Set #2 - for Wednesday April 16

Read RHK Ch. 19 (Tuesday lecture will help you to solve problems marked red)

Solve

From RHK **Ch. 19** Exercises 2, 9, 24, 27 Problems 3, 5, 12

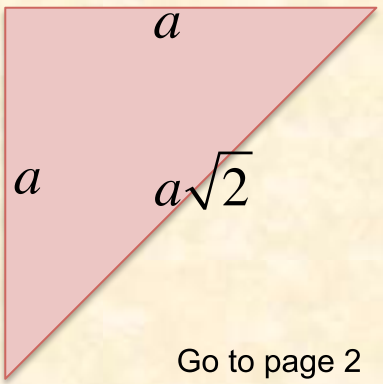
From K&K **Ch. 5** Problem 5.8

Problem 1. Consider a square drumhead vibrating in the degenerate (12)-(21) mode with frequency ω_{12} (= ω_{21}). Assume that this vibration mode has the form

$$z(x,y,t) = \left[A\sin\left(\frac{2\pi x}{a}\right)\sin\left(\frac{\pi y}{a}\right) + B\sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{2\pi y}{a}\right)\right]\cos(\omega_{12}t)$$

Find the nodal lines (lines of zero displacement: $z = 0$) for the four special cases $A = 0$; $B = 0$; $A - B = 0$; $A + B = 0$. The length of the side of the square is a. That is, for each case you set $z = 0$ and see what relationship you get for x and y. Plot result on x-y plane These are the nodal lines.

Problem 2. Find the lowest frequency of a drumhead in the shape of an isosceles triangle with sides $a,\ a,\ a\sqrt{2}$



Go to page 2

Set #2 - for Wednesday April 16

Problem 3. Use index notations to show the results

$$(a)\ \vec{\nabla} A e^{i(\vec{k}\cdot\vec{r}-\omega t)} = i\vec{k} A e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$(b)\ \vec{\nabla}\cdot\vec{A} e^{i(\vec{k}\cdot\vec{r}-\omega t)} = i\left(\vec{k}\cdot\vec{A}\right) e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$(c)\ \vec{\nabla}\times\vec{A} e^{i(\vec{k}\cdot\vec{r}-\omega t)} = i\left(\vec{k}\times\vec{A}\right) e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$(d)\ \vec{\nabla}^2\vec{A} e^{i(\vec{k}\cdot\vec{r}-\omega t)} = -k^2\vec{A} e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

Problem 4. Verify by direct substitution that $p(\vec{r},t) = Ae^{i(\vec{k}\cdot\vec{r}-\omega t)}$, where A is a constant, is a solution of the 3-D wave equation $\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$, provided $\omega = c\left|\vec{k}\right|$.

Problem 5. Show that for waves with spherical symmetry: $p = p(r, t)$,

(a) The wave equation can be written as

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial p}{\partial r}\right) - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0.$$

(b) Further

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}\left(rp\right) - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0.$$

(c) Show that $p = \frac{A}{r}\sin\left(\vec{k}\cdot\vec{r} - \omega t\right)$ with $\omega = ck$ is a solution of (b) above, and that in general

$$p(r,t) = \frac{1}{r} f\left(kr - \omega t\right) + \frac{1}{r} g\left(kr + \omega t\right)$$

Hint: Let $\psi(r,t) = rp(r,t)$ and solve the wave equation satisfied by $\psi(r,t)$.

Good Luck, Dr. B