GENERALIZED QUANTUM THEORIES

	Non-Relativistic Quantum Mechanics	Gauge Field Theory	Single Relativistic World Line	General Relativity
Fine-Grained Histories $\{f\}$	Paths $x(t)$ that move forward in time.	Four-dimensional single-valued configurations of the potential $A^\mu(x)$.	Paths in spacetime $x^\alpha(\lambda)$ that move both forward and backward in time and multiplier $N(\lambda)$.	Four-dimensional manifolds, M with metrics $g_{\alpha\beta}(x)$, and matter fields $\phi(x)$.
Coarse Grainings $\{c_\alpha\}$	Partitions of the paths into classes.	Partitions of the potential into *gauge invariant* classes.	Partitions of the paths into *reparametrization invariant* classes.	Partitions of manifolds, metrics, and fields into *diffeomorphism invariant* classes.
	e.g., (i) By the position of crossing a surface of constant time, (ii) By whether a path crosses a given spacetime region or does not.	*e.g.*, By the values of the field averaged over a given spacetime region, or any other gauge-invariant functional.	*e.g.*, (i) By whether the path crosses a given spacetime region or does not, (ii) By values of the "proper time" $\int N d\lambda$.	*e.g.*, By whether a 4-geometry contains a spacelike surface with a given 3-geometry or or does not.
Decoherence Functional	$\langle \text{end}''\|C_\alpha\|\text{end}'\rangle = \Sigma_{f\epsilon c_\alpha} \exp(iS[f])$ $\langle\phi_i\|C_\alpha\|\psi_j\rangle = \phi_i \circ \langle \text{end}''\|C_\alpha\|\text{end}'\rangle \circ \psi_j$ $D(\alpha',\alpha) = \mathcal{N}\Sigma_{ij} p''_i \langle\phi_i\|C_{\alpha'}\|\psi_j\rangle \langle\phi_i\|C_\alpha\|\psi_j\rangle^* p'_j$			
	$\circ = L_2$ inner product on functions of x.	$\circ = L_2$ inner product on "true degrees of freedom".	$\circ =$ Klein-Gordon inner product on a surface. in spacetime.	$\circ =$ DeWitt inner product on a surface in superspace.

X. Summation

These lectures have developed generalized quantum frameworks for non-relativistic quantum mechanics, field theory, and a single relativistic world line in which quantum theory is put into fully spacetime form both with respect to dynamics and alternatives. These frameworks motivate the proposal of Section VIII for a quantum framework for cosmology incorporating a quantum dynamics of spacetime geometry. The three basic elements of a generalized quantum theory are compared for these frameworks in the table above.

To conclude we summarize the main points concerning the quantum mechanics of cosmology developed in these lectures in a short list:

- Quantum mechanics is formulated for a closed system — the universe. Decoherence rather than measurement distinguishes those alternatives which may consistently be assigned probabilities from those which may not. The framework may thus be applied to make predictions of alternatives of interest to cosmology in the very early universe or on very large distance scales which are not part of any measurement situation. The familiar Copenhagen quantum mechanics of measured subsystems is an approximation to this more general quantum theory of closed systems that is appropriate when the decoherence of the alternatives of the apparatus that register the results of the measurement can be idealized as exact.

- The sum-over-histories approach to quantum mechanics is used to formulate the quantum mechanics of cosmology in fully spacetime form. Dynamics is expressed in terms of sums over fine-grained histories that are four-dimensional manifolds, metrics, and matter field configurations. Alternatives are defined by partitions (coarse-grainings) of these four-dimensional, fine-grained histories into exhaustive sets of exclusive diffeomorphism invariant classes. The analogs of "unitary evolution" and "reduction of the wave packet" are given a unified sum-over-histories expression. The formulation is manifestly four-dimensionally diffeomorphism invariant if the formal diffeomorphism invariance of the functional integrals defining sums-over-geometries can be relied upon.

- The alternatives to which this quantum theory assigns probabilities, if they decohere, are at once more general and more restricted than the "observables" that are often considered in other formulations. Four-dimensional diffeomorphism invariant *alternatives on a spacelike surface*, for example, usually are restricted to classical constants of the motion that commute with the constraints. The present formulation considers the much larger, more realistic, and more accessible class of diffeomorphism invariant *spacetime alternatives*. However, in its present form the theory considers only alternatives describable in spacetime form as partitions of the unique fine-grained set of histories of the sum-over-histories formulation. Alternatives analogous to all the Hermitian observables of transformation theory are considered approximately by expressing them in spacetime form. A spacetime description is adequate for our experience and for cosmology. It remains to be seen whether it is fundamental, as assumed here, or whether the theory can be extended to an even richer class of alternatives.

