Quantum Semiclass. Opt. **8** (1996) 277–282. Printed in the UK

Measuring a single quantum trajectory

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Abstract. We propose an experiment on single two-level atoms (or ions) which would demonstrate the physical significance of quantum trajectory descriptions for spontaneous decay. We predict that the experiment will reveal the reality of neoclassical decay of the atomic inversion before the emission of a photon.

1. Introduction

Single-quantum-trajectory descriptions of open systems arise in a natural way when the possible histories of the reservoir are taken into account [1, 2]. A recent increase in the interest in quantum trajectories has been stimulated by the possibility of economic numerical simulations of dissipative processes in quantum optics [3–11]. The idea is that the single quantum trajectories are designed in such a way that the average over many such trajectories provides the solution of the master equation. The numerical advantage is that a simulation of a single quantum trajectory requires the determination of the n components of the wavefunction of the reduced system instead of the $n \times n$ components of the corresponding density matrix. Of course, many single quantum trajectories have to be simulated in order to obtain a proper averaged result but still, for large n, this approach can be much more efficient than a direct calculation of the density matrix evolution. At the same time, quantum trajectory descriptions provide a new insight into the statistical nature of the evolution of open systems, in the light of quantum measurement theory.

Several recipes have been developed for single quantum trajectories of a system coupled to the radiation field. The evolution of the state vector representing a quantum trajectory is described by an evolution equation with two terms; one term corresponds to a deterministic evolution, the other term has a stochastic nature, and corresponds to quantum jumps or diffusion of the wavefunction [5, 9, 12]. The different recipes depend on the specific way in which the data is collected from the radiation field. Even if the detection device yields a zero recording the evolution is affected by the specific detection scheme.

A natural question that arises is whether or not such single quantum trajectories do have any physical reality. A pragmatic point of view is that the starting point of each recipe is the master equation which, by definition, only contains information on the evolution of an ensemble, and not on an individual member of this ensemble. Hence, the role of quantum trajectories is simply to decompose the density matrix in pure states, and only the total density matrix has a clear physical significance. On the other hand one can argue that, since the master equation itself is derived from a full quantum mechanical treatment without dissipation (i.e. the total atom–field system is taken into account), an unravelling in single quantum trajectories for well specified states of the radiation field might well be physically realistic.

This paper is written from the latter point of view. We are not interested in systems with a large dimension n of the state space, nor in the numerical aspects of the trajectory

1355-5111/96/010277+06$19.50

method. Instead, we will consider the case of dissipation in a two-level system due to spontaneous decay and discuss the physical relevance of the corresponding quantum trajectory description. In particular, we will address observable effects and propose an experiment.

2. Quantum trajectories and the neoclassical radiation theory

In a previous paper we pointed out that for a two-level atom coupled to the vacuum radiation field which is monitored by an ideal photodetector a single quantum trajectory is a combination of neoclassical decay and quantum jumps [13]. This can be demonstrated as follows. Suppose that at time t the state vector $|\Psi(t)\rangle$ of the coupled atom–field system is given by

$$|\Psi(t)\rangle = |\psi(t)\rangle \otimes |0\rangle \tag{1}$$

where the atomic-state vector $|\psi(t)\rangle$ is a superposition of the ground state $|g\rangle$ and the excited state $|e\rangle$,

$$|\psi(t)\rangle = \alpha_{\mathrm{g}}(t)|g\rangle + \alpha_{\mathrm{e}}(t)|e\rangle \tag{2}$$

and the quantized radiation field is in its ground state $|0\rangle$. A single quantum trajectory is obtained by considering a sequence of atomic evolutions over many time intervals which are much smaller than the spontaneous lifetime Γ^{-1} of the atom, $\mathrm{d}t \ll \Gamma^{-1}$.

During each time interval there are, according to the master equation (see [5, 13]), two ways in which the atom can evolve. (i) The atom can make a jump to the ground state, that corresponds to the emission and detection of a photon for which there is a probability of

$$\mathrm{d}p = \Gamma|\alpha_{\mathrm{e}}(t)|^2\,\mathrm{d}t \ll 1\,. \tag{3}$$

(ii) The atomic state evolves into

$$|\psi(t+\mathrm{d}t)\rangle = \eta\alpha_{\mathrm{g}}(t)|g\rangle + \eta\left(1 - \frac{\Gamma}{2}\,\mathrm{d}t - \mathrm{i}\omega_0\,\mathrm{d}t\right)\alpha_{\mathrm{e}}(t)|e\rangle \tag{4}$$

with ω_0 the atomic transition frequency. Here we introduced an extra normalization factor η,

$$\eta = \frac{1}{\sqrt{1-\mathrm{d}p}}\,. \tag{5}$$

In the second case no detection of a photon has taken place for which there is the complementary probability of (3), i.e. $1 - \mathrm{d}p$.

