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Level Truncation and Rolling the Tachyon in the Lightcone Basis for Open String Field Theory

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Abstract: A recent paper by Gross and Erler (hep-th/0406199) showed that by making a certain well-defined, unitary transformation on the mode basis for the open bosonic string—one that identifies the lightcone component of position with the string midpoint—it is possible to render the action for cubic string field theory local in lightcone time. In this basis, then, cubic string field theory possesses a well-defined initial value formulation and a conserved Hamiltonian. With this new understanding it seems natural to study time dependent solutions representing the the decay of an unstable D-branes. In this paper we study such solutions using level truncation of mode oscillators in the lightcone basis, finding both homogenous solutions by perturbatively expanding the string field in modes e^{nt}, and inhomogenous solutions by integrating the equations of motion on a lattice. Truncating the theory to level $(\tilde{2},\tilde{4})$ in α^+ oscillators, we find time dependent solutions whose behavior seems to converge to that of earlier solutions constructed in the center of mass basis, where the cubic action contains an infinite number of time derivatives. We further construct time-dependent inhomogeneous solutions including all fields up to level $(\tilde{2},\tilde{4})$. These solutions at the outset display rather erratic behavior due to an unphysical instability introduced by truncating the theory at the linear level. However upon truncating away the field responsible for the instability, we find more reasonable solutions which may possibly represent an approximation to tachyon matter. We conclude with some discussion of future directions.

Keywords: String Field Theory, Tachyon Condensation.

Contents

1. Introduction

String field theory has seen a remarkable revival following the conjectures of Sen[1], which have revealed (upon extensive numerical work) that string field theory provides a framework where quite distinct string backgrounds can be formulated in terms of a single set of underlying degrees of freedom. The most famous example of this is the closed string vacuum solution of open bosonic string field theory[2]. This solution describes the state of the open string after all unstable branes in the bosonic theory are allowed to decay, and as such must represent the vacuum of the closed bosonic string without any open string excitations. Indeed, numerical analysis in the level truncation scheme has both revealed the absence of open string states[3, 4] and verified that the difference in energies between the unstable D25-brane vacuum and the stable vacuum is precisely the energy density of the space filling D25-brane, within a fraction of a percent[5, 6]. For a useful review of open bosonic string field theory and related work see ref.[7].

String theory must eventually be able to address cosmological questions, and hence it seems crucial to understand the role of time and dynamical solutions in the theory. String field theory, incorporating some measure of background independence, seems to be a natural framework in which to contemplate these questions. One particularly simple problem is the nature of the time-dependent decay of the unstable D25-brane in the bosonic theory. Sen[8] proposed a remarkably simple boundary conformal field theory describing such a process, whereby the tachyon rolls homogenously off the unstable maximum towards the closed string vacuum, but does not cross over in finite time. At late times, the resulting "tachyon matter" describes a pressureless gas[9] whose cosmological implications have been explored quite extensively in the literature[**?**].

Surprisingly, open bosonic string field theory has proven to be quite inept at recovering this physically interesting solution. Studies of the rolling tachyon process in string field theory have approached the problem from many perspectives: inverse Wick rotating a marginal solution[8, 10], perturbative expansion of the field equations in modes[11], and studies of time-dependence in p-adic models[10, 12]. All of this work has revealed a consistent picture, though one drastically different from Sen's: the tachyon rolls off the unstable maximum, speeds quickly through the closed string vacuum and then far up the steep side of the potential, in fact quite a bit further than the height of the unstable vacuum, after which a sequence of oscillations of diverging amplitude ensues. Strangely, in this process the tachyon can roll arbitrarily far into the negative, unbounded side of the cubic-like potential, yet somehow feel the urge to turn around and roll back up towards the unstable vacuum! The pressure of the string field does not seem to vanish at late times, but rather oscillates with diverging amplitude[12].

