Set #7 - for Wednesday February 26

Read RHK Ch. 14
K&K Ch. 9: 9.1-9.3

Solve

From RHK **Ch. 14** Exercises 2, 10, 11, 15, 20, 23; Problems 2, 11, 31
From K&K **Ch. 9** Problem 9.1

Problem 1.
(a) Show that in a chute through the earth along a chord line, rather than along a diameter, the motion of an object will be simple harmonic; assume a uniform Earth density.
(b) Find the period.
(c) Will the object attain the same maximum speed along a chord as it does along a diameter?

Problem 2. Consider the action functional:

$$S[x(t)] = \int_0^T \left[\left(\frac{dx(t)}{dt} \right)^2 + x^2(t) \right] dt$$

Find the curve $x(t)$ satisfying the conditions $x(0) = 0$, $x(T) = \sinh T$, which makes $S[x(t)]$ an extremum. What is the extremum value of $S[x(t)]$? Is it a maximum or a minimum?

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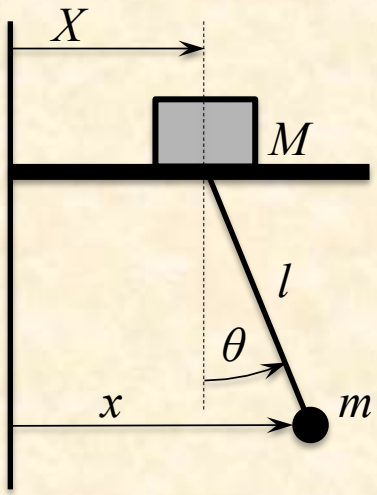
Problem 3. A mass M is free to slide on a frictionless air track, as shown below. Suspended by a pivot and a very light rod of length l is another mass m (point mass).



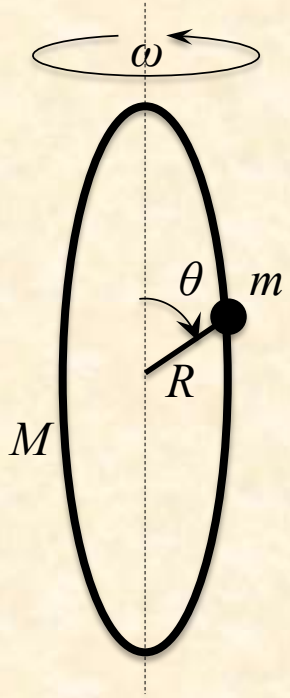
a) Construct a Lagrangian for this system.

b) Write down the differential equations of motion for this system. (There must be one differential equation for each generalized coordinate.)

c) Find the frequency of small oscillations for the mass m.

d) Is the total energy of this system conserved? If so, write down an equation for energy conservation in terms of the generalized coordinates.

Problem 4. A bead of mass m is constrained to move without friction on a circular ring of mass M which is rotating at a constant angular velocity ω about the vertical diameter as shown. The radius of the ring is R.



a) Write down a suitable Lagrangian for this system.

b) Write down the differential equation for the motion of the bead.

c) Show that if $\omega^2 < g/R$, the bead can have two equilibrium positions. Show that if $\omega^2 > g/R$, the bead has three equilibrium positions.

Good Luck, Dr. B