Equation (4) gives rise to the evolution equation for the amplitudes

$$\frac{\mathrm{d}}{\mathrm{d}t}\alpha_{\mathrm{e}}(t) = -\mathrm{i}\omega_0\alpha_{\mathrm{e}}(t) - \tfrac{1}{2}\Gamma\alpha_{\mathrm{e}}(t)|\alpha_{\mathrm{g}}(t)|^2 \tag{6}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\alpha_{\mathrm{g}}(t) = \tfrac{1}{2}\Gamma\alpha_{\mathrm{g}}(t)|\alpha_{\mathrm{e}}(t)|^2\,. \tag{7}$$

These nonlinear differential equations describe the evolution of the atom during a time interval in which no jump happens to occur.

There are three crucial steps in obtaining a single quantum trajectory, which cannot be derived from the master equation itself but rely on the underlying evolution of the combined atom–field system. First, the physical interpretation of quantum jumps and smooth deterministic evolution is attached to the individual terms in the master equation. Second, it is argued that a quantum jump to the ground state is accompanied by the emission of a photon which does not act back on the atom at later times, and that the jump-free evolution does not yield such a photon. (iii) According to step (ii), the two evolutions are experimentally distinguishable by monitoring the radiation field. Hence in each of the two cases the state vector should be normalized at all times.

From a computational point of view normalization is only an unwanted complication and it is omitted in some recipes for single quantum trajectories. From a physical point of view, however, the normalization is essential since it accounts for the act of a measurement.

From equations (6) and (7) we find the nonlinear differential equations

$$\frac{\mathrm{d}}{\mathrm{d}t}N(t) = -\frac{\Gamma}{2}[1 - N^2(t)] \tag{8}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}M(t) = \frac{\Gamma}{2}M(t)N(t) \tag{9}$$

for the atomic inversion $N(t) = |\alpha_\mathrm{e}(t)|^2 - |\alpha_\mathrm{g}(t)|^2$ and the atomic coherence $M(t) = 2\alpha_\mathrm{g}(t)\alpha_\mathrm{e}^*(t)$, respectively. The solutions of (8) and (9) are

$$N(t) = -\tanh\frac{\Gamma}{2}(t - t_0) \tag{10}$$

$$M(t) = \cosh^{-1}\frac{\Gamma}{2}(t - t_0)M(t_0) \tag{11}$$

with t_0 the instant that the inversion passes zero.

We have pointed out that this description is identical to the neoclassical radiation theory for ‘spontaneous’ emission [13]. This theory, put forth by Crisp and Jaynes [14] and Stroud and Jaynes [15] is based on classical radiation reaction on an oscillating dipole; the radiation field is not quantized and the electron wavefunction is interpreted as a charge density instead of a probability distribution.

The addition of a stochastic quantum jump to the ground-state transforms, after averaging over many trajectories, the neoclassical tanh decay and $\cosh^{-2}$ lineshape, defined as the squared amplitude of the Fourier transform of $M(t)$, into the QED exponential decay and Lorentzian line shape. This also clarifies the status of the neoclassical evolution as a conditional evolution of the system. The influence of radiation reaction on the evolution of the system during a zero-result measurement has been discussed by Dicke [16]. The sound theoretical basis for the jump-free decay, also derived from a full QED calculation in [13], strongly suggests that a single quantum trajectory is more than a useful numerical expedient.

3. Proposal for an experiment

To guide the discussion on the above topic it is very useful to consider a specific experiment. Inspired by the modern developments in single-atom or ion experiments we propose an experiment in order to test the physical reality of the single-quantum-trajectory description given above. Recall that this physical reality depends on the specific measurement performed on the radiation field. In the proposed experiment we will consider ideal broad-band photodetection, where the emitted photons are detected with 100% efficiency. Realistic detection efficiencies can be included by adding classical stochasticity in the description. Then a null measurement would still allow the case that a photon has been emitted, with a known classical probability. In this paper we omit this complication for clarity of the argument.

In our proposed experiment, a two-level atom (or ion) is prepared in a well defined superposition of the ground state and the excited state. This can be done by passing the atom in its ground state through a resonant laser field, so that it acquires a Θ_0 pulse, with $0 < \Theta_0 < \pi$. Next, the atom is allowed to evolve freely in a surrounding where a possible spontaneously emitted photon is recorded with 100% efficiency. If after a time T no photon has been detected, then one might be tempted to argue that the atomic state has remained unchanged, since no energy has been lost. However, the picture of quantum trajectories indicates that the atomic inversion should have decayed according to (10) with t_0 determined by

$$N(0) = \tanh\left(\frac{\Gamma t_0}{2}\right) = -\cos\Theta_0 . \tag{12}$$

The two amplitudes immediately after the pulse can be taken as

$$\alpha_e(0) = i\sin\frac{\Theta_0}{2} \qquad \alpha_g(0) = \cos\frac{\Theta_0}{2} . \tag{13}$$

