A SPREADING LAYER ORIGIN FOR DWARF NOVA OSCILLATIONS
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Draft version October 6, 2004

ABSTRACT

Dwarf nova outbursts often show coherent \((Q \sim 10^4 - 10^6)\) sinusoidal oscillations with the largest pulsed fraction in the extreme ultraviolet. Called dwarf nova oscillations (DNOs), they have periods of \(P \sim 3 - 40\) s and scale with luminosity as \(P \propto L^{-\beta}\) with \(\beta \approx 0.1 - 0.2\). We propose that DNOs may be produced by nonradial oscillations in a thin hydrostatic layer of freshly accreted material, the “spreading layer” (SL), at the white dwarf (WD) equator. This would naturally explain a number of key properties of DNOs, including their frequency range, sinusoidal nature, sensitivity to accretion rate, and why they are only seen during outburst. Since their discovery (Warner & Robinson 1972), DNOs have roughly tracked the surface Keplerian period of the accreting white dwarf (WD) \(\left[\frac{P_K}{P}\right] \approx \frac{1}{\beta}\); see Figure 19 of Patterson 1981, or Knigge et al. 1998), so that it is generally believed that the oscillations are created near or on the WD surface.

A number of explanations have been proposed for DNOs, but none have quantitatively explained the majority of their key features. The characteristic period of the DNOs, \(P \sim P_K\), implies a pulsational, rotational, or even perhaps inner-accretion disk origin. Since the period changes on the same timescale as accretion, it is improbable that the entire WD takes part in creating DNOs, ruling out global g-modes or the WD spin modulated by hot spots. This led Paczynski (1978) to suggest that if accretion torques were causing the period drifts then only a small amount of material is involved in making DNOs \((\lesssim 10^{24} \, \text{g})\). Papaloizou & Pringle (1978) studied rotationally modified global g-modes and r-modes on WDs. Since truly global modes are not sensitive to the instantaneous accretion rate, they proposed that DNOs are high radial-order modes that have large amplitudes near the WD surface. Their work explains many of the properties of DNOs, but some questions still remain, including: (1) what physical mechanism favors modes that are strongly concentrated in the outer layers of the star, (2) why the radiating area associated with DNOs is such a small fraction of the total WD surface area \((\sim 10^{-4} - 10^{-2})\) as inferred from the EUV emission of SS Cyg during DN outbursts; Córdova et al. 1980; Mauche & Robinson 2001, hereafter MR01; Mauche 2004), and (3) why the modes are only excited during outbursts. These failings led others to suggest alternatives, including magnetic accretion onto a slipping belt (Warner & Woudt 2002) and non-axisymmetric bulges on the inner part of accretion disks (Popham 1999).

Inogamov & Sunyaev (1999) presented a new way of studying disk accretion close to an accreting neutron star (NS), which Piro & Bildsten (2004; hereafter PB04) extended to the case of WDs. Instead of using a boundary layer model, they follow the latitudinal flow of accreted material over the stellar surface, starting at the equator and streaming up towards the pole, using a spreading layer (SL) model. In the case of WDs the covering fraction (which is equivalent to the spreading angle) is found to be small, \(f \sim 10^{-3} - 10^{-1}\) (for \(M \approx 10^{16} - 10^{18} \, \text{g s}^{-1}\)), so that the size of the freshly accreted layer is set by the thickness of the accretion disk.

The material in the SL is much hotter in temperature and lower in density than the underlying WD. This contrast allows waves in the SL to travel freely, unencumbered by the material below. We propose that DNOs are shallow surface waves in the layer of recently accreted material confined to the WD equator. We argue that the \(m = 0\) mode provides the period which is most often identified as a DNO, and we show that the \(m = -1\) prograde mode explains the occurrence of some of the higher frequency DNOs, including the extra periods seen from SS Cyg and VW Hyi (MR01; Woudt & Warner 2002). We also discuss the \(m = 1\) retrograde mode and whether it also corresponds to an observed frequency.

In §2 we describe a simple model to estimate the shallow surface wave periods and show how they scale with \(M\) and the WD mass and radius. We then compare these periods to surface conditions on WDs.
observed DNOs from CVs §3. We conclude in §4 with a summary of the DNO properties successfully understood using our model along with a discussion of problems that are still left unanswered.

