phi t_b\rangle$ is the ket whose wave function is $\phi(x_b, t_b)$ and $|\chi t_b\rangle$ is the ket whose wave function is $\chi(x_b, t_b)$.

a) Show that the resulting amplitude, whose absolute value square gives the probability desired, is given by the expression

$$\langle \chi\, t_b \,|\, \psi\, t_a \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \chi^*(x_b, t_b) K(b, a) \psi(x_a, t_a) dx_a dx_b$$

where $K(b, a)$ is the propagation kernel for the system. This integral is called the *transition amplitude* to go from the state $\psi(x_a, t_a)$ to $\chi(x_b, t_b)$.

b) Assume that a simple harmonic oscillator of mass m has a sharp value x_0 of the position at $t = 0$. Calculate the transition probability that at time $0 < t < \frac{\pi}{\omega}$ the oscillator will be in the ground state $\psi_0(x, t)$.

Problem 4 (Feynman and Hibbs Problem 6-4)

using arguments similar to those leading to Eq. (6.19), show that the wave function $\psi(b)$ satisfies the integral equation

$$\psi(b) = \phi(b) - \frac{i}{\hbar} \int K_0(b, c) V \psi(c) d\tau_c$$