This type of time dependent behavior is at the very least strange, if not catastrophic. It is not difficult to see where the problem lies. As is well known, the cubic vertex describing the interaction of the open bosonic string contains an infinite number of derivatives in both time and space. As a result, the kinetic energy for the truncated open string theory (as well as for p-adic models) is not positive definite; this explains in particular how the tachyon can roll up to a height even *greater* than that of the unstable vacuum where it originated, yet apparently manage to conserve energy. The type of instability we're witnessing is in fact generic in any theory whose Lagrangian depends nontrivially on any more than first time derivatives, as was discovered over 1.5 centuries ago by Ostrogradski[13]. To see the problem, consider a Lagrangian which depends on coordinates, velocities, accelerations etc. up to some order N, $L(q, \dot{q}, ..., q^{(N)})$. Ostrogradski tells us we can find the Hamiltonian by defining phase space variables,

$$Q_n = q^{(n-1)} \qquad P_n = \sum_{k=n}^{N} \left(-\frac{d}{dt}\right)^{k-n} \frac{\partial L}{\partial q^{(k)}} \qquad n = 1, 2, ...N$$

representing the $2N$ initial conditions necessary to specify a solution to the Euler-Lagrange equations. The Hamiltonian is then[1],

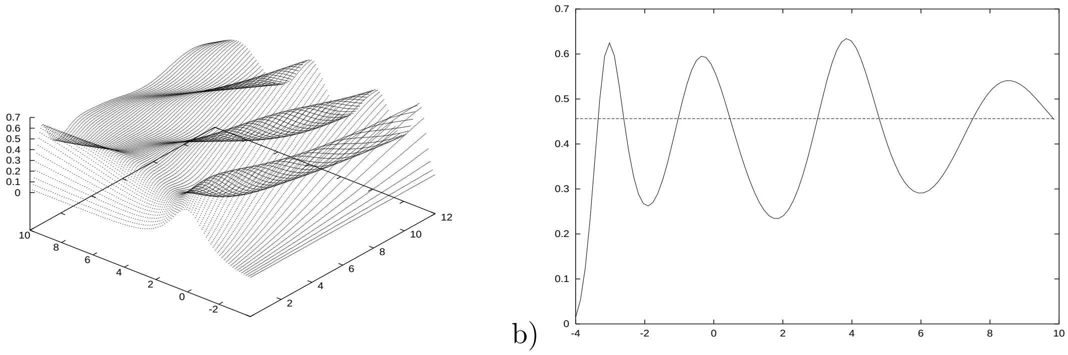
$$H = \sum_{n=1}^{N-1} P_n Q_{n+1} + P_N \dot{Q}_N - L(Q_1, ..., Q_N, \dot{Q}_N)$$

Since P_n and Q_{n+1} for $n < N$ are independent phase space variables, and nothing else in the Hamiltonian depends on P_n, it is clear that the first term in this equation can be made arbitrarily negative and the Hamiltonian is unbounded from below. So truncated open string field theory and p-adic models are probably sick theories, and the erratic behavior of their solutions should be no surprise[2].

A recent paper by Gross and the author[16] proposed an alternative. There, we showed that by making a certain well-defined, unitary transformation on the mode basis for the

[1]If the theory is nondegenerate, we can invert the relation for P_N to write $q^{(N)} = \dot{Q}_N(Q_1, ..., Q_N, P_N)$

[2]There has been some hope that the $N \to \infty$ higher derivative limit may fail in truncated string field theory and p-adic string theory, and that requirements of analyticity may constrain the initial value problem to the point where the theory has a more or less the usual canonical structure and a stable Hamiltonian. This possibility does not seem to be borne out by the analyzes of references [14, 15].



a) b)

Figure 1: a) ϕ graphed vertically, as a function of (x^+, x^-). x^- increases to the left, and "time" x^+ increases to the right. b) The profile of ϕ as a function of x^- at $x^+ = 12$. The horizontal line represents the closed string vacuum.

open bosonic string—one that identifies the lightcone component of position with the string midpoint—it is possible to render the action for cubic string field theory local in lightcone time. (In the usual mode basis, both space and time are identified with the open string center of mass.) Thus, it would seem that the *exact* string field theory has both a well-defined initial value formulation and a Hamiltonian free of higher derivative instabilities. This suggests that the unfortunate time-dependent behavior we're witnessing may be an unphysical artifact of the level truncation scheme, and the exact theory admits more reasonable time-dependent solutions, perhaps even ones resembling Sen's tachyon matter.

