Classical Physics and Hamiltonian Quantum Mechanics as Relics of the Big Bang*

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Abstract

In a fundamental formulation of the quantum mechanics of a closed system, such as the universe as a whole, three forms of information are needed to make predictions of the probabilities of alternative histories. These are the action functional of the elementary particles, the quantum initial condition of the universe, and the information about our specific history. We discuss the origin of the "quasiclassical domain" of familiar experience and Hamiltonian quantum mechanics with its preferred time in such a formulation of quantum cosmology. It is argued that these features of the universe are not general properties of quantum theory, but rather approximate features emergent after the Planck time because of the character of the specific initial condition and dynamics of our universe.

1. Introduction

An unavoidable inference from the physics of the last sixty years is that we live in a quantum mechanical universe — a universe in which the process of prediction conforms to that framework we call quantum mechanics on all scales from those of the elementary particles to those of the universe itself. We perhaps have little direct evidence of peculiarly quantum phenomena on very large and even familiar scales today, but there is no evidence that the phenomena that we do see cannot be described in quantum mechanical terms and explained by quantum mechanical laws. In the earliest universe, the subject of this conference, we have a physical system of the largest possible size for which a quantum mechanical description is likely to be essential, especially at times earlier than the Planck time. The nature of this quantum mechanical description and its observable consequences are the subject of quantum cosmology.

The most general objects of prediction in quantum mechanics are the probabilities of alternative histories of the universe. Three forms of information are needed to make such predictions. These are the action functional of the elementary particles, the initial quantum state of the universe, and the information about our specific history on which probabilities for future prediction are conditioned. These are sufficient for every prediction in science and there are no predictions which do not, at this fundamental level, involve all three forms of information. A theory of the action functional of the fundamental fields is the goal of elementary particle physics. The equally fundamental, equally necessary, theory of the initial condition of the universe is the objective of quantum cosmology.[1] In a fundamental theory these may even be related objectives.

To make contact with observation, a theory of the initial condition must seek to explain correlations among observations today. What are these observations likely to be? On the largest scales there are the features of the universe whose explanation cosmology has usually traced to the intitial condition. These include the approximate homogeneity and isotropy, the approximate spatial flatness, the simple spatial topology, and the spectra of deviations from exact homogeneity and isotropy which we can see today as large scale structure and earlier as anisotropies in the background radiation. On very small scales, Sidney Coleman and Stephen Hawking have described at this conference how certain coupling constants of the elementary particles could be quantum probabilistic with a probability distribution that may depend, in part, on the initial condition. In this talk, however, I shall discuss two much more familiar features of the universe, accessible at ordinary scales, which must owe their origin, at least in part, to the quantum initial condition. These are the applicability of classical physics over much of the late universe, including especially the existence of classical spacetime, and the applicability of the Hamiltonian form of quantum mechanics. I shall argue that, in a fundamental formulation of the quantum process of prediction, these familiar features of the universe call for explanation just as much as do the large scale structural characteristics alluded to above, although there may be a very wide range of initial conditions of the universe from which they follow.

For a quantum mechanical system to exhibit classical behavior there must be some restriction on its state and some coarseness in how it is described. This is clearly illustrated in the quantum mechanics of a single particle. Ehrenfest's theorem shows that generally

$$M\frac{\mathrm{d}^2\langle x\rangle}{\mathrm{d}t^2} = \left\langle -\frac{\partial V}{\partial x}\right\rangle. \tag{1.1}$$

However, only for special states, typically narrow wave packets, will this become an equation of motion for $\langle x\rangle$ of the form

$$M\frac{\mathrm{d}^2\langle x\rangle}{\mathrm{d}t^2} = -\frac{\partial V(\langle x\rangle)}{\partial x}. \tag{1.2}$$

For such special states, successive observations of position in time will exhibit the classical correlations predicted by the equation of motion (1.2) provided that these observations are coarse enough so that the properties of the state which allow (1.2) to replace the general relation (1.1) are not affected by these observations. An exact determination of position, for example, would yield a completely delocalized wave packet an instant later and (1.2) would no longer be a good approximation to (1.1). Thus, even for large systems, and in particular for the universe as a whole, we can expect classical behavior only for certain initial states and then only when a sufficiently coarse grained description is used. If classical behavior is in general a consequence only of a certain class of states in quantum mechanics, then, as a particular case, we

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can expect to have classical spacetime only for certain states in quantum gravity. The classical spacetime geometry we see all about us in the late universe is not property of every state in a theory where geometry fluctuates quantum mechanically but traceable fundamentally to restrictions on the initial condition. Such restrictions are likely to be generous in that, as in the single particle case, many different states will exhibit classical features. The existence of classical spacetime and the applicability of classical physics are thus not likely to be very restrictive conditions on constructing a theory of the initial condition. However, they are such manifest and accurate features of the late universe that it is important to understand quantitatively the class of initial conditions with which they are consistent. Any initial condition predicted by theory must lie in this class.

A feature of the late universe which is closely related to the existence of classical spacetime is the applicability of Hamiltonian quantum mechanics. Time plays a special role in the familiar Hamiltonian formulation of quantum mechanics. Time is the only observable for which there are no interfering alternatives as position is an interfering alternative for momentum. Time is the only observable not represented in the formalism as an operator but rather enters the theory as a parameter describing evolution. Thus, just for its formulation, Hamiltonian quantum mechanics requires a fixed background spacetime to supply the preferred time. This we can expect only for special states of the universe and then only approximately.