2. NONRADIAL OSCILLATION PERIODS IN THE SPREADING LAYER

Patterson (1981) showed that (at the time) all known DNOs on WDs with measured masses have \( P \gtrsim P_\kappa \). This relation is an important constraint for any explanation of DNOs, so we begin by showing why a SL mode should mimic such a period. The geometry of the SL is a thin, quickly spinning, layer in hydrostatic balance, which is predicted both observationally (Mauche 2004) and theoretically (PB04) to cover a fraction of the surface area \( f \sim 10^{-3} - 10^{-1} \). In situations where there is a large entropy contrast between a surface layer and the underlying material, nonradial oscillations can be confined to high altitude regions with little or no pulsational energy extending deeper into the WD. When the horizontal wavelength is much greater than the layer depth, these shallow surface waves have a frequency

\[
\omega^2 = g_{\text{eff}} h k^2,
\]

where \( g_{\text{eff}} = GM/R^2 - v_\phi^2/R - v_\theta^2/R \approx GM/R^2 - v_\phi^2/R \) is the surface gravitational acceleration decreased by centrifugal effects, \( n = P/(g_{\text{eff}} a) \) is the pressure scale height of the layer, and \( k \) is the transverse wavenumber. The SL can be thought of as a waveguide with latitudinal width \( 2\lambda \) and azimuthal length \( 2\pi \), but since \( f \ll 1/\pi \) the lateral contribution dominates so that \( k \sim 1/(\pi R) \).

Setting \( g_{\text{eff}} = AGM/R^2 \), where \( \lambda \ll 1 \) is a dimensionless parameter that depends on the spin of the layer, we rearrange the terms in equation (1) to find

\[
\omega = \left( \frac{GM}{R^2} \right)^{1/2} \left( \frac{\lambda h}{f R^2} \right)^{1/2},
\]

which shows that the mode’s frequency is the Keplerian frequency times a factor less than unity (as long as \( \lambda h \ll f R^2 \)), and therefore explains why these shallow waves are consistent with the findings of Patterson (1981).

To find how these modes scale with \( M \) and the WD mass we consider a simple model to estimate \( f \) and \( h \). The theoretical studies of PB04 suggest that in the \( M \) range of dwarf novae the covering fraction, \( f \), may be set by the thickness of the accretion disk resulting in (Shakura & Sunyaev 1973; in the limit of gas pressure much greater than radiation pressure and using Kramer’s opacity)

\[
f = 1.8 \times 10^{-2} \alpha_{\text{disk},2}^{-1/10} M_{17}^{1/20} M_i^{-3/8} R_9^{1/8},
\]

where \( \alpha_{\text{disk}} \) is the viscosity parameter for the accretion disk and \( \alpha_{\text{disk},2} \equiv \alpha_{\text{disk}}/10^{-2} \), \( M_{17} \equiv M/(10^{17} \text{ g} \text{s}^{-1}) \), \( M_i \equiv M/M_\odot \), \( R_9 \equiv R/(10^9 \text{ cm}) \), and we set the factor \( [1 - (r/R)^{-1/2}] \approx 1 \).

As long as the WD is rotating at much less than breakup (Gänsicke et al. 2001), half of the accretion luminosity is released in a layer at the WD surface, so that the flux of this layer is given by \( 4\pi f R^2 F = GMM/(2R) \). In a one-zone layer the radiative flux equation can be integrated to give

\[
F = acT_0^3/(3\kappa y),
\]

where \( a \) is the radiation constant, \( \kappa \) is the opacity (set to 0.34 cm\(^2\) g\(^{-1}\)) for Thomson scattering with a solar composition, and \( y \) is the column depth of the layer (measured in units of \( g \text{ cm}^{-2} \)). The flux is the timescale for viscous dissipation in the SL. The viscosity between the fresh, quickly spinning material and the underlying WD is given by \( \nu = \alpha_{\text{SL}} v_{\phi} h \) (PB04), where \( \alpha_{\text{SL}} \) is the SL’s viscosity parameter, and \( v_{\phi} \approx (GM/R)^{1/2} \) is the initial azimuthal velocity of the material in the SL. Using an ideal gas equation of state, we find the lowest order mode, which does not propagate in the azimuthal manner \( m = 0 \), has a period

\[
P_{m=0} = 30 s \alpha_{\text{disk},2}^{-2/15} \alpha_{\text{SL},3}^{-1/6} M_{17}^{2/15} M_i^{-1/3} R_9^{19/12},
\]

where \( \alpha_{\text{SL},3} = \alpha_{\text{SL}}/10^{-3} \) and \( \lambda_1 = 3 \times 10^{-1} \). Equation (4) provides both a scaling and period suggestive of DNOs. We discuss these similarities in more detail when we compare to observations in §3. If we instead take \( \nu = \alpha_{\text{SL}} c_{\kappa} \) (in analogy to a Shakura & Sunyaev accretion disk), where \( c_{\kappa} \) is the speed of sound, we find \( P \propto M^{-13/140} M^{-27/56} R_{89/56} \), also similar to observations. This shows the robustness of this idea in replicating the general scalings of DNOs.

