Set #3 - for Wednesday April 23

Read RHK Ch. 19, 15

Solve

From RHK **Ch. 19** Exercise 43 Problems 17 (Read RHK p. 442, 443), 19
From RHK **Ch. 15** Problems 1, 3, 7, 8, 12, 13

Problem 1. Show that the intensity of a sound wave (a) when expressed in terms of the pressure amplitude p_0 is given by $I = \frac{p_0^2}{2\rho_0 c}$, where c is the speed of the wave and ρ_0 is the standard density of air, and (b) when expressed in terms of the displacement amplitude ξ_0, is given by $I=2\pi^2\rho_0 c\xi_0^2 f^2$, where f is the frequency of the wave. (c) If two sound waves, one in air and one in water are equal in intensity, what is the ratio of the pressure amplitude of the wave in water to that of the wave in air? (d) If the pressure amplitudes are equal instead, what is the ratio of the intensities of the waves?

Problem 2. As a model of how a radar speed detector operates, consider a transmitter of high frequency f_0 emitting a wave that strikes an automobile moving away from the transmitter with a speed v. The wave is reflected back toward the transmitter and is Doppler-shifted. This reflected wave is mixed with the original transmitted wave and the beat frequency measured. Find an equation giving the beat frequency b in terms of the original frequency f_0, the speed of the wave c, and the speed of the automobile, v.

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Problem 3. Use index notations to show the results

(a) $\nabla \times (\phi \vec{V}) = \phi \nabla \times \vec{V} + \nabla \phi \times \vec{V}$

(b) $\nabla \cdot (\nabla \times \vec{V}) = 0$

(c) $\nabla \times (\nabla \phi) = \vec{0}$

(d) $\nabla \cdot (\nabla \phi) = \nabla^2 \phi$

(e) $\nabla(\nabla \cdot \vec{V}) = \nabla^2 \vec{V} + \nabla \times (\nabla \times \vec{V})$

(f) $\nabla^2 (\nabla \cdot \vec{V}) = \nabla \cdot (\nabla^2 \vec{V})$

Problem 4a Show that the displacement field $\vec{\xi}$ satisfies the vector wave equation:

$$\nabla(\nabla \cdot \vec{\xi}) - \frac{1}{c^2}\frac{\partial^2 \vec{\xi}}{\partial t^2} = 0$$

where $c = \sqrt{\frac{B}{\rho_0}}$ and $\rho_0 = \text{const.}$

Problem 4b-d A sound pressure wave is described by the equation: $p = A\sin(x + y + z - at)$ where A and a are known positive quantities.

4b Find a unit vector in the direction of propagation of this wave.

4c Find the wavelength and the velocity of propagation.

4d Find the displacement wave $\vec{\xi}(\vec{r}, t)$ and verify that it satisfies the wave equation in part (4a) above, and that it does also satisfy:

$$\nabla^2 \vec{\xi} - \frac{1}{c^2}\frac{\partial^2 \vec{\xi}}{\partial t^2} = 0$$

Problem 4e Show that if the displacement field $\vec{\xi}$ of a sound wave is such that $\nabla \times \vec{\xi} = \vec{0}$, then the wave equation satisfied in general by $\vec{\xi}$ that you derived in (4a) can be rewritten as it is done in (4d).

Good Luck, Dr. B