Classical physics *is* applicable over a wide domain in the universe. Here in the late universe the geometry of spacetime, viewed sufficiently coarsely, *is* classical, definite and evolving by Einstein's equation. Hamiltonian quantum mechanics with its preferred time *is* correct for field theory on all accessible scales. The point of view I shall describe in this talk is that these familiar, homey, features of our world are most fundamentally seen not as exact properties of the basic theory but rather as approximate, emergent properties of the late universe appropriate to the particular initial condition which our universe does have. Put more crudely, I shall argue that the classical domain with its classical spacetime and the Hamiltonian form of quantum mechanics with its preferred time are relics of the big bang.

To exhibit the classical domain and Hamiltonian quantum mechanics as emergent features of the universe we need a generalization of the Copenhagen framework for quantum mechanics on at least two counts. First, the various Copenhagen formulations of quantum mechanics characteristically *posited* the existence of the classical domain (or a classically behaving "observer") as an additional assumption beyond the framework of wave function and Schrödinger equation that was necessary to interpret the theory. Second, these Copenhagen formulations *assumed* the preferred time of Hamiltonian quantum mechanics. What has been built in we cannot expect to get out. A more general framework is necessary.

In this talk I shall describe some routes towards these necessary generalizations. The generalizations I shall describe stress the consistency of probability sum rules as the primary criterion for assigning probabilities to histories rather than any notion of "measurement". They stress the initial condition of the universe as the ultimate origin within quantum mechanics of the classical domain. They stress the sum-over-

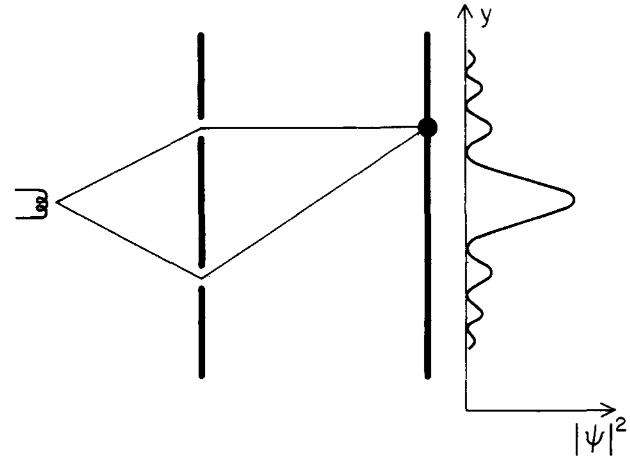

Fig. 1. The two-slit experiment. An electron gun at right emits an electron traveling towards a screen with two slits, its progress in space recapitulating its evolution in time. When precise detections are made of an ensemble of such electrons at the screen it is not possible, because of interference, to assign a probability to the alternatives of whether an individual electron went through the upper slit or the lower slit. However, if the electron interacts with apparatus that measures which slit it passed through, then these alternatives decohere and probabilties can be assigned.

histories formulation of quantum mechanics as a potentially more general and generally covariant framework for a quantum mechanics of spacetime. The work on these generalization cannot be said to be complete but the directions seem promising to me. To keep the discussion manageable I shall take the discussion in two steps. First, in Sections 2–4, I shall neglect gross fluctuations in the geometry of spacetime; later, in Sections 5 and 6, I shall return to the generalization needed to accommodate them.

2. Post-Everett quantum mechanics

We begin with a brief review of the post-Everett formulation of the quantum mechanics of closed systems such as the universe as a whole. As described above, we shall first assume a fixed background spacetime which supplies a preferred family of timelike directions. This, of course, is an excellent approximation on accessible scales for times later than 10^{-43} s after the big bang. The familiar apparatus of Hilbert space, states, Hamiltonian and other operators may then be applied to process of prediction. Indeed, in this context the quantum mechanics of cosmology is in no way distinguished from the quantum mechanics of a large isolated box, perhaps expanding, but containing both the observed and its observers.

The quantum mechanical framework that I shall describe has its origins in the work of Everett [2] and has been developed by many. In its recent developments it incorporates ideas of Zeh [3], Joos and Zeh [4], Zurek [5], Griffiths [6], and Omnès [7]. The particular development I shall follow is due to Murray Gell-Mann and myself [8, 9].

A characteristic feature of a quantum mechanical theory is that not every history which can be described can be assigned a probability. Nowhere is this more clearly illustrated than in the two-slit experiment (Fig. 1). In the usual discussion, if we have not measured which slit the electron passed through on its way to being detected at the screen, then we are not permitted to assign probabilities to these alternative histories. It would be inconsistent to do so since the correct probability sum rules would not be satisfied. Because of interference, the probability to arrive at y is not the sum of the probabilities to arrive at y going through the upper and the lower slit:

$$p(y) \neq p_U(y) + p_L(y) \quad (2.1)$$

because

$$|\psi_L(y) + \psi_U(y)|^2 \neq |\psi_L(y)|^2 + |\psi_U(y)|^2. \quad (2.2)$$

If we *have* measured which slit the electron went through, then the interference is destroyed, the sum rule obeyed, and we can meaningfully assign probabilities to these alternative histories.

It is a general feature of quantum mechanics that a rule is needed to determine which histories can be assigned probabilities. As the two-slit example illustrates, in the Copenhagen formulations, probabilities are assigned to histories which are *measured.* This is a rule which assumes a division of the universe into one subsystem which is measured or observed and another which does the measuring or observing. Further, to define measurement, the Copenhagen formulations had, in one way or another, to posit as fundamental the classical world that we see all about us. We can have none of this in cosmology. In a theory of the whole thing there can be no fundamental division into observer and observed. There is no fundamental reason for a closed system to exhibit classical behavior generally in any variables. Measurements and observers cannot be fundamental notions in a theory which seeks to describe the early universe where neither existed. We need a more general rule for assigning probabilities in quantum cosmology.