The arguments from above suggest that the SL can contain additional modes, most notably those that propagate in the azimuthal direction. These modes have an observed frequency of \( \omega_{\text{obs}} = |\omega - m \omega_{\text{SL}}| \), where \( \omega_{\text{SL}} \) is the spin of the SL and \( m \) is the azimuthal wavenumber. Since the layer is spinning quickly, it is possible that Coriolis effects will modify \( k \), which has been studied in the limit of a thin layer using the “traditional approximation” (see Bildsten, Ushomirsky & Cutler 1996 and references therein). Using this analysis, Coriolis effects are negligible since \( f \approx \omega/(2\omega_{\text{SL}}) \). If we set \( P_{\text{SL}} = 2\pi/\omega_{\text{SL}} \), where \( P_{\text{SL}} \gtrsim P_\kappa \), the next lowest order modes (still \( n = 1 \)) have periods

\[
\frac{1}{P_{\text{m}=\pm 1}} = \left[ \frac{1}{P_{\text{m}=0}} \mp \frac{1}{P_{\text{SL}}} \right].
\]

Since there are a few DNOs with \( P \lesssim P_\kappa \) (as we discuss in §3), we propose that the prograde \( m = 1 \) mode is also necessary for explaining some short period DNOs. It is less clear whether the \( m = 1 \) retrograde mode is consistent with an oscillation observed during DN, and in §3 we speculate whether this mode may be related to the long period DNOs (lpDNOs; Warner, Woudt & Pretorius 2003).

3. DNOs IN THE CV POPULATION

We compare the most recent compilations of DNO periods (Table 1 of Warner 2004) and WD masses (Ritter & Kolb 2003) of CVs in Figure 1. The period error bars indicate the range of periods that have been seen from each object, while the mass error bars indicate the errors. We plot systems that have shown both high and low DNO periods as two separate points (these CVs are SS Cyg, CN Ori, and VW Hyi). The heavy, dashed line denotes the surface Keplerian period, \( P_\kappa \), for a given WD mass (using the mass-radius relation of Truran & Livio 1986, from private communication with Eggleton). This demonstrates that there are DNOs both above and below \( P_\kappa \), which is why we consider SL nonradial oscillations with \( m = 0 \) and \( m = -1 \). For each mode we present periods for accretion rates \( \dot{M} = 10^{16} - 10^{18} \text{ g} \text{s}^{-1} \), the range expected during a DN outburst. To calculate the \( m = -1 \) mode we must assume a period for the SL’s spin, \( P_{\text{SL}} \), so we plot mode periods for both \( P_{\text{SL}} = P_\kappa \) and \( P_{\text{SL}} = 2P_\kappa \). This shows that our model is insensitive to the choice of the SL spin rate.

Besides predicting multiple classes of DNOs with different period ranges, we also predict that each of these groups should
have a different dependence on $\dot{M}$. This can be seen from the size of each shaded band, which is much wider for the $m = 0$ mode than the $m = -1$ mode. On average the DNOs with $P \gtrsim P_K$ show less variation with $M_\ast$, qualitatively consistent with the $m = -1$ mode of our model.

To investigate the $M$ dependence in more detail we plot our predicted DNO periods versus $M$ for a $M = 1.0 M_\odot$ WD in Figure 2. This illustrates the shallower dependence for the $m = -1$ mode. The separation and relative slopes of the $m = -1$ and $m = 0$ modes are suggestive of the frequency doubling that was seen by MR01 during the 1996 outburst of SS Cyg. In both the beginning and tail of the burst they saw a DNO with $P = 6.59 - 8.23$ s and following a $P \propto L^{-\beta}$ power law with $\beta = 0.097$. Near the burst peak, $P$ suddenly shifted to 2.91 s and then decreased with a shallower power law with $\beta = 0.021$. Indeed, we find similar periods and power laws when we use a larger WD mass.

In Figure 2 we also plot the $m = 1$ retrograde mode for $P_{SL} = (1 - 1.5) P_K$. Due to the difference taken in equation (5), this mode has a much more complicated dependence on $M$, no longer being a power law nor monotonic. This mode has a higher period than typical DNOs, and may be relevant for the lpDNOs. Warner (2004) identified 17 CVs as showing such oscillations. The lpDNOs typically have larger amplitudes than DNOs and, like DNOs, are not seen in every DN outburst. Often both of these oscillations are seen simultaneously. To positively identify these as modes requires checking for the predicted scalings between $P$ and $\dot{M}$. This test may negate a mode explanation for lpDNOs since Warner, Woudt & Pretorius (2003) claim to find no such correlation for these oscillations and thus favor a spin related mechanism. On the other hand, on separate occasions there have been $32 - 36$ s (Robinson & Nather 1979) and $83 - 110$ s (Mauche 2002b) oscillations seen from SS Cyg, in the domain expected for these modes. This wide spread of periods may be explained by the $m = 1$ mode’s steeper dependence on $M$. The $32 - 36$ s oscillations showed much less coherence than typical DNOs, which could be due to the retrograde oscillations beating against the accretion disk. Other CVs such as VW Hya show similar modes in the range of $\sim 90$ s (Haefner, Schoembs & Vogt 1977, 1979), which may be of similar origin.