- Formally, the generalized quantum mechanics of spacetime is free from the "problem of time". No preferred family of spacelike surfaces was needed either to define the fine-grained histories, or quantum evolution, or the alternatives for which probabilities are predicted. These were specified directly in four-dimensional, geometrical, terms. This does not mean that the notion of time has been eliminated from this framework, for this is a quantum theory of space*time*! But this generalized quantum framework for spacetime neither requires nor specifies a preferred family of spacelike surfaces.

- Familiar Hamiltonian quantum mechanics of matter fields, with its preferred time(s), is an approximation to this generalized quantum mechanics of spacetime. The approximation is appropriate for decoherent coarse-grainings that specify coarse-grained geometries that display the classical correlations predicted by Einstein's equation with high probability. The classical geometries that summarize these correlations supply the notion of time for an approximate Hamiltonian quantum mechanics of matter fields.

Such classical behavior of geometry is an emergent feature of the boundary conditions in cosmology. Having generalized Hamiltonian quantum mechanics to deal with quantum spacetime, we recover known it in a suitable limit.

- A significant advantage of any sum-over-histories formulation of quantum mechanics is that the classical limit may be analyzed directly. That is especially important in quantum cosmology where we expect that most of the predictions of particular theories of the initial condition that can be confronted with observation will be semiclassical in nature. A system behaves semiclassically when, in a suitably coarse-grained decoherent set of histories, the probability is high for histories correlated by deterministic laws. These probabilities are supplied by this generalized quantum framework providing criteria for when the semiclassical approximation is appropriate. The wave function that specifies the initial condition does not have a direct probabilistic interpretation in this framework. However, assuming their decoherence, the probabilities for histories can be used to provide a justification for the familiar rules that have been used to extract semiclassical predictions directly from wave functions of semiclassical form.

- A lattice version of this generalized quantum mechanics can be constructed using the methods of the Regge calculus with fine-grained histories that are four-dimensional simplicial geometries. Such quantum models are a natural cut-off version of general relativity. They supply a finite and tractable arena in which to examine the low energy, large scale predictions of specific proposals for initial condition and with which to test the sensitivity of these predictions to the nature of quantum gravity at smaller scales.

- This sum-over-histories formulation of the quantum mechanics of cosmological spacetimes is a *generalization* of familiar quantum mechanics that neither utilizes states on spacelike surfaces nor even permits their construction in general. It is therefore different from the usual versions of Dirac or ADM quantum mechanics which are formulated in terms of states on a spacelike surface. Constraints do not play a primary role in constructing quantum dynamics. States satisfying the constraints are used to specify the initial and final conditions of a quantum cosmology but it is only in this sense that "true physical degrees of freedom" are defined. However, should a preferred time be discovered in classical general relativity nothing necessarily needs to be changed in this formulation of the quantum mechanics of spacetime as long as that preferred structure is expressible in terms of the metric. Further, should experiment show that quantum theory singles out a preferred family of spacelike surfaces not distinguished by the classical theory it is still possible to construct a generalized quantum mechanics on the principles described here, by suitably restricting the set of fine-grained histories.

This short list of attractive features does not mean that the generalized quantum mechanics of spacetime that we have described is correct! That determination is, in principle, a matter for experiment and observation. Of course, we are unlikely to have such experimental checks any time in the near future and much remains to be done to complete the theory. The main issue, of course, its to provide a complete and manageable quantum theory of gravity whose consequences can be investigated with the generalized quantum framework developed here. Once that is done problems such as the exact nature of the fine-grained histories and the diffeomorphism invariance of the functional integrals defining sums over these histories may be addressed more precisely. In the meantime, we may analyze these questions in the context of models which capture some of the features of the expected quantum theory of gravity.

As far as quantum cosmology is concerned, the main result of these investigations is to show that the rules for semiclassical prediction that are commonly employed can be put on a firmer probabilistic footing in a generalized quantum framework that does not require a preferred notion of time or or a definition of measurement.