I shall now describe the rules which specify which histories of a closed system may be assigned consistent probabilities in the post-Everett formulation and what these probabilities are. They are essentially the rules of Griffiths [6] further developed by Omnès [7] and independently but later arrived at by Gell-Mann and the author [8]. The idea is simple: Probabilities can be assigned to those coarsely described histories for which the probability sum rules are obeyed as a consequence of the particular initial state the closed system does have.

To describe the rules in detail, it is convenient to begin with Feynman's sum-over-histories formulation of quantum mechanics since histories are our concern. There, all quantum amplitudes are expressed as functionals of completely fine-grained histories specified by giving a set of generalized coördinates $q^i(t)$ as functions of time. These might be the values of fundamental fields at different points of space, for example.

Completely fine-grained histories cannot be assigned probabilities; only suitable coarse-grained histories can. Examples of coarse graining are: (1) Specifying the q^i not at all times but at a discrete set of times. (2) Specifying not all the q^i at any one time but only some of them. (3) Specifying not definite values of these q^i but only ranges of values. An exhaustive set of ranges at one time consists of regions $\{\Delta_\alpha\}$ which make up the whole space spanned by the q^i as α passes over all values. An exhaustive set of coarse-grained histories is then defined by sets of such exhaustive ranges $\{\Delta_\alpha^i\}$ at times t_i, $i = 1, \ldots, n$.

The important theoretical construct for giving the rule that determines whether probabilities may be assigned to a given set of alternative coarse-grained histories, and what these

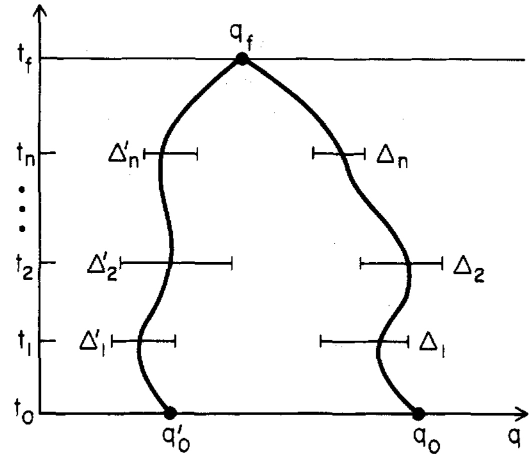

Fig. 2. The sum-over histories construction of the decoherence functional.

probabilities are, is the decoherence functional, D[(history)′, (history)]. This is a complex functional on any pair of histories on any pair of histories in the set. In the sum-over-histories framework for completely fine-grained history segments between an initial time t_0 and a final time t_f, it is defined as follows:

$$D[q'^i(t), q^i(t)] = \delta(q_f'^i - q_f^i) \times \exp\{i(S[q'^i(t)] - S[q^i(t)])/\hbar\}\rho(q_0'^i, q_0^i). \quad (2.3)$$

Here, ρ is the initial density matrix of the universe in the q^i representation, $q_0'^i$ and q_0^i are the initial values of the complete set of variables, and $q_f'^i$ and q_f^i are the final values necessarily common to both histories. The decoherence functional for coarse-grained histories is obtained from (2.3) according to the principle of superposition by summing over all that is not specified by the coarse graining. Thus,

$$D([\Delta_{\alpha'}], [\Delta_\alpha]) = \int_{[\Delta_{\alpha'}]} \delta q' \int_{[\Delta_\alpha]} \delta q \delta(q_f'^i - q_f^i) \times e^{i\{(S[q'^i] - S[q^i])/\hbar\}} \rho(q_0'^i, q_0^i). \quad (2.4)$$

More precisely, the integral is as follows (Fig. 2): It is over all histories $q'^i(t)$, $q^i(t)$ that begin at $q_0'^i$, q_0^i respectively, pass through the ranges $[\Delta_{\alpha'}]$ and $[\Delta_\alpha]$ respectively, and wind up at a common point q_f^i at any time $t_f > t_n$. It is completed by integrating over $q_0'^i$, q_0^i, q_f^i. The three forms of information necessary for prediction – state, action, and specific history are manifest in this formula as ρ, S, and the sequence of ranges $[\Delta_\alpha]$ respectively.

The connection between coarse-grained histories and completely fine-grained ones is transparent in the sum-over-histories formulation of quantum mechanics. However, the sum-over-histories formulation does not allow us to consider coarse-grained histories of the most general type. For the most general histories one needs to exploit the transformation theory of quantum mechanics and for this the Heisenberg picture is convenient. In the Heisenberg picture an exhaustive set of alternatives at one time corresponds to a set of projection operators $\{P_\alpha^k(t)\}$ satisfying

$$\sum_\alpha P_\alpha^k(t) = 1, \quad P_\alpha^k(t)\, P_\beta^k(t) = \delta_{\alpha\beta} P_\beta^k(t). \quad (2.5)$$

Here, k labels the set of alternatives, α the particular alternative, and t the time. The operators representing the same alternatives at different times are connected by

$$P_\alpha^k(t) = e^{iHt/\hbar} P_\alpha(0)\, e^{-iHt/\hbar} \quad (2.6)$$

where H is the total Hamiltonian. Time sequences of such sets of alternatives define sets of alternative histories for the univese. A particular history is one particular sequence of alternatives $(P^1_{\alpha_1}(t_1), P^2_{\alpha_2}(t_2), \ldots, P^n_{\alpha_n}(t_n))$ corresponding to a particular sequence of α's. We abbreviate such an individual history by $[P_\alpha]$. With this notation the decoherence functional in the Heisenberg picture may be written

$$D([P_{\alpha'}], [P_\alpha]) \;=\; \mathrm{Tr}\,[P^n_{\alpha'_n}(t_n) \ldots P^1_{\alpha'_1}(t_1)\rho P^1_{\alpha_1}(t_1) \ldots P^n_{\alpha_n}(t_n)], \tag{2.7}$$