4. DISCUSSION AND CONCLUSIONS

We propose that DNOs in outbursting CVs are nonradial oscillations in a hot layer of freshly accreted material near the WD equator. A large number of DNO properties are then simply understood: (1) the highly sinusoidal nature of the oscillations is consistent with nonradial oscillations, (2) the periods can change on the timescale of accretion because there is little mass in the layer ($\lesssim 10^{21}$ g; PB04), (3) the periods vary inversely with $M$ because they have the temperature scaling of shallow surface waves, (4) the covering fraction is naturally small for the SL, (5) the DNOs are only seen during DN outbursts because this is the only time when an optically thick layer of material can build up at the equator, and (6) the largest pulsed amplitude is in the EUV, consistent with the SL temperature.

In support of our hypothesis we presented a simple, phenomenologically motivated model that quantitatively explains many of the DNO’s features. We compared our model with the overall population of DNOs, both as a function of WD
mass and accretion rate. In each case it is necessary to consider both the \( m = 0 \) mode and the \( m = -1 \) (prograde) mode to explain the range of periods observed and the different \( P-M \) scalings. The majority of DNOs are consistent with the period of the \( m = 0 \) mode, and this may be related to its latitudinal propagation. The speed of shallow surface waves is \( \approx gh \) so that they slow down as the scale height decreases. This steers an initially azimuthally propagating mode to instead travel perpendicular to the SL edge, similar to ocean waves at the beach, and may explain why \( m = 0 \) modes are favored over \( m \neq 0 \) modes. In the 1996 DN outburst of SS Cyg (MR01), the \( m = -1 \) mode is only seen near the outburst peak. This may indicate that a wide enough region without differential rotation is only present when there is sufficient spreading at high \( M \).

There are still many difficulties that must be answered about this idea for explaining DNOs. From the SL model (PB04) we borrowed the concept of hot material in hydrostatic balance covering a small fractional area of the WD, but we ignored important details of these calculations, such as differential rotation and the change of scale height with latitude. The data do not require such additional complications, so we refrain from including them for now. In a more sophisticated model it would probably still be true that \( k \propto 1/(fR) \), but an eigenvalue calculation would determine the constant of proportionality. A full calculation may also help explain the variety of scalings. The majority of DNOs are consistent with the model predicts a single power law index for the \( m = 0 \) mode.

Another important property of WDs that would affect the modes is a magnetic field. A strong field inhibits shearing between the SL and WD, and modifies the frequency of shallow surface waves. It is therefore interesting that no intermediate polars (IPs) have shown DNOs or IpDNOs. Even LS Peg and V795 Her, neither of which are IPs, but both show polarization modulations (Rodríguez-Gil et al. 2001, 2002) indicative of a reasonably strong magnetic field, are without DNOs or IpDNOs. Further studies should also work toward an understanding of the excitation mechanism for the modes, which would help explain the high coherence typical of DNOs. Material deposited at the WD equator spreads fairly quickly, \( 10 - 100 \) \( s \) (PB04), so that this material must be moving through the oscillating region on timescales of order the mode period. In light of the many strengths of the SL mode explanation of DNOs we do not abandon it because of this problem, but this is definitely a weakness that puts limits any proposed excitation mechanism.

Our explanation of DNOs raises interesting questions about the relationship between oscillations originating from accreting compact objects. Mauche (2002b) showed that there is a strong correlation in the high to low oscillation frequency ratio of WDs, NSs, and black holes (BH). Using this picture, DNOs are associated with the kilohertz QPOs of low mass X-ray binaries. Suggestively, the Fourier frequency resolved spectroscopy of NSs (Gilfanov, Revnivtsev & Molkov 2003) imply that both the normal branch oscillations and the kilohertz QPOs are created in the NS boundary layer, strengthening the possibility that our SL mode model may also apply in this case. For a typical NS mass and radius our model results in frequencies in the range expected for kHz QPOs, but it does not explain their complicated \( P-L \) relation (the “parallel tracks”): van der Klis 2000). Interestingly, DNOs may also show the parallel tracks phenomenon, as seen in three observations of SS Cyg (Mauche 2002a), once again supporting the correlation. On the other hand, continuing the analogy to WD and BH oscillations is problematic because in the case of BHs there is no surface for nonradial oscillations.

We thank Phil Arras and Christopher Mauche for many helpful discussions. We also thank the referee for asking the right questions, which inspired a significantly revised manuscript. This work was supported by the National Science Foundation under grants PHY99-07949 and AST02-05956, and by the Joint Institute for Nuclear Astrophysics through NSF grant PHY02-16783.

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