Set #1 - for Wednesday April 9

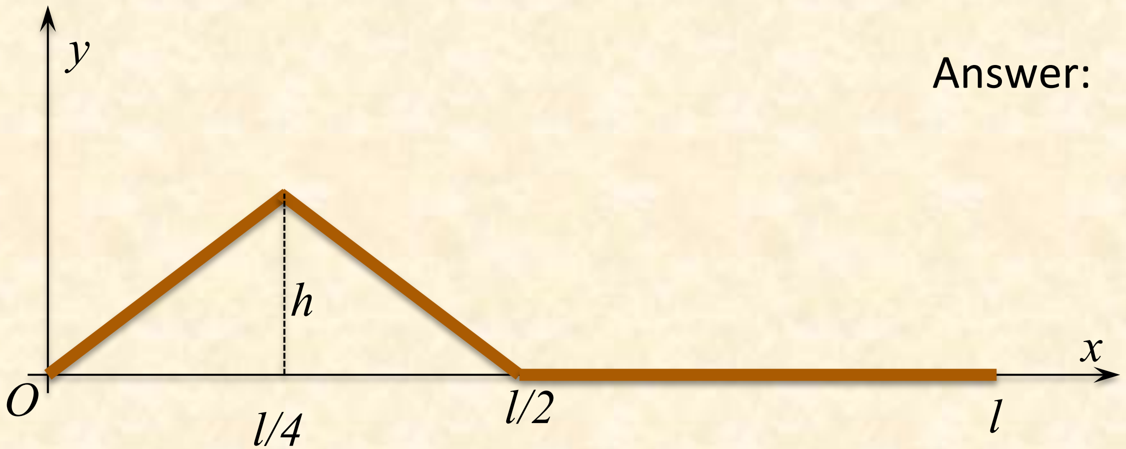
Read RHK Ch. 18; next will be Ch. 19: 19.1-19.6

Solve

From RHK **Ch. 18** Exercise 29 Problems 11, 13, 16, 19, 22

From K&K **Ch. 5.2** Problem 5.2

Problem 1. At $t = 0$, a string of length l and fixed endpoints has zero initial velocity and a displacement $y(x, 0)$ as shown. (This initial displacement might be caused by stopping the string at the center and plucking half of it.) Find the displacement as a function of x and t as a sum of normal modes.



Answer: $$y(x,t) = \frac{8h}{\pi^2}\sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi x}{l}\right)\cos\left(\frac{n\pi vt}{l}\right),$$

$$A_n = \frac{2\sin\left(\frac{n\pi}{4}\right) - \sin\left(\frac{n\pi}{2}\right)}{n^2}$$

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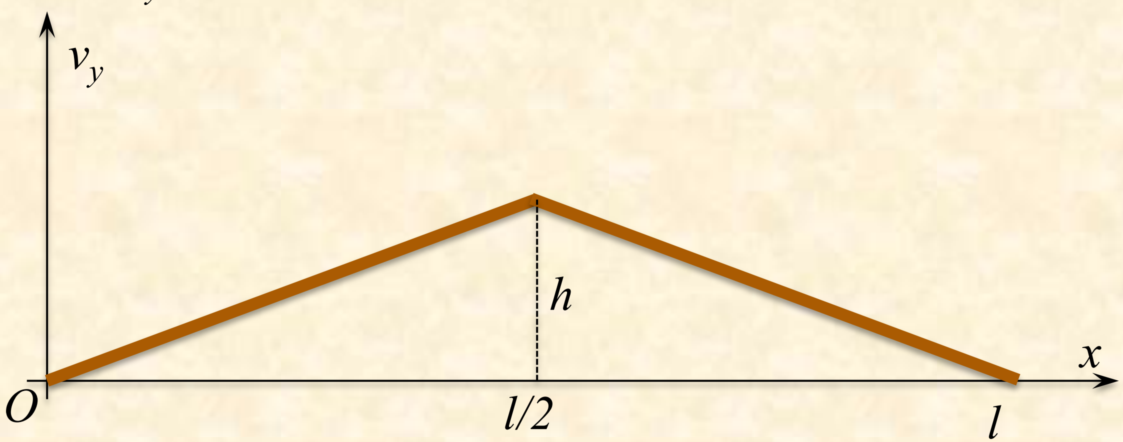
Problem 2. A string of length l is initially stretched straight; its ends are fixed. At time $t = 0$, its points are given the transverse velocity $v_y(x)$ as indicated in the diagram (for example, by hitting the string).

(a) Determine the shape of the string at time t. That is, find the displacement y as a function of x and t in the form of an infinite sum of **normal modes**.

(b) Plot $v_y(x,0)$ using the first three terms in the expansion above.



Problem 3. Prove the orthogonality relations used when you solved the wave equation in terms of standing waves, or normal modes:

(a) $$\int_0^l \sin\left(\frac{n\pi x}{l}\right)\sin\left(\frac{m\pi x}{l}\right)dx = \frac{l}{2}\delta_{nm}, \qquad n,m \in N$$

(b) $$\int_0^l e^{\frac{2in\pi x}{l}} e^{-\frac{2im\pi x}{l}} dx = l\delta_{nm}, \qquad n,m \in N$$

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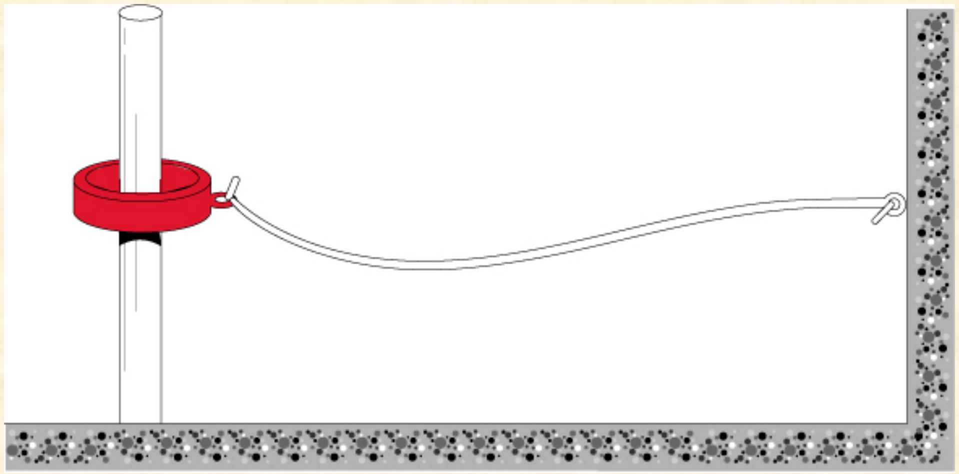
Problem 4. Consider standing waves in a string of length L. One of the ends is fixed and the other is free, that is, there is no vertical force at the free end. This is realized in practice by having one end of the string attached to a small ring that can move up and down a frictionless vertical stick. At a free end, the slope of the string must be zero. Why?

(a) Find a formula for the frequency of the standing waves in terms of L, the mass of the string m and the tension F_{T}.

(b) Write the expression for the normal modes of vibration representing standing waves.



Good Luck, Dr. B