Beyond theories of the initial condition, it is possible that these ideas may be useful in formulating a quantum theory of gravity which must necessarily predict the quantum behavior of spacetime geometry in a suitable limit. Thus, while we have learned little about a correct quantum theory of gravity in these lectures, we may have learned something of how to formulate questions to ask of it.

Acknowledgments

The author is grateful to Bernard Julia for much encouragement in the preparation of these lectures and much patience in their completion. In many places the lectures reflect the discussions that the author had with the students and other lecturers at the school. Thanks are due to B. Julia and J. Zinn-Justin for the stimulating and hospitable atmosphere at Les Houches. Over a longer period of time the author has benefited with conversations from physicists too numerous to mention on the subjects of these lectures. Special thanks should be made to M. Gell-Mann whose joint work with the author on the quantum mechanics of closed systems forms the basis of much of this material, to J. Halliwell for many discussions on quantum cosmology, to K. Kuchař for many discussions on the problem of time in quantum gravity, and to C. Teitelboim and R. Sorkin for conversations on the sum-over-histories formulation of quantum mechanics. Thanks are due to A. Barvinsky, J. Halliwell, C. Isham, R. Laflamme, D. Page, R. Sorkin, and R. Tate for critical readings of the manuscript, and special thanks to K. Kuchař who criticized almost every paragraph. Their constructive criticisms are reflected throughout although they are not, of course, responsible for the remaining errors nor do they necessarily endorse every point of view expressed here. The preparation of these lectures as well as the work they describe was supported in part by the US National Science Foundation under grant PHY-90-08502.

Notation and Conventions

For the most part we follow the conventions of Misner, Thorne, and Wheeler [1] with respect to signature, curvature, and indices. In particular:

Signature — $(-,+,+,+)$ for Lorentzian spacetimes.

Indices — Greek indices range over spacetime from 0 to 3. Latin indices range over space from 1 to 3. Indices on tensors are often suppressed where convenient.

Units — In Sections VI-VIII we use units in which $\hbar = c = 1$. In Section IX we include $\hbar$ explicitly but set $c = 1$; The length ℓ is $\ell = (16\pi G)^{\frac{1}{2}} = 1.15 \times 10^{-32}$cm which is $(4\pi)^{\frac{1}{2}}$ times the Planck length.

Coördinates and Momenta — The four coördinates of spacetime $\{x^\alpha\}$ are frequently abbreviated just as x. Similarly, conjugate momenta $\{p_\alpha\}$ are abbreviated as p. Spatial coördinates $\{x^i\}$ are written $\mathbf{x}$ and spatial momenta $\{p_i\}$ as $\mathbf{p}$. Thus $p \cdot x = p_\alpha x^\alpha$ and $\mathbf{p} \cdot \mathbf{x} = p_i x^i$. Similarly, configuration space coördinates $\{q^i\}$ are written as q, conjugate momenta $\{p_i\}$ as p, and $p \cdot q = p_i q^i$.

Vectors — Four-vectors $a^\alpha, b^\alpha, \cdots$ are written $a, b, c \cdots$ and their inner products as $a \cdot b$, etc. Three-vectors are written as $\vec{a}, \vec{b}, \vec{c} \cdots$ and their inner products as $\vec{a} \cdot \vec{b}$, etc. Thus, in the case of displacement vectors and their conjugate momenta we use $\mathbf{p} \cdot \mathbf{x} = \vec{p} \cdot \vec{x}$ interchangeably.

Covariant Derivatives — ∇_α denotes a spacetime covariant derivative and D_i a spatial one. $\nabla^2 = \nabla_\alpha \nabla^\alpha$. In flat space ∇f is $\nabla_\alpha f$ and $\vec{\nabla} f$ is the usual three-dimensional gradient.

Traces and Determinants — Traces of second rank tensors $K_{\alpha\beta}$ are written as $K = K^\alpha_\alpha$ except when the tensor is the metric in which case g is the determinant of $g_{\alpha\beta}$ and h the determinant of spatial metric h_{ij};

Extrinsic Curvatures — If n_α is the unit normal to a spacelike hypersurface in a Lorentzian spacetime, we define its extrinsic curvature to be

$$K_{ij} = -\nabla_i n_j \ .$$

Intrinsic Curvatures — Intrinsic curvatures are defined so that the scalar curvature of a sphere is positive.

