Set #5 - for Wednesday May 7

Read RHK Ch. 16

Solve

From RHK **Ch. 16** Exercises 10, 23 Problems 3, 4, 8, 10, 13, 14
From Tipler **Ch. 2** Exercises 11, 12 (page 86)

Problem 1. We showed in class that the velocity field for a stationary, incompressible, irrotational flow can be described by $\vec{v}(\vec{r}) = \nabla\varphi(\vec{r})$ where $\varphi(\vec{r})$ is a scalar function called the velocity potential. Furthermore $\varphi(\vec{r})$ obeys the equation (Laplace's equation)

$$\nabla^2\varphi(\vec{r}) = 0$$

Consider the different flows described by the velocity potentials shown below:

(a) $\varphi(\vec{r}) = -Ax$

(b) $\varphi(\vec{r}) = -B/r$

(c) $\varphi(\vec{r}) = C/r$

(d) $\varphi(\vec{r}) = -\frac{D}{r} + \frac{E}{\sqrt{(x-x_0)^2 + y^2 + z^2}}$

A, B, C, D, E, x_0 are positive constants and $r = \sqrt{x^2 + y^2 + z^2}$

(A) For each of these flows, do the following:
(i) Check if the given expression $\varphi(\vec{r})$ satisfies Laplace's equation.
(ii) Find the velocity field $\vec{v}(\vec{r})$
(iii) Sketch the velocity field $\vec{v}(\vec{r})$ and briefly describe in words what is happening in this flow.

(**B**) Find the acceleration field for flow (b).

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Problem 2. A velocity field is given by $\vec{v}(\vec{r}) = \langle 0,\ 2z,\ y^2 \rangle$. Obtain equation for the streamline passing through a point $\vec{r} = \langle 0,\ 3,\ 2 \rangle$.

Problem 3. Evaluate the generic Gaussian integral

$$I_n = \int_0^\infty x^n e^{-\lambda x^2} dx$$

Hint 1: For $n = 0$, consider $(I_0)^2$ and rewrite this integral in polar coordinates.
Hint 2: Consider derivatives of I_0 with respect to λ.

Problem 4. Consider an incompressible steady flow with the velocity field parallel to x-axis. Assume that the value of the velocity depends on y-coordinate only. Figure 16-40 (RHK, p 371) is a possible example of this kind of flow. The flow is maintained by the constant pressure gradient in x-direction $\nabla p = \langle -p', 0, 0 \rangle$, where p' is a positive constant. Pressure stays positive in the region of our observation. Viscosity of the fluid is η. Do not take gravity into account.

(a) Write Navier-Stokes' equation in the region.
(b) Obtain general solution of the equation.

Good Luck, Dr. B