In this paper we investigate time-dependent solutions by truncating string field theory in this new "lightcone" basis. To get a feeling for what is involved, consider the action truncated to level 0 in the lightcone basis:

$$S_0 = \int dx \left[\phi(-\partial_+\partial_- + \tfrac{1}{2}\,\partial_\perp^2 - 1)\phi + \frac{2g\kappa}{3}\left(e^{-\frac{1}{2}V_{00}\partial_\perp^2}\phi\right)^3 \right] \tag{1.1}$$

where $x^+ = \frac{1}{\sqrt{2}}(x^0 + x^1)$ $x^- = \frac{1}{\sqrt{2}}(x^0 - x^1)$ and $x_\perp = \{x^2, x^3, ...\}$, g is the open string coupling, and κ, V_{00} are numerical constants. The field ϕ here is a linear combination of the tachyon and lightcone derivatives of an infinite number of massive fields. There are two things to notice about this action. First, the Lagrangian is first order in lightcone time derivatives ∂_+, which appear only in the kinetic term. Second, the action is not Lorentz invariant, as can be seen by the derivative dependence in the interaction. Lorentz invariance, in fact, is only preserved in the level truncation scheme in the infinite level limit. The curious absence of Lorentz invariance, at the deepest level, is really connected to the fact that string theory does not admit a formulation in terms of spacetime fields with completely local interactions[16]; we can only achieve locality along a single null direction. Thus, the loss of Lorentz invariance at finite level is the price we have to pay for having a formulation of string field theory which is not at the outset rendered pathological by higher derivative instabilities and the absence of a well-posed initial value problem.

We can easily imagine finding a dynamical solution to eq.1.1 representing the decay of a D-25 brane. In fact, looking for solutions independent of $x_\perp$, eq.1.1 reduces to that of an ordinary ϕ^3 theory in two dimensions. To let the D25-brane decay, all we have to do is set boundary conditions on the initial like-like surface $x^+ = 0$ which displace ϕ off of the unstable maximum $\phi = 0$ towards the "closed string vacuum" at $\phi = \frac{1}{g\kappa}$, and then integrate the field equations. A numerical solution for Gaussian initial conditions is plotted in figure 1. The field simply flows off of the unstable maximum and oscillates in a completely innocuous way around the stable vacuum. Of course, this is simply a solution to ϕ^3 theory, and we wouldn't expect much uniquely stringy physics to emerge, but as a first pass it isn't bad.

This paper is organized as follows. In section 2 we review some features of the lightcone formalism of ref.[16] which will be relevant for our analysis. In section 3 we study the effect of level truncation on the linear term in the string field theory action. We find that when truncating the linear term we both loose linear solutions to the exact theory and introduce some spurious solutions associated with α^- excitations. These unstable solutions result from the fact that the truncated equations of motion "drive themselves" at their resonant frequency. In section 4 we investigate time dependent solutions representing D25-brane decay using a modified level expansion where we include only α^+_{2n} excitations. The motivation for this particular truncation scheme is that in the infinite level limit the solutions should behave similarly to those constructed in the center of mass basis, which contains an infinite number of time derivatives. Indeed, in this truncation we find that the tachyon quickly passes through the closed string vacuum and then up the steep side of the potential to a height *greater* than that of the unstable vacuum where it originated. We explain in detail how in this truncation it is possible to approximate solutions "natrual" in one basis by fields in the other. In section 5 we study D25-brane decay including all fields up to level $(\tilde{2}, \tilde{4})$. We find that, due to a spurious linear instability introduced by level truncation, the solutions at this level behave somewhat erratically. However, by setting the field responsible for this instability to zero, we find much more reasonable solutions. The picture that emerges seems to indicate that, in comparison to solutions generated by including only scalars at level $\tilde{2}$, the tachyon rolls faster towards the closed string vacuum initially but then rapidly decelerates and does not become as large before turning around for the first time. We end with some concluding remarks.