Momentum Space Normalization — We use Lorentz invariant normalization for momentum states of a relativistic particle and include factors of 2π and $\hbar$ as follows:

$$\langle \mathbf{p}\,'' | \mathbf{p}\,' \rangle = (2\pi\hbar)^3 (2\omega_p)\, \delta^{(3)} \left(\mathbf{p}\,'' - \mathbf{p}\,' \right)$$

where $\omega_p = \sqrt{\mathbf{p}^{\,2} + m^2}$. Similarly in the non-relativistic case

$$\langle \mathbf{p}\,'' | \mathbf{p}\,' \rangle = (2\pi\hbar)^3 \delta^{(3)} \left(\mathbf{p}\,'' - \mathbf{p}\,' \right) \ .$$

This convention means that sums over momenta occur as $d^3p/[(2\omega_p)(2\pi\hbar)^3]$ or as $d^3p/(2\pi\hbar)^3$ respectively.

Klein-Gordon Inner Product —

$$i \int_t d^3x \phi^*(x) \frac{\overleftrightarrow{\partial}}{\partial t} \psi(x) = i \int_t d^3x \left[\phi^*(x) \frac{\partial \psi(x)}{\partial t} - \frac{\partial \phi^*(x)}{\partial t} \psi(x) \right] \ .$$

The Feynman Propagator —

$$\Delta_F(x) = \hbar^2 \int \frac{d^4p}{(2\pi\hbar)^4} \frac{e^{ip\cdot x/\hbar}}{p^2 + m^2 - i\epsilon} \ .$$

REFERENCES

1. C.W. Misner, K.S. Thorne, and J.A. Wheeler, *Gravitation*, W.H. Freeman, San Francisco (1970).
2. Y. Aharonov, P. Bergmann, and J. Lebovitz, *Phys. Rev.* **B134**, 1410, (1964).
3. Y. Aharonov and L. Vaidman, *J. Phys. A* **24**, 2315, (1991).
4. R. Arnowitt, S. Deser, and C.W. Misner, in *Gravitation: An Introduction to Current Research*, ed. by L. Witten, John Wiley, New York (1962).
5. M.E. Agishtein and A.A. Migdal, *Nucl. Phys. B* **385**, 395, (1992).
6. A. Ashtekar, *Lectures on Non-Perturbative Quantum Gravity*, World Scientific, Singapore (1991).
7. A. Barvinsky, *Operator Ordering in Theories Subject to Constraints of the Gravitational Type*, University of Alberta preprint, Thy-11-92.
8. I. A. Batalin and G. Vilkovisky, *Phys. Lett.* **69B**, 309, (1977).
9. P. Bergmann and A. Komar, *Int. J. Theor. Phys.* **5**, 15, (1972).
10. D. Bohm, *Quantum Theory*, Prentice-Hall, Englewood Cliffs, N.J. (1951), p. 608.
11. D. Boulware and S. Deser, *Ann. Phys. (N.Y.)* **89**, 193, (1975).
12. N. Bohr, *Atomic Physics and Human Knowledge*, Science Editions, New York (1958).
13. N. Bohr and L. Rosenfeld, *Det Kgl. Danske Vidensk. Selskab Mat.-Fys. Medd.* **12**, nr. 8, (1933).
14. A. Caldeira and A. Leggett, *Physica* **121A**, 587, (1983).
15. R.H. Cameron, *J. Math. and Phys.* **39**, 126, (1960).
16. C. Caves, *Phys. Rev. D* **33**, 1643, (1986).
17. C. Caves, *Phys. Rev. D* **35**, 1815, (1987).
18. C. Caves, (to be published).
19. S. Coleman, *Nucl. Phys.* **B310**, 643, (1988).
20. L. Cooper and D. VanVechten, *Am. J. Phys.* **37**, 1212, (1969).
21. P.C.W. Davies, *The Physics of Time Asymmetry*, University of California Press, Berkeley (1976).
22. S. Deser, *J. Grav. and Rel.* **1**, 9, (1970).
23. B. DeWitt, *Phys. Rev.* **160**, 1113, (1967).
24. B. DeWitt, *Physics Today* **23**, no. 9, (1970).

25. B. DeWitt, in *Proceedings of the Fourth Seminar on Quantum Gravity*, ed. by M.A. Markov, V.A. Berezin, and V.P. Frolov, World Scientific, Singapore (1988) and unpublished NSF proposals.