In the Heisenberg picture a completely fine-grained set of histories is defined by giving a complete set of projections at each and every time. Every possible set of alternative histories may then be otained by coarse graining the various fine-grained sets, that is by using P's in the coarser grained sets which are sums of those in the finer grained sets. Thus, if $\{[\bar{P}_\beta]\}$ is a coarse graining of the set of histories $\{[P_\alpha]\}$, we write

$$D([\overline{P_{\beta'}}], [\overline{P_\beta}]) \;=\; \sum_{\substack{\text{all } P_{\alpha'} \\ \text{not fixed by } [\overline{P_{\beta'}}]}} \;\; \sum_{\substack{\text{all } P_\alpha \\ \text{not fixed by } [\overline{P_\beta}]}} D([P_{\alpha'}], [P_\alpha]). \tag{2.8}$$

A set of coarse-grained alternative histories is said to decohere when the off-diagonal elements of D are sufficiently small to be considered vanishing for all practical purposes:

$$D([P_{\alpha'}], [P_\alpha]) \approx 0, \quad \text{for any } \alpha'_k \neq \alpha_k. \tag{2.9}$$

This is a generalization of the condition for the absence of interference in the two-slit experiment (approximate equality of the two sides of (2.2)).

The rule for when probabilities can be assigned to histories of the universe is then this: To the extent that a set of alternative histories decoheres, probabilities can be assigned to its individual members. The probabilities are the diagonal elements of D. Thus,

$$\begin{aligned} p([P_\alpha]) &= D([P_\alpha], [P_\alpha]) \\ &= \mathrm{Tr}\,[P^n_{\alpha_n}(t_n) \ldots P^1_{\alpha_1}(t_1)\rho P^1_{\alpha_1}(t_1) \ldots P^n_{\alpha_n}(t_n)] \end{aligned} \tag{2.10}$$

when the set decoheres.

The probabilities defined by (2.10) obey the rules of probability theory as a consequence of decoherence. The principal requirement is that the probabilities by additive on "disjoint sets of the sample space". For histories this means the sum rules

$$p([\overline{P_\beta}]) \approx \sum_{\substack{\text{all } P_\alpha \text{ not} \\ \text{fixed by } [\overline{P_\beta}]}} p([P_\alpha]). \tag{2.11}$$

These relate the probabilities for a set of histories to the probabilities for all coarser grained sets that can be constructed from it. For example, the sum rule eliminating all projections at only one time is, in an obvious notation:

$$\begin{aligned} &\sum_{\alpha_k} p(\alpha_n t_n, \ldots \alpha_{k+1} t_{k+1}, \alpha_k t_k, \alpha_{k-1} t_{k-1}, \ldots, \alpha_1 t_1) \\ &\qquad \approx p(\alpha_n t_n, \ldots \alpha_{k+1} t_{k+1}, \alpha_{k-1} t_{k-1}, \ldots, \alpha_1 t_1). \end{aligned} \tag{2.12}$$

Given this discussion, the fundamental formula of quantum mechanics may be reasonably taken to be

$$D([P_{\alpha'}], [P_\alpha]) \approx \delta_{\alpha'_1\alpha_1} \ldots \delta_{\alpha'_n\alpha_n}\, p([P_\alpha]) \tag{2.13}$$

for all $[P_\alpha]$ in a set of coarse-grained alternative histories. Vanishing of the off-diagonal elements of D gives the rule for when probabilities may be consistently assigned. The diagonal elements give their values.

Decoherent histories of the universe are what we may utilize in the quantum mechanical process of prediction for they may be assinged probabilities. Decoherence thus generalizes and replaces the notion of "measurement", which served this role in the Copenhagen interpretations. Decoherence is a more precise, more objective, more observer-independent idea and gives a definite meaning to Everett's branches. If their associated histories decohere, we may assign probabilities to various values of reasonable scale density fluctutions in the early universe whether or not anything like a "measurement" was carried out on them and certainly whether or not there was an "observer" to do it.

3. Origins of decoherence

The decoherence of a set of alternative histories is not a property of their coarse graining alone. As the formula (2.7) for D shows, it depends on all three forms of information necessary to make predictions about the universe and in particular on its quantum initial condition. Given ρ and H, we would compute which sets of alternative histories decohere and there would be a great many such sets.

