

Research Statement

Richard Eager¹

Department of Physics

University of California, Santa Barbara, CA 93106, USA

1 Overview of Research Interests

My research interests are focused on deepening our understanding of strongly coupled quantum field theories. I am interested in quantum field theories with either additional conformal symmetry or supersymmetry. Theories with additional symmetry provide an ideal “toy-model” for applications to particle physics and condensed matter. However, much remains to be understood, even for theories with both supersymmetry and conformal symmetry. I seek general tools that enable us to study broad families of superconformal field theories.

String theory offers several tools to study quantum field theories. Among the most important tools are weak-strong dualities that relate strongly coupled field theories to weakly coupled field theories. An important example is gauge-gravity duality. Polchinski conjectures that “any CFT that has a large- N expansion, and in which all single-trace operators of spin greater than two have parametrically large dimensions, has a local bulk dual” [1]. My current focus has been on $\mathcal{N} = 1$ superconformal field theories, which are an excellent starting point for proving Polchinski’s conjecture.

2 How Much of AdS/CFT Can We Prove?

Maldacena’s discovery of the AdS/CFT correspondence revolutionized string theory, providing a new weak-strong duality between gauge theories and gravitational theories. Shortly thereafter, the AdS/CFT conjecture was generalized to a duality between an $\mathcal{N} = 1$ gauge theory and a supergravity theory.

One prediction of the generalized AdS/CFT conjecture is that the a -anomaly coefficient of the field theory is inversely proportional to the volume of the Sasaki-Einstein manifold, L^5 .

$$\text{Vol}(L^5) \propto \frac{\pi^3}{4} \frac{N^2}{a}.$$

Gubser verified this in the case of the conifold, which was a significant vindication of the dual gauge theory proposed by Klebanov and Witten. In [2], I proved this prediction using four key ingredients:

- The volume of any Sasaki-Einstein manifold, L , can be determined entirely in terms of the Hilbert series of Calabi-Yau singularity $X = C(L)$, which is the cone over L .
- The Hilbert series H_Q of a quiver gauge theory with an AdS_5 -dual can be succinctly encoded by the inverse of a polynomial in the adjacency matrix, M_Q , of the quiver

$$H(Q; t) = \frac{1}{1 - M_Q(t) + t^2 M_Q^T(t^{-1}) - t^2}. \quad (2.1)$$

- The leading asymptotic behavior of the Hilbert series is governed by the smallest eigenvalue of the adjacency matrix.

¹reager@physics.ucsb.edu

- The smallest eigenvalue is proportional to the a -central charge and can be computed using identities between anomalies in 4D $\mathcal{N} = 1$ superconformal field theories.

I plan to extend these methods to prove other aspects of the AdS/CFT correspondence. In particular, I hope to prove that the R-charges of baryonic operators match the AdS/CFT prediction [3] as well.

3 Geometry and $1/N^2$ Corrections to AdS/CFT

With M. Gary and M. Roberts [4], I show that the subleading terms in the Hilbert series of a quiver gauge theory contain information about the integrated Riemann curvature squared in addition to the volume of the Sasaki-Einstein manifold L^5 ,

$$\lim_{s \rightarrow 0} \frac{(2\pi)^3}{27} H_Q(e^{-s}) = \frac{\text{vol}(L^5)}{s^3} + \frac{\text{vol}(L^5)}{s^2} + \left(\frac{91}{216} \text{vol}(L^5) + \frac{1}{1728} \text{Riem}^2(L^5) \right) \frac{1}{s} + \mathcal{O}(1). \quad (3.1)$$

Previously, the only computation of these curvature terms needed the explicit form of the Sasaki-Einstein metric [5]. Since the leading coefficient term is inversely related to the a -anomaly, it is natural to conjecture that the sub-leading terms are related to the $1/N^2$ corrections to the a - and c -anomalies. Recently, [5] showed that the $1/N^2$ correction to $a - c$ contains a contribution from the integrated curvature squared term. I plan to extend these two papers to prove that anomaly matching in *AdS/CFT* holds to order $1/N^2$.

4 New Tools for Constructing Gauge Theories

A central goal of my research is the development of computationally effective tools for determining the low energy effective physics of a stack of D-branes at a Calabi-Yau singularity. The low energy physics is described by a four dimensional supersymmetric gauge theory whose form depends crucially on the Calabi-Yau geometry. In my research, I use diverse mathematical tools, including the derived category of coherent sheaves and matrix factorizations mixed with insights from gauge theory to determine the matter content of the gauge theory and its superpotential.