26. B. DeWitt and R.N. Graham, eds. *The Many Worlds Interpretation of Quantum Mechanics*, Princeton University Press, Princeton (1973).

27. C. DeWitt-Morette, A. Maheshwari, and B. Nelson, *Phys. Rev.* **50**, 255, (1979).

28. P.A.M. Dirac, *Lectures on Quantum Mechanics*, Yeshiva University Press, New York (1964).

29. H. Dowker and J. Halliwell, *Phys. Rev.* **D46**, 1580, (1992).

30. H. Everett, *Rev. Mod. Phys.* **29**, 454, (1957).

31. L. Faddeev, *Teor. i Mat. Fiz.* **1**, 3, (1969) [*Theoretical and Mathematical Physics* **1**, 1, (1970)].

32. W. Feller, *An Introduction to Probability Theory and Its Applications*, John Wiley, New York (1957).

33. R.P. Feynman, *Rev. Mod. Phys.* **20**, 267, (1948).

34. R.P. Feynman, *Phys. Rev.* **76**, 769, (1950).

35. R.P. Feynman, *Phys. Rev.* **84**, 108, (1951).

36. R.P. Feynman, and A. Hibbs, *Quantum Mechanics and Path Integrals*, McGraw-Hill, New York (1965).

37. R.P. Feynman and J.R. Vernon, *Ann. Phys. (N.Y.)* **24**, 118, (1963).

38. E. Fradkin and G. Vilkovisky, *Phys. Rev. D* **8**, 4241, (1973).

39. E. Fradkin and G. Vilkovisky, *Phys. Lett.* **55B**, 224, (1975).

40. E. Fradkin and G. Vilkovisky, *CERN Report TH-2332*, (unpublished) (1977).

41. J. Friedman and I. Jack, *J. Math. Phys.* **27**, 2973, (1986).

42. T. Fukuyama and M. Morikawa, *Phys. Rev.* **D39**, 462, (1989).

43. C. Garrod, *Rev. Mod. Phys.* **38**, 483, (1966).

44. M. Gell-Mann, unpublished (1963).

45. M. Gell-Mann, *Physics Today* **42**, no. 2, 50, (1989).

46. M. Gell-Mann and J.B. Hartle in *Complexity, Entropy, and the Physics of Information, SFI Studies in the Sciences of Complexity*, Vol. VIII, ed. by W. Zurek, Addison Wesley, Reading or in *Proceedings of the 3rd International Symposium on the Foundations of Quantum Mechanics in the Light of New Technology* ed. by S. Kobayashi, H. Ezawa, Y. Murayama, and S. Nomura, Physical Society of Japan, Tokyo (1990).

47. M. Gell-Mann and J.B. Hartle in the *Proceedings of the 25th International Conference on High Energy Physics, Singapore, August, 2-8, 1990*, ed. by

K.K. Phua and Y. Yamaguchi (South East Asia Theoretical Physics Association and Physical Society of Japan) distributed by World Scientific, Singapore (1990).

48. M. Gell-Mann and J.B. Hartle, *Phys. Rev. D* **47**, 3345, (1993).

49. M. Gell-Mann and J.B. Hartle, in *Proceedings of the NATO Workshop on the Physical Origins of Time Assymmetry, Mazagon, Spain, September 30-October4, 1991* ed. by J. Halliwell, J. Perez-Mercader, and W. Zurek, Cambridge University Press, Cambridge (1993).

50. U. Gerlach, *Phys. Rev.* **117**, 1929, (1969).

51. R. Geroch, *Noûs* **18**, 617, (1984).

52. R. Geroch, *J. Math. Phys.* **8**, 782, (1967).

53. S. Giddings and A. Strominger, *Nucl. Phys.* **B307**, 854, (1988).

54. R. Griffiths, *J. Stat. Phys.* **36**, 219, (1984).

55. H.J. Groenewold, *Proc. Akad. van Wetenschappen*, Amsterdam, Ser. B, **55**, 219 (1952).

56. S. Habib and R. Laflamme, *Phys. Rev D* **42**, 4056, (1990).

57. P. Hájíček and K. Kuchař, *Phys. Rev. D* **41**, 1091, (1990).

58. J. Halliwell, *Phys. Rev. D* **36**, 3626, (1987).

59. J. Halliwell, *Phys. Rev. D* **39**, 2912, (1989).

60. J. Halliwell, in *Quantum Cosmology and Baby Universes: Proceedings of the 1989 Jerusalem Winter School for Theoretical Physics*, eds. S. Coleman, J.B. Hartle, T. Piran, and S. Weinberg, World Scientific, Singapore (1991) pp. 65-157.