We are not likely to carry out a calculation of all decohering sets of alternative histories for the universe anytime in the near future. It is therefore important to investigate specific mechanisms for decoherence in more restrictive circumstances. Specific examples of decoherence have been discussed by many authors, among them Joos and Zeh [4], Zurek [5], Caldeira and Leggett [10], and Unruh and Zurek [11]. Typically these discussions have considered coarse grainings defined by projection operators which project onto a few particular degrees of freedom of a system while ignoring the rest. The simplest model consists of a single oscillator interacting linearly with a large number of others. A coarse graining is used which follows the coördinate of the distinguished oscillator and ignores the coördinates of the others. Let x be the coördinate of the special oscillator, M its mass, ω_R its frequency renormalized by its interactions with the others, and S_{free} its free action. Consider the special case where the density matrix of the whole system, referred to an initial time, factors into the product of a density matrix $\bar{\rho}(x, y)$ of the distinguished oscillator and another for the rest. Then, generalizing slightly a treatment of Feynman and Vernon [12], we can write the decoherence functional defined by (2.4) for this coarse graining as

$$\begin{aligned} D([\Delta_{\alpha'}], [\Delta_\alpha]) &= \int_{[\Delta_{\alpha'}]} \delta x' \int_{[\Delta_\alpha]} \delta x \delta(x'_f - x_f) \\ &\quad \times \exp\{\mathrm{i}(S_{\text{free}}[x'(t)] - S_{\text{free}}[x(t)] \\ &\quad + W[x'(t), x(t)])/\hbar\}\bar{\rho}(x'_0, x_0) \end{aligned} \tag{3.1}$$

The sum over the paths of the rest of the oscillators has been carried out and is summarized by the Feynman–Vernon influence functional $\exp(\mathrm{i}W[x'(t), x(t)])$.

The case when the rest of the oscillators are in an initial thermal state has been extensively investigated by Caldeira and Leggett [10]. In the simple limit of a uniform cut-off continuum of oscillators and in the Fokker–Planck limit of high temperature, they find

$$\begin{aligned} W[x'(t), x(t)] &= -M\gamma \int \mathrm{d}t\, [x'\dot{x}' - x\dot{x} + x'\dot{x} - x\dot{x}'] \\ &\quad + \mathrm{i}\,\frac{2M\gamma kT}{\hbar} \int \mathrm{d}t\, [x'(t) - x(t)]^2 \end{aligned} \tag{3.2}$$

where γ summarizes the interaction strengths of the distinguished oscillator with the rest. The real part of W contributes dissipation to the equations of motion. The imaginary part squeezes the trajectories $x(t)$ and $x'(t)$ together, thereby accomplishing decoherence on the characteristic time scale

$$t_{\text{decoherence}} \gtrsim \frac{1}{\gamma}\left[\left(\frac{\hbar}{\sqrt{2MkT}}\right)\cdot\left(\frac{1}{d}\right)\right]^2. \tag{3.3}$$

As emphasized by Zurek [13], this squeezing can be very rapid when compared with characteristic dynamical timescales ($t_{\text{decoherence}}/t_{\text{dynamical}} \sim 10^{-40}$ for typical "macroscopic" values). In such models coherent phase information is lost by the creation of correlations with variables which are then ignored in the coarse-graining and so summed over in constructing the decoherence functional.

What such models convincingly show is that decoherence is frequent and widespread in the universe. Joos and Zeh [4] calculate that a superposition of two positions of a grain of dust, 1 mm apart, is decohered simply by the scattering of the cosmic background radiation on the timescale of a nanosecond. So widespread is this phenomena with the initial condition and dynamics of our universe that we may meaningfully speak of habitually decohering variables such as the center of mass positions of massive bodies.

4. Quasiclassical domains

As observers of the universe, we deal with coarse grainings that are appropriate to our limited sensory perceptions, extended by instruments, communication, and records, but in the end characterized by a great amount of ignorance. Yet we have the impression that the universe exhibits a finer grained set of decohering histories, independent of us, defining a sort of "quasiclassical domain", governed largely by classical laws, to which our senses are adapted while delaying with only a small part of it. No such coarse graining is determined by pure quantum theory alone. Rather, like decoherence, the existence of a quasiclassical domain in the universe must be a consequence of its initial condition and the Hamiltonian describing its evolution.

Roughly speaking, a quasiclassical domain should be a set of alternative decohering histories, maximally refined consistent with decoherence, with individual histories exhibiting as much as possible patterns of classical correlation in time. To make the question of the existence of one or more quasiclassical domains into a *calculable* question in quantum cosmology we need criteria to measure how close a set of histories comes to constituting a "quasiclassical domain". A quasiclassical domain cannot be a *completely* fine-grained description for then it would not decohere. It cannot consist *entirely* of a few "classical variables" repeated over and over because sometimes we may measure something highly quantum mechanical. These variables cannot be *always* correlated in time by classical laws because sometimes quantum mechanical phenomena cause deviations from classical physics. We need measures for maximality and classicality [8].

It is possible to give crude arguments for the type of habitually operators we expect to occur over and over again in a set of histories defining a quasiclassical domain. Such habitually decohering operators are called "quasiclassical operators". In the earliest instants of the universe the operators defining spacetime on scales well above the Planck scale emerge from the quantum fog as quasiclassical [14]. Any theory of the initial condition that does not imply this is simply inconsistent with observation in a manifest way. Then, where there are suitable conditions of low temperature, density, etc., various sorts of hydrodynamic variables may emerge as quasiclassical operators. These are integrals over suitably small volumes of densities of conserved or nearly conserved quantities. Examples are densities of energy, momentum baryon number, and, in later epochs, nuclei and even chemical species. The sizes of the volumes are limited above by maximality and are limited below by classicality because they require sufficient "inertia" to enable them to resist deviations from predictability caused by their interactions with one another, by quantum spreading, and by the quantum and statistical fluctuations resulting from the interactions with the rest of the universe that accomplish decoherence. Suitable integrals of densities of approximately conserved quantities are thus candidates for habitually decohering quasiclassical operators. These "hydrodynamic variables" *are* among the principle variables of classical physics.

It would be in such ways that the classical domain of familiar experience could be an emergent property of the early universe, not genrrally in quantum mechanics, but as a consequence of our specific initial condition and the Hamiltonian describing evolution.

