

<b>Read</b>	RHK	Ch. 20	Ohanian	Ch. 2, 2.1-2.6
	K&K	Ch. 12	Feynman Vol. I,	Ch. 16, 17

**Solve**

From K&K **Ch. 11, 12** Problems 11.4, 12.5, 12.6, 12.9, 12.11, 12.12, **Extra Credit:** 12.10  
 From Ohanian **Ch. 2** Problems 7, 21, 23

**Problem 1.** An observer  $A$  is in a rocket ship that passes the Earth at a speed of  $0.6c$ . An observer  $B$  on Earth sets her watch so that it reads zero, the same as  $A$ 's watch when the ship passes. If observer  $B$  looks at  $A$ 's watch through a telescope, what time does  $B$  see on  $A$ 's watch when  $B$ 's watch reads 30.0 seconds?

**Problem 2.** (Solve K&K problems 12.11 and 12.12 first.) The twin paradox: Assume that a rocket ship leaves the Earth in the year 2015. One of a set of twins born in 1995 remains on Earth; the other rides in the rocket. The rocket ship is so constructed that it has an acceleration  $g = 9.8\text{m/s}^2$  in its own rest frame. It accelerates in a straight line path for 5 years (by its own clocks), decelerates for 5 years, turns around, accelerates for 5 years, decelerates for 5 years, and lands on Earth. The twin in the rocket ship is 40 years old. What year is it on Earth?

*Hint:* Show that in Earth's frame, the rocket acceleration is:

and integrate.

$$\frac{dv}{dt} = g \left( 1 - \frac{v^2}{c^2} \right)^{\frac{3}{2}}$$

**Problem 3.** All 4-vectors expressed in contravariant components transform under Lorentz transformation like  $x^\mu$ :  $\tilde{a}^0 = \gamma(a^0 - \beta a^1)$ ;  $\tilde{a}^1 = \gamma(a^1 - \beta a^0)$ ;  $\tilde{a}^2 = a^2$ ;  $\tilde{a}^3 = a^3$

- (a) How do covariant components transform under Lorentz transformation? Write expressions.  
 (b) Show that scalar product of two 4-vectors is invariant under the transformation:  $a^\mu b_\mu = \tilde{a}^\mu \tilde{b}_\mu$ .  
 (c) Explain, why components of velocity  $u_x, u_y, u_z$  are not the components of a 4-vector.  
 (d) 4-velocity is defined as  $w^\mu = \frac{dx^\mu}{d\tau}$ . Express all components of 4-velocity in terms of  $u_x, u_y, u_z$ .  
 (e) Differential operator  $\frac{\partial}{\partial x^\mu}$  is an extended version of the “nabla” operator in 3 dimensions.

What do you think is right index notation for the operator  $\frac{\partial}{\partial x^\mu} = \partial^\mu$  or  $\frac{\partial}{\partial x^\mu} = \partial_\mu$ ?

*Hint:* Consider a scalar function  $\varphi(x^\mu)$ . Express scalar  $d\varphi$  in terms of partial derivatives of  $\varphi$  and  $dx^\mu$  and make your conclusion about covariant or contravariant nature of  $\frac{\partial}{\partial x^\mu}$  operator.

**Problem 4.** A baby Barbara was born in Santa Barbara on May 22 at 4:50 pm. Two more babies Spartacus and Timotha were born on the opposite side of the Earth soon after Barbara. These events were recorded using our Pacific Standard Time. Consider Barbara’s event with each of the additional two events.

- (a) Make a numeric example of a possible time when baby Spartacus was born. Your number should correspond to a spacelike interval between the events. Describe the frame of reference where the events are simultaneous.  
 (b) Make a numeric example of a possible time when Baby Timotha was born. Your number should correspond to a timelike interval between the events. Describe the frame of reference where these events are at the same space point.