

Read RHK Ch. 15 (including 15.6), 16.1, 16.2

Solve

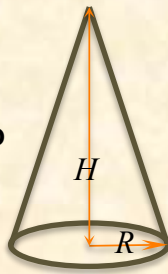
From RHK **Ch. 15** Problems 9, 10, 11, 19

From RHK **Ch. 16** Exercise 5

Problem 1. The thin-walled cone shown below is completely filled with water.

(a) Find the force on the bottom of the cone due to the water.

(b) If you place this cone on the flat support of a spring scale, what does the scale read? Neglect the weights of the wall and the base of the cone. Explain the difference, if any, in the results you got in (a) and (b).



Problem 2. Newtonian Stars: Consider a star of mass M and radius R . Assume the star interior is in a state of hydrostatic equilibrium and the density is uniform.

(a) Show that the gravitational potential is given by

$$\phi(r) = \frac{GM}{2R^3} (r^2 - 3R^2), \quad 0 \leq r \leq R$$

(b) Consider the hydrostatic equation derived in class. Obtain p as a function of r .

Problem 3. Use the subscript notation and show the vector calculus identity:

$$(\vec{v} \cdot \nabla) \vec{v} = \vec{\Omega} \times \vec{v} + \frac{1}{2} \nabla (v^2)$$

Vector $\vec{\Omega}$ is called vorticity.

Problem 4. Calculate vorticity for a bucket of water on a turntable if the table rotates with the angular velocity $\vec{\omega}$. Water rotates together with the bucket with the same angular velocity.

Problem 5. Prove: If $\nabla \cdot \vec{F} = 0$ (solenoidal vector field), then

$$\int_V \vec{F} \cdot \vec{r} dV = \frac{1}{2} \oint_S r^2 \vec{F} d\vec{A}$$

Problem 6. Verify by direct substitution that $p(\vec{r}, t) = f(\phi)$, where f is an arbitrary twice differentiable function of ϕ , and $\phi = \vec{k} \cdot \vec{r} - \omega t$, is a solution of the 3-D wave equation

$$\nabla^2 p - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = 0$$

provide $\omega = c |\vec{k}|$.

Problem 7. Given a sound wave with spherical symmetry,

(a) Show: $\nabla p(\vec{r}, t) = \hat{r} \frac{\partial p(r, t)}{\partial r}$.

(b) Show that the wave is longitudinal. That is, the displacement is $\vec{\xi}(\vec{r}, t) = \hat{r} f(r, t)$, f is a scalar function.