5. Generalized quantum mechanics

I would now like to turn to the generalization of quantum mechanics which may be needed to resolve the conflict between the need for a preferred time variable in Hamiltonian quantum mechanics and the inability of any generally covariant quantum theory of spacetime to supply one. To start, it is useful to consider what we might mean most generally by a quantum mechanical theory [9]. Roughly speaking, by a quantum mechanics we mean a theory that admits a notion of fine and coarse-grained histories, the amplitudes for which are connected by the principle of superposition and for which there is a rule (decoherence) for when coarse-grained histories can be assigned probabilities obeying the sum rules of probability calculus. More precisely, from the discussion in the preceeding section its possible to abstract the following three elements of quantum mechanics in general:

(1) *The fine-grained histories*: The sets of fine-grained, exhaustive, alternative histories of the universe that are the most refined description to which one can contemplate assigning probabilities.

(2) *A notion of coarse graining*: A coarse graining of an exhaustive set of histories is a partition of that set into exhaustive and exclusive classes $\{h\}$. Various possible coarse-grained sets of alternative histories may be constructed by coarse graining the fine-grained sets or by further coarse graining of an already coarse-grained set.

(3) *A decoherence functional*: The decoherence functional, $D(h, h')$, is defined on each pair of coarse-grained histories in an exhaustive set, $\{h\}$, for all possible sets including the completely fine-grained ones. It must satisfy the

following properties:

(i) *Hermiticity*:

$$D(h, h') = D^*(h', h), \tag{5.1a}$$

(ii) *Positivity*:

$$D(h, h) \geqslant 0, \tag{5.1b}$$

(iii) *Normalization*:

$$\sum_{h,h'} D(h, h') = 1, \tag{5.1c}$$

(iv) *The principle of superposition*: If $\{\bar{h}\}$ is a coarser graining of a coarse-grained set $\{h\}$ then the decoherence functional for the coarser grained set is related to that of the finer grained set by:

$$D(\bar{h}, \bar{h}') = \sum_{\substack{\text{all } h \\ \text{in } \bar{h}}} \sum_{\substack{\text{all } h' \\ \text{in } \bar{h}'}} D(h, h'). \tag{5.1d}$$

These three elements are sufficient for the process of prediction. Decoherence can be defined and probabilities assigned according to the fundamental formula

$$D(h, h') \approx \delta_{hh'}\, p(h). \tag{5.2}$$

As a consequence of the four requirements (5.1) on the decoherence functional these probabilities will obey the rules of probability calculus. With these probabilities the thoery becomes predictive.

In Hamiltonian quantum mechanics the three elements are as follows: (1) The set of fine-grained histories are defined by sequences of sets projections onto *complete* sets of states, one set at each time. (2) Coarse grainings of sets of histories are defined by sets of projections which are sums of projections in finer grained ones. (3) The decoherence functional is (2.7). However, Hamiltonian quantum mechanics is not the only way of constructing a theory with the three elements of generalized quantum mechanics. More general possibilities may be considered, and, as we shall argue below, may be useful in constructing a generally covariant quantum theory of spacetime.

An interesting example of a generalized quantum mechanics is provided by field theory in the kind of wormhole spacetime discussed by Morris, Thorne, Yurtsver [15] and others that is illustrated in Fig. 3. These are not the four-dimensional wormholes discussed in connection with the value of the cosmological constant. They are handles on three-dimensional space. The topology of spacetime is $\boldsymbol{R} \times M^3$ with M^3 being multiply connected.

Imagining that before some time $t = t_s$ the wormhole mouths are at rest with respect to one another. At time t_s they begin to rotate about one another and continue until a moment of time symmetry when they reverse their motion eventually coming to relative rest at time t_e. Before t_s and after t_e there are no closed timelike lines and it is possible to define surfaces of constant time that foliate those portions of spacetime. In between t_s and t_e, however, because of time dilation in the rotating wormhole mouth, there are closed timelike lines, as Fig. 3 illustrates. By going through the wormhole throat it is effectively possible to go backward in time. Such wormhole spacetimes are time orientable but not causal.

It is clear that there is no straightforward Hamiltonian quantum mechanics in a wormhole spacetime between the surfaces t_s and t_e. What would be the surfaces of the preferred time? How would unitary evolution of arbitrary states in the Hilbert space be defined in the presence of closed timelike lines?

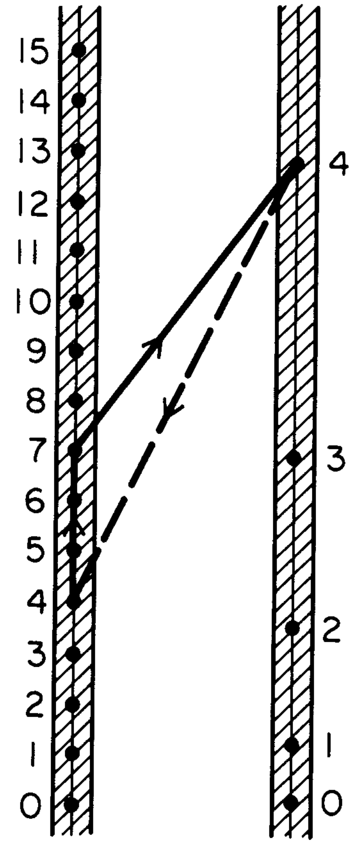

Fig. 3. Closed timelike lines in a wormhole geometry. The figure shows a spacetime diagram (time upward) with two wormhole mouths (the shaded regions). The wormhole geometry is multiply connected so that it is possible to pass nearly simultaneously from points in one wormhole mouth to another. The wormhole mouth on the left remains at rest in an inertial frame. The one at right is initially at rest with respct to the first at $t = 0$ but then begins to rotate about it. The figure shows the corotating frame and the readings of a clock at the centre of each wormhole mouth. As a consequence of time dilation in the rotating mouth, this spacetime has closed timelike curves of which one is shown. The dotted segment represents the nearly instantaneous passage through the wormhole throat.