61. J. Halliwell, *Phys. Rev.* **D46**, 1610, (1992).

62. J. Halliwell and J.B. Hartle, *Phys. Rev. D* **43**, 1170, (1991).

63. J. Halliwell and J.B. Hartle, *Phys. Rev. D* **41**, 1815, (1990).

64. J.J. Halliwell and J. Louko, *Phys. Rev.* **D46**, 39, (1989).

65. J. Halliwell and M. Ortiz, *Sum-Over-Histories Origin of the Composition Law of Relativistic Quantum Mechanics*. MIT Preprint, CTP2134.

66. H. Hamber, *Phys. Rev.* **D45**, 507, (1992) and *Phases of Simplicial Quantum Gravity in Four-Dimensions: Estimates for Critical Exponents*, (to be published in *Nucl. Phys. B*

67. A. Hanson, T. Regge, and C. Teitelboim, *Constrained Hamiltonian Systems*, Accademia Nazionale dei Lincei, Roma (1976).

68. J.B. Hartle, *J. Math. Phys.* **26**, 804, (1985).

69. J.B. Hartle *Class. & Quant. Grav.* **2**, 707, (1985).

70. J.B. Hartle, in *Gravitation in Astrophysics* ed. by J.B. Hartle and B. Carter, Plenum Press, New York (1986).

71. J.B. Hartle, *Phys. Rev. D* **37**, 2818, (1988).

72. J.B. Hartle, *Phys. Rev. D* **38**, 2985, (1988).

73. J.B. Hartle, in *Proceedings of the 5th Marcel Grossmann Conference on Recent Developments in General Relativity*, ed. by D. Blair and M. Buckingham, World Scientific, Singapore (1989) or in *Conceptual Problems of Quantum Gravity*, ed. by A. Ashtekar and J. Stachel, Birkhauser, Boston (1990).

74. J.B. Hartle in *Elementary Particles and the Universe: Essays on Honor of Murray Gell-Mann*, ed. by J. Schwarz, Cambridge University Press, Cambridge (1991).

75. J.B. Hartle, *The Quantum Mechanics of Cosmology*, in *Quantum Cosmology and Baby Universes: Proceedings of the 1989 Jerusalem Winter School for Theoretical Physics*, ed. by S. Coleman, J.B. Hartle, T. Piran, and S. Weinberg, World Scientific, Singapore (1991) pp. 65-157.

76. J.B. Hartle, *Phys. Rev.* **D44**, 3173, (1991).

77. J.B. Hartle, *Vistas in Astronomy* **37**, 569, (1993); or in *Topics in Quantum Gravity and Beyond (Essays in Honor of Louis Witten on His Retirement)* ed. by F. Mansouri and J. J. Scanio, World Scientific, Singapore (1993), or in *Proceedings of the IVth Summer Meeting on the Quantum Mechanics of Fundamental Systems*, Centro de Estudios Cientificos de Santiago, Santiago, Chile, December 26-30, 1991, (to be published).

78. J.B. Hartle, in *Directions in General Relativity, Volume 1: A Symposium and Collection of Essays in honor of Professor Charles W. Misner's 60th Birthday*, ed. by B.-L. Hu, M.P. Ryan, and C.V. Vishveshwara, Cambridge University Press, Cambridge (1993).

79. J.B. Hartle, in *Directions in General Relativity, Volume 2: A Collection of Essays in honor of Professor Dieter Brill's 60th Birthday*, ed. by B.-L. Hu and T.A. Jacobson, Cambridge University Press, Cambridge (1993).