From this time on, the two amplitudes evolve according to the above quantum trajectory description as

$$\alpha_e(t) = i\sin\frac{\Theta(t)}{2} \qquad \alpha_g(t) = \cos\frac{\Theta(t)}{2} \tag{14}$$

with $\Theta(t)$ determined by the decay of the inversion (equation (10)), so that

$$\cos\Theta(t) = \tanh\left(\frac{\Gamma}{2}(t - t_0)\right) . \tag{15}$$

In order to probe whether or not the atomic state really evolves according to this trajectory, a second pulse $\pi - \Theta(T)$ is applied at time T. This pulse should transform the atomic state to the pure excited state, which yields a 100% probability for an ideal detector to record an emitted photon at some time following the second pulse. If no state evolution had taken place for an atom that did not yield a photon detection in $[0, T]$, the transformation to the excited state would require a pulse $\pi - \Theta_0$, which can be substantially different from $\pi - \Theta(T)$ for a proper choice of Θ_0 and T.

In fact, the difference between the two outcomes is maximal when Θ_0 is close to π, and if T amounts to many lifetimes Γ^{-1}. However, the measurement is only performed on the selected sub-ensemble of atoms that did not yield a photon detection between the two pulses. The complementary fraction has to be discarded. This latter fraction amounts to

$$\sin\left(\frac{\Theta_0}{2}\right)\left(1 - e^{-\Gamma T}\right) \tag{16}$$

so that the relevant sub-ensemble becomes prohibitively small when $\Theta_0 \approx \pi$ and $\Gamma T \gg 1$. Therefore, it seems best to select Θ_0 in such a way that during just a few lifetimes the

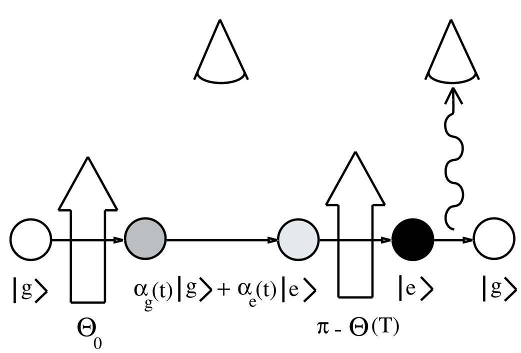

Figure 1. Schematic drawing of the proposed experiment.

difference between Θ_0 and $\Theta(T)$ is appreciable. A reasonable choice is $\Theta_0 = 3\pi/4$, $T = 2t_0$. Then the difference between Θ_0 and $\Theta(T)$ is $\pi/2$, which is large enough to also allow discrimination with realistic non-ideal photodetectors and pulses with limited accuracy. On the other hand, the relevant sub-ensemble still contains about one out of every six atoms. Figure 1 shows a schematic drawing of the proposed experiment.

4. Discussion and conclusions

In this paper we proposed to perform an experiment on single two-level atoms (or ions) which would provide information on the smooth atomic evolution when no photon is observed (by ideal photodetectors). We predict that this experiment will indicate the presence of the neoclassical decay during the jump-free evolution. Furthermore, it would refute the pragmatic interpretation of quantum trajectories, which states that only the average over such trajectories is physically relevant.

It is worthwhile noticing that the energy of a single trajectory is not conserved. During the jump-free evolution, the expectation value of the atomic inversion decreases, and the field energy is constant. On the other hand, when the atom jumps to the ground state, the field gains more energy than the atom loses. When a superposition state (equation (1)) is interpreted as a classical probability distribution over the two atomic states, this would not reflect a change of the physical state, but simply a change of our knowledge about that state. Then the non-observance of a photon would be viewed as an information-gathering process, which enhances the probability that the atom is actually in the ground state. However, this interpretation is refuted in the proposed experiment, since a coherent pulse cannot fully transform a mixed state into the excited state.

In quantum theory, however, the non-conservation of energy during a single trajectory is a common feature of a measurement process. It arises from the fact that the state of the system is not an eigenstate of the quantity measured, whereas a measurement must produce an eigenvalue. Consequently, conservation laws hold only for the average over many measurement results, and the only justified interpretation of single quantum trajectories seems to rely on a full quantum mechanical treatment.

To understand how the neoclassical theory can still be valid for a quantum trajectory up to the photon emission, we recall that the decay in this theory originates from the radiation reaction by the classical radiation field. Using simple Feynman diagrams, it was shown in [13] that the quantum treatment of radiation reaction yields the same result as the classical description, provided that the state is properly normalized. In the quantum treatment the decay of the excited state occurs via repeated application of the interaction Hamiltonian, involving the successive creation and annihilation of a photon. From this observation one can conclude that quantization of the radiation field is not essential for the description of the atomic evolution as long as the detector yields a zero recording. However, to obtain a general description of the evolution of the total atom–field-detector system quantization of both the atom and the field is essential.

Acknowledgments

This work is part of the research programme of the Foundation for Fundamental Research on Matter (FOM) and was made possible by the financial support from the Netherlands Organization for Scientific Research (NWO).

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