A generalized quantum mechanics of the kind we have been discussing, however, may be constructed for this example using a sum-over-histories decoherence functional. The three ingredients would be the following*

(1) *Fine-grained histories*: For the fine grained histories we may take single-value field configurations, $\phi(x)$, in the wormhole spacetime.

(2) *Coarse grainings*: The fine-grained histories may be partitioned according to their values on spacetime regions. Select a set of spacetime regions, specify an exhaustive set of ranges for the average values of the field in these regions, and one has partitioned the four-dimensional field configurations into classes, $\{h\}$, that have the various possible values of the average field. For example one might specify the spatial field configurations on an initial constant time surface with $t < t_s$ and on a final constant time surface with $t > t_e$. The resulting probabilties would be relevant for defining the S-matrix for scattering from the wormhole.

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* This generalization was developed in discussions with G. Horowitz.

(3) *Decoherence functional*: A sum-over-histories decoherence functional is

$$D(h, h') = \int_h \delta\phi \int_{h'} \delta\phi' \, \delta[\phi_f(x) - \phi_f'(x)] \times \exp\{i(S[\phi(x)] - S[\phi(x')])/\hbar\} \rho_0[\phi_0(x), \phi_0'(x)]. \quad (5.3)$$

Here, the integrations are over single-valued field configurations between some initial constant time surface $t_0 < t_s$ and some final constant time surface $t_f > t_e$. $\phi_0(x)$ and $\phi_0'(x)$ are the spatial configurations on the initial surface; their integral is weighted by the density matrix ρ_0. $\phi_f(x)$ and $\phi_f'(x)$ are the spatial configurations on the final surface; their coincidence is enforced by the functional δ-function. The integral over $\phi(x)$ is over the class of field configurations in the class h. For example, if h specifies the average value of the field is some region to lie in a certain range, then the integral is only over $\phi(x)$ that have such average values. Formally, this decoherence functional satisfies four conditions of eq. (5.1).

With the generalized quantum mechanics based on the three elements described above probabilities can be assigned to coase-grained sets of field histories in the wormhole spacetime. These probabilities obey the standard probability sum rules. There is no equivalent Hamiltonian formulation of this quantum mechanics because this wormhole spacetime, with is closed timelike lines provides no foliating family of spacelike surfaces to define the required preferred time. Nevertheless, the generalized theory is predictive. What has been lost in this generalization is any notion of "state at a moment of time" and of its unitary evolution in between the surfaces t_e and t_s. This is perhaps not surprising for a region of spacetime that has no well defined notion of "at a moment of time".

6. A quantum mechanics for spacetime

I would now like to sketch how a generalized quantum mechanics for spacetime might be constructed that does not break general covariance by singling out a preferred family of spacelike surfaces for the distinguished time variable that would be needed in Hamiltonian formulation. I would then like to discuss how Hamiltonian quantum mechanics could be an approximation to this more general framework appropriate because of the classical spacetime of the late universe [16].

A history in cosmology is a cosmological four-geometry with a four-dimensional matter field configuration upon it. An example is the classical Friedman evolution of a closed universe from the big bang to a big crunch. In quantum mechanics all the possible histories, classical and non-classical, must be assigned amplitudes. Classical or non-classical, cosmological histories may be thought of as successions of three-dimensional geometries. The Friedmann universe is a three-sphere expanding and contracting according to the Einstein equation. Non-classical histories can have arbitrarily varied histories of expansion and contraction. Thus, cosmological histories may be thought as *paths* in the superspace of three-geometries and three dimensional matter configurations (Fig. 4).

One natural way of defining a coarse graining of cosmological histories is to utilize a family of regions in superspace, $\{R_\alpha\}$, and partition the paths according to how they pass through them. A history which passes through, say each of three regions at least once has at least spacelike surfaces with geometries and matter field configurations specified to

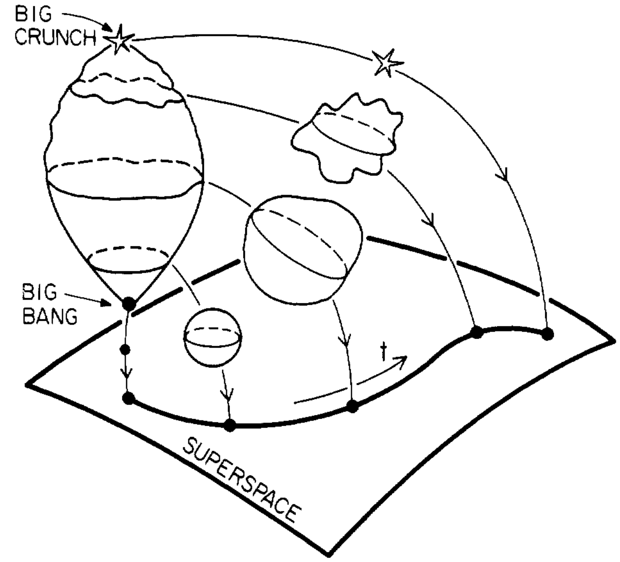

Fig. 4. Superspace. A cosmological history is a four-dimensional cosmological spacetime with matter fields upon it. A two dimensional representation of such a history is shown in the upper left of this figure proceeding from a big bang to a big crunch. A cosmological history can be thought of as a succession of three-dimensional geometries and spatial matter field configuration. Superspace is the space of such three-dimensional geometries and matter field configurations. A "point" in superspace is a particular three-geometry and spatial matter field configuration. The succession of three-geometries and matter fields that make up a four-geometry and field history, therefore, trace out a path in superspace.