In [6], I show that the endomorphism algebra of a tilting object can be used to derive the gauge theory dual to the $L^{a,b,c}$ family of singularities. This world-sheet description [7] matches all of the examples derived from a space-time perspective in [8, 9, 10]. The labeling of gauge groups by modules leads to a simple description of a duality cascade confirming a conjecture of Butti, Forcella, and Zaffaroni [10]. Surprisingly, my construction of the $L^{a,b,c}$ gauge theories [6] was based on a construction used by David Speyer [11] to study the octahedron recurrence. Speyer's work provides a combinatorial interpretation of non-linear recurrence relations in terms of perfect matchings. Currently, I am working on generalizing Speyer's construction to recursively determine the finite pyramid partition functions for the $L^{a,b,c}$ geometries. In string theory, these partition functions encode the BPS partition function of D0-D2-D6 bound states.

In joint work in progress with S. Franco and M. Romo, we study how finite pyramids arise from framed quivers. By varying the framing of the quiver, we can change the shape of the corresponding pyramids. The shape of the pyramid encodes a quantized version of the Kähler parameters of the original Calabi-Yau geometry. I intend to use these pyramids to provide a finite analog of [12], connecting Seiberg duality of quiver gauge theories to the Kontsevich-Soibelman and Denef-Moore wall-crossing formulas.

5 Future Directions

While string theory suggests that any four-dimensional $\mathcal{N} = 1$ superconformal quiver field theory should be realizable in string theory, until recently most evidence was based on the preponderance of examples. In [2], I show that there is a simple way to determine the Hilbert series of a Calabi-Yau singularity from the generalized adjacency matrix of the quiver gauge theory. In favorable cases, this is enough information to realize the gauge theory in string theory. Using ideas from representation theory, I hope to show how any superconformal quiver gauge theory can be constructed in string theory. One key ingredient will be the careful matching of superpotential deformations of the quiver gauge theory to fluxes and complex structure moduli in the *AdS* dual theory.

I have been able to reconstruct volume and curvature data of the *AdS/CFT* dual geometry using the spectrum of mesonic operators in the quiver field theory. The mesonic operators are a special class of single-trace operators in the chiral ring. I plan on rephrasing my results entirely in terms of the chiral ring of the field theory. This will further the study of all $\mathcal{N} = 1$ superconformal field theories, including those without a Lagrangian description. This will enhance our understanding of the *AdS/CFT* correspondence and will provide a solid starting point for extending our understanding of gauge-gravity duality to theories without supersymmetry or conformal invariance.

References

- [1] I. Heemskerk, J. Penedones, J. Polchinski, and J. Sully, “Holography from Conformal Field Theory,” *JHEP* **10** (2009) 079, [arXiv:0907.0151 \[hep-th\]](#).
- [2] R. Eager, “Equivalence of A-Maximization and Volume Minimization,” [arXiv:1011.1809 \[hep-th\]](#).
- [3] D. Berenstein, C. P. Herzog, and I. R. Klebanov, “Baryon spectra and AdS/CFT correspondence,” *JHEP* **06** (2002) 047, [arXiv:hep-th/0202150](#).
- [4] R. Eager, M. Gary, and M. M. Roberts, “Can you hear the shape of dual geometries?,” [arXiv:1011.5231 \[hep-th\]](#).
- [5] J. T. Liu and R. Minasian, “Computing $1/N^2$ corrections in AdS/CFT,” [arXiv:1010.6074 \[hep-th\]](#).
- [6] R. Eager, “Brane Tilings and Non-Commutative Geometry,” [arXiv:1003.2862 \[hep-th\]](#).
- [7] P. S. Aspinwall, “D-Branes on Toric Calabi-Yau Varieties,” [arXiv:0806.2612 \[hep-th\]](#).
- [8] S. Franco *et al.*, “Gauge theories from toric geometry and brane tilings,” *JHEP* **01** (2006) 128, [arXiv:hep-th/0505211](#).
- [9] S. Benvenuti and M. Kruczenski, “From Sasaki-Einstein spaces to quivers via BPS geodesics: Lpqr,” *JHEP* **04** (2006) 033, [arXiv:hep-th/0505206](#).
- [10] A. Butti, D. Forcella, and A. Zaffaroni, “The dual superconformal theory for $L(p,q,r)$ manifolds,” *JHEP* **09** (2005) 018, [arXiv:hep-th/0505220](#).
- [11] D. E. Speyer, “Perfect matchings and the octahedron recurrence,” *J. Algebraic Combin.* **25** (2007) no. 3, 309–348, [math.CO/0402452](#).
- [12] M. Aganagic and K. Schaeffer, “Wall Crossing, Quivers and Crystals,” [arXiv:1006.2113 \[hep-th\]](#).