2. Review

Let us review the basics of the lightcone basis introduced in ref.[16, 17]. A string field $|\Psi\rangle$ is an element of the state space $\mathcal{H}_{\rm BCFT}$ of a combined matter-ghost boundary conformal field theory describing an open bosonic string living on a space-filling D-25 brane. The usual basis for $\mathcal{H}_{\rm BCFT}$ is given by the mode oscillators α^μ_n, b_n, c_n acting on the vacuum $|k\rangle$ describing the open string tachyon at momentum k ($\alpha^\mu_0 = p^\mu$). Consider a change of basis generated by the unitary transformation,

$$U = \exp\left[-p_+ \sum_{n=1}^{\infty} \frac{(-1)^n}{2n}(\alpha^+_{2n} - \alpha^+_{-2n})\right]. \tag{2.1}$$

Under this change of basis the matter oscillators and zero-modes transform as,

$$\begin{aligned}\tilde{\alpha}_n^- &\equiv U\alpha_n^- U^{-1} = \alpha_n^- + \cos\tfrac{n\pi}{2}\, p_+ \qquad n \neq 0 \\ \tilde{x}^+ &\equiv U x^\mu U^{-1} = x^+ i\sqrt{2}\sum_{n=1}^{\infty}\frac{(-1)^n}{2n}(\alpha_{2n}^+ - \alpha_{-2n}^+) = x^+(\tfrac{\pi}{2}).\end{aligned} \tag{2.2}$$

The ghosts and other Lorentz components of α^μ are unaffected by this transformation. To simplify notation, we will denote all components of the matter oscillators in this basis using a tilde $\tilde{\alpha}^\mu$, even though only the minus component of the even oscillators is affected. The vacuum $|k\rangle$ transforms into a state $|k\rangle'$:

$$|k\rangle' \equiv \exp\left[k_+ \sum_{n=1}^{\infty}\frac{(-1)^n}{2n}\alpha_{-2n}^+\right]|k\rangle, \tag{2.3}$$

Since the transformation is unitary, this basis satisfies the usual properties,

$$\begin{aligned}[\tilde{\alpha}_m^\mu, \tilde{\alpha}_{-n}^\nu] &= m\eta^{\mu\nu}\delta_{mn} \qquad [b_m, c_{-n}] = \delta_{mn} \\ \tilde{\alpha}_n^\mu|k\rangle' &= b_n|k\rangle' = c_n|k\rangle' = 0 \qquad n > 0 \\ p_\mu|k\rangle' &= k_\mu|k\rangle' \qquad b_0|k\rangle' = 0 \\ (\tilde{\alpha}_n)^+ &= \tilde{\alpha}_{-n} \qquad (b_n)^+ = b_{-n} \qquad (c_n)^+ = c_{-n}\end{aligned} \tag{2.4}$$

The zeroth matter Virasoro generator can be expressed,

$$\begin{aligned}L_0 &= \tfrac{1}{2}p^2 + p_+\sum_{n=1}^{\infty}(-1)^n(\tilde{\alpha}_{2n}^+ + \tilde{\alpha}_{-2n}^+) + \sum_{n=1}^{\infty}\tilde{\alpha}_{-n}\cdot\tilde{\alpha}_n \\ &= \tilde{L}_0|_0 + \tfrac{1}{2}p_\perp^2 + p_+P^+(\tfrac{\pi}{2})\end{aligned} \tag{2.5}$$

where $P(\frac{\pi}{2})$ is the momentum of the string midpoint, and $\tilde{L}_0$ is simply L_0 with the replacement $\alpha \to \tilde{\alpha}$, $|_0$ means we evaluate the operator at zero momentum. The zeroth ghost Virasoro takes the usual form,

$$L_0^{gh} = \sum_{n=1}^{\infty} n(b_{-n}c_n + c_{-n}b_n)$$

The BPZ inner product $\langle , \rangle$ satisfies the familiar relations:

$$\begin{aligned}\langle\tilde{\alpha}_{-m}^\mu\Psi, \Phi\rangle &= (-1)^{m+1}\langle\Psi, \tilde{\alpha}_m^\mu\Phi\rangle \\ \langle b_{-m}\Psi, \Phi\rangle &= (-1)^m(-1)^\Psi\langle\Psi, b_m\Phi\rangle \\ \langle c_{-m}\Psi, \Phi\rangle &= (-1)^{m+1}(-1)^\Psi\langle\Psi, c_m\Phi\rangle\end{aligned} \tag{2.6}$$

where $(-1)^\Psi$ denotes the Grassmann parity of Ψ. The two string vertex $\langle V_2|$ is as in the old basis after the replacement of the oscillators and vacua with their tilded counterparts.

String interactions are defined by the cubic vertex $\langle V_3| \in \mathcal{H}_{BCFT}\otimes\mathcal{H}_{BCFT}\otimes\mathcal{H}_{BCFT}$ which in the matter sector takes the form,

$$\begin{aligned}\langle V_3^m| = \kappa\int dk^1dk^2dk^3\delta(k^1+k^2+k^3)\langle +, k^1|'\langle +, k^2|'\langle +, k^3|' \\ \exp\left[-\tfrac{1}{2}V_{00}^{AB}k_\perp^A\cdot k_\perp^B - V_{m0}^{AB}\tilde{a}_m^A\cdot k_\perp^B - V_{m0}^{AB}\tilde{a}_m^{-,A}k_-^B - \tfrac{1}{2}V_{mn}^{AB}\tilde{a}_m^A\cdot\tilde{a}_n^B\right]\end{aligned} \tag{2.7}$$

where $\tilde{a}_m = \frac{\tilde{\alpha}_m}{\sqrt{m}}$, the capital indices A, B refer to the state space and range from 1 to 3, and the mode number indices m, n range from 1 to ∞, all repeated indices summed. Note that the exponential has no dependence on k_+, meaning that the vertex contains no derivatives with respect to $\tilde{x}^+$ and is therefore local in lightcone time. The ghost component of the vertex in Siegel gauge takes the usual form, which we write down for reference:

$$\langle V_3^{gh}|b_0^{(1)}b_0^{(2)}b_0^{(3)} = \langle -|\langle -|\langle -|\exp\left[X_{mn}^{AB}c_m^A b_n^B\right]$$

Of course, $\langle V_3| = \langle V_3^m|\langle V_3^{gh}|$. Explicit expressions for the constants κ, V, and X (the latter two are called "Neumann coefficients") were calculated in ref.[18] and can be found for convenient reference for example in ref.[7].

At zero momentum, the lightcone basis is identical to the center of mass basis. Therefore, since the oscillator expression for the closed string vacuum is identical in either basis, at this level either basis would seem equally appropriate for investigating dynamics about the vacuum.

For a real string field $|\Psi\rangle$ in Siegel gauge $b_0|\Psi\rangle = 0$, the string field theory action takes the form,

$$S = \langle\Psi|c_0(L_0 + L_0^{gh} - 1)|\Psi\rangle + \frac{2g}{3}\langle V_3||\Psi\rangle|\Psi\rangle|\Psi\rangle \tag{2.8}$$

The assumption of reality amounts to the condition,

$$\langle\Psi, A\rangle = \langle\Psi|A\rangle$$

for any string field $|A\rangle$. The equations of motion derived from eq.2.8 are,

$$\left[\tfrac{1}{2}p