an accuracy determined by the sizes of the regions. A *set* of regions defines a *partition of the paths* into exhaustive and exclusive classes. For example, with two regions there is the class of paths which go through both regions at least once, the class of paths which go through the first region at least once but never the second, the class which goes through none of the regions, and so forth. A sum-over-histories decoherence functional on the various classes $\{h\}$ defining such a coarse-grained set is then naturally constructed as follows:

$$D(h, h') = \int_{h,\mathscr{C}} \delta g \delta\phi \int_{h',\mathscr{C}} \delta g' \delta\phi' \exp\{i(S[g, \phi] - S[g', \phi'])/\hbar\} \quad (6.1)$$

The sum over (g, ϕ) is over histories which start from prescribed initial conditions (say, the "no boundary" proposal) and proceed through regions $\{R_\alpha\}$ as specified by the class h to a final condition representing complete ignorance. The sum over (g', ϕ') is similarly over histories in the partition h'. The initial density matrix and the final δ-function which occur in other sum-over-histories expressions like (2.4) are here expressed in terms of conditions on the paths, $\mathscr{C}$. With appropriate conditions, this construction satisfies the requirements for a decoherence functional discussed above. With this decoherence functional the fundamental formula (5.2) can be used to identify decoherent sets of histories and assign them probabilities which obey the rules of probability theory to the level that decoherence is enforced.

There is, in general, no possible choice of time variable such that this quantum mechanics of spacetime can be put into the Hamiltonian form. For that to be the case we would need a time function on superspace whose constant time surface the histories cross once and only once. There is none. Put differently, there is no geometrical quantity which uniquely labels a spacelike hypersurface. The volume of the universe, for example, may single out just a few surfaces in a *classical* cosmological history, but in quantum mechanics we must consider all possible histories, and a non-classical history may have arbitrarily many surfaces of a given volume.

However, while we do not recover a Hamiltonian formulation precisely and generally we may recover it *approximately* for special coarse grainings in restricted domains of superspace and particular initial conditions. Suppose for example, the the initial condition was such that for coarse grainings defined with sufficiently unrestrictive regions, in a regime of three-geometries much larger than the Planck scale, only a single spacetime geometry $\hat{g}$ contributed to the geometrical sums (5.1) defining the decoherence functional. Then we would have

$$D(h, h') = \int_{h,\mathscr{C}} \delta\phi \int_{h',\mathscr{C}} \delta\phi' \exp\{i(S[\hat{g}, \phi] - S[\hat{g}, \phi'])/\hbar\}. \tag{6.2}$$

This decoherence functional defines the quantum mechanics of a field theory of ϕ in the background spacetime $\hat{g}$. Any family of spacelike surfaces in this background picks out a unique field configuration since the sums are over fields which are single-valued on spacetime. There is a notion of causality, and we recover a sum-over-histories expression of the field theory of ϕ in the background spacetime $\hat{g}$ in a particular initial cosmological state. This does have an equivalent Hamiltonian formulation.

It could be in this way that the familiar Hamiltonian framework quantum mechanics emerges an approximation appropriate to the existence of an approximate classical spacetime – an approximation which is not generally valid in quantum theories, but appropriate to our special place late in a universe with particular initial conditions.

7. Conclusions

In the history of physics, ideas that were once seen to be fundamental, general, and inescapable parts of the theoretical framework are sometimes later seen to be consequent, special, and but one possibility among many in a yet more general theoretical frameowork. This is often for the following reason: The idea was not a truly general feature of the world, but only *perceived* to be general because of our special place in the universe and the limited range of our observations [17]. Examples are the earth-centered picture of the solar system, the Newtonian notion of time, the exact status of the laws of the thermodynamics, the Euclidean laws of spatial geometry, and classical determinism. In veiw of this history, it is appropriate to ask of any current theory "which ideas are truly fundamental and which are 'excess baggage' that can be viewed more successfully as but one possibility out of many in a yet more general theoretical framework?" In cosmology it is especially appropriate to ask this question. We live in a special position in the universe, not so much in place, as in time. We are late, living ten and some billion years after the big bang, a time when many interesting possibilties for physics could be realized which are not easily accessible now. Moreover, we live in a special universe whose smooth, perhaps comprehensible, initial condition is but one of the many we could imagine.

This lecture has advanced the point of view that there are two features of common quantum mechanics usually taken to be fundamental that may be special, approximate, emergent features of the late epoch of a universe with our kind of initial condition. These are the "quasiclassical domain" of familiar experience and the Hamiltonian framework of quantum mechanics with its preferred time variable. Before the Planck time there are unlikely to have been classically behaving variables of any sort. In particular it is unlikely for there to have been a classically behaving background spacetime to supply the preferred time of Hamiltonian quantum mechanics. However, although a "quasiclassical domain", so central to the Copenhagen interpretations of quantum mechanics, may not be a general feature of all epochs of all universes it may be seen as an approximte feature of the late epoch of this universe in a more general post-Everett formulation. Hamiltonian quantum mechanics, with its preferred time, may not be the most general formulation of quantum mechanics, but it may be an approximation to a more general sum-over-histories formulation appropriate to the late epochs of a universe, like ours, whose intial condition implies classical spacetime there.

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