80. J.B. Hartle (unpublished).

81. J.B. Hartle (unpublished).

82. J.B. Hartle (unpublished).

83. J.B. Hartle and S.W. Hawking, *Phys. Rev. D* **28**, 2960, (1983).

84. J.B. Hartle and K. Kuchař, *Phys. Rev.* **D34**, 2323, (1986).

85. J.B. Hartle and R. Sorkin, *Gen. Rel. Grav.* **13**, 541, (1981).

86. S.W. Hawking, *Phys. Lett.* **B196**, 337, (1983).

87. M. Henneaux, *Phys. Rep.* **126**, 1, (1985).

88. M. Henneaux and C. Teitelboim, *Ann. Phys. (N.Y.)* **143**, 127, (1983).

89. M. Henneaux, C. Teitelboim, and J.D. Vergara, *Nucl. Phys. B* **387**, 391, (1992).

90. K. Hepp, *Comm. Math. Phys.* **35**, 265, (1974).

91. G. Horowitz, *Class. Quant. Grav.* **8**, 587, (1991).

92. C. Isham, in *Recent Aspects of Quantum Fields*, ed. by H. Mitter and H. Gausterer, Springer-Verlag, Berlin (1992)

93. C. Isham, in *Integrable Systems, Quantum Groups, and Quantum Field Theories*, ed. by L.A. Ibort and M.A. Rodriguez, Kluwer Academic Publishers, London (1993).

94. E. Joos and H.D. Zeh, *Zeit. Phys.* **B59**, 223, (1985).

95. C. Kiefer, *Class. Quant. Grav.* **4**, 1369, (1987).

96. K. Kuchař, in *Relativity Astrophysics and Cosmology*, ed. by W. Israel, D. Reidel, Dordrecht (1974).

97. K. Kuchař in *Quantum Gravity 2*, ed. by C. Isham, R. Penrose, and D. Sciama, Clarendon Press, Oxford (1981).

98. K. Kuchař, *J. Math. Phys.* **22**, 2640, (1981).

99. K. Kuchař, unpublished lecture notes, University of Utah (1981) or in *Proceedings of the 13th International Conference on General Relativity and Gravitation, Cordoba, Argentina, 1992*, ed. by C. Kozameh *et. al.* (to be published).

100. K. Kuchař, *Phys. Rev.* **D34**, 3044, (1986); *ibid.* **D35**, 596, (1987).

101. K. Kuchař, in *Proceedings of the 4th Canadian Conference on General Relativity and Relativistic Astrophysics*, ed. by G. Kunstatter, D. Vincent, and J. Williams, World Scientific, Singapore, (1992).

102. J. Lee and R. Wald, *J. Math. Phys.* **31**, 725, (1990).

103. T.D. Lee, *Physics Reports* **9C**, 144, (1974).

104. L. Landau and E. Lifshitz, *Quantum Mechanics*, Pergamon, London (1958).

105. F. London and E. Bauer, *La théorie de l'observation en mécanique quantique*, Hermann, Paris (1939).

106. J.J. Loschmidt, *Wiener Ber.* **73**, 128, (1876); *ibid.* **75**, 67, (1877).

107. F. Lund and T. Regge, *Simplicial Approximation to Some Homogeneous Cosmologies* (unpublished).

108. J.E. Marsden and F. Tipler, *Physics Reports* **66**, 109, (1980).

109. M.B. Mensky, *Phys. Rev. D* **20**, 384, (1979); *Theor. Math. Phys.* **75**, 357, (1988).

110. V.F. Mukhanov, in *Proceedings of the Third Seminar on Quantum Gravity*, ed. by M.A. Markov, V.A. Berezin, and V.P. Frolov, World Scientific, Singapore (1985).

111. E. Nelson, *J. Math. Phys.* **5**, 332, (1964).

112. T. Newton and E.Wigner, *Rev. Mod. Phys.* **21**, 400, (1949).

113. R. Omnès, *J. Stat. Phys.* **53**, 893, (1988); *ibid* **53**, 933, (1988); *ibid* **53**, 957, (1988); *ibid* **57**, 357, (1989); *Rev. Mod. Phys.* **64**, 339, (1992).

114. R. Omnès, *J. Stat. Phys.* **57**, 357, (1989).

115. R. Omnès, *Rev. Mod. Phys.* **64**, 339, (1992).

116. T. Padmanabhan, *Phys. Rev. D* **39**, 2924, (1989).

117. T. Padmanabhan and T.P. Singh, *Class. and Quant. Grav.* **7**, 411, (1990).

118. R. Penrose in *General Relativity: An Einstein Centenary Survey* ed. by S.W. Hawking and W. Israel, Cambridge University Press, Cambridge (1979).

119. A. Peres, *Nuovo Cimento* **26**, 53, (1962).

120. T. Piran and R. Williams, *Phys.Rev. D* **33**, 1622, (1986).

121. T. Regge, *Nuovo Cimento* **19**, 558, (1961).

122. C. Rovelli, *Phys. Rev.* **D42**, 2638, (1990); *ibid.* **D43**, 442, (1991).

123. A. Schmid, *Ann. Phys.* **173**, 103, (1987).

124. K. Schleich and D. Witt, *Nucl. Phys. B* **402**, 411, (1993); *ibid.* **402**, 469, (1993).

125. L. Schulman, *Techniques and Applications of Path Integration*, John Wiley, New York, (1981).

126. B. Simon, *Functional Integration in Quantum Physics*, Academic Press, New York (1979).

127. R. Sorkin, *On the Role of Time in the Sum-over-histories Framework for Quantum Gravity*, paper presented at the conference on The History of Modern Gauge Theories held in Logan Utah, July 1987, to be published in *Int. J. Theor. Phys.*.

128. J. Stachel in *From Quarks to Quasars*, ed. by R.G. Colodny, University of Pittsburg Press, Pittsburgh (1986) p. 331ff.

129. C. Teitelboim, *Phys. Rev. D* **25**, 3159, (1983); *ibid.* **28**, 297, (1983); *ibid.* **28**, 310, (1983).

130. C. Teitelboim, *Phys. Rev. D* **25**, 3159, (1983).

131. C. Teitelboim, *Phys. Rev. Lett.* **50**, 795, (1983).

132. H.F. Trotter, *Proc. Am. Math. Soc.* **10**, 887, (1959).

133. N.C. Tsamis and R. Woodard, *Phys. Rev. D* **36**, 3641, (1987).

134. W. Unruh in *New Techniques and Ideas in Quantum Measurement Theory*, ed. by D.M. Greenberger, *Ann. N.Y. Acad. Sci.* **480**, New York Academy of Science, New York (1986).

135. W. Unruh, in *Gravitation: A Banff Summer Institute*, ed. by R. Mann and P. Wesson, World Scientific, Singapore (1991).

136. W. Unruh and W. Zurek, *Phys. Rev. D* **40**, 1071, (1989).

137. A. Vilenkin, *Phys. Rev. D* **37**, 888, (1988).

138. J.A. Wheeler, *Rev. Mod. Phys.* **29**, 463, (1957).

139. J.A. Wheeler in *Batelle Rencontres*, ed. by C. DeWitt and J.A. Wheeler, Benjamin, New York (1968).

140. J.A. Wheeler in *Problemi dei fondamenti della fisica*, Scuola internazionale di fisica “Enrico Fermi”, Corso 52, ed. by G. Toraldo di Francia, North-Holland, Amsterdam (1979).

141. E. Wigner, *Am. J. Phys.* **31**, 6, (1963).

142. R. Williams and P. Tuckey, *Class. Quant. Grav.* **9**, 1409, (1992).

143. N. Yamada and S. Takagi, *Prog. Theor. Phys.* **85**, 985, (1991).

144. N. Yamada and S. Takagi, *Prog. Theor. Phys.* **86**, 599, (1991).

145. N. Yamada and S. Takagi, *Prog. Theor. Phys.* **87**, 77, (1992).

146. H.D. Zeh, *Found. Phys.* **1**, 69, (1971).

147. H.D. Zeh, *Phys. Lett. A* **116**, 9, (1986).

148. H.D. Zeh, *Phys. Lett. A* **126**, 311, (1988).

149. H.D. Zeh, *The Physical Basis of the Direction of Time*, Springer, Berlin, (1989).

150. W. Zurek, *Phys. Rev. D* **24**, 1516, (1981); *ibid.* **26**, 1862, (1982).

151. W. Zurek, in *Non-Equilibrium Quantum Statistical Physics*, ed. by G. Moore and M. Scully, Plenum Press, New York (1984).

152. W. Zurek, *Physics Today* **44**, 36, (1991).

153. W. Zurek, in *Proceedings of the NATO Workshop on the Physical Origins of Time Assymmetry*, Mazagon Spain, Sept 30–Oct 4, 1991, ed. by J. Halliwell, J. Pérez-Mercader, and W. Zurek, Cambridge University Press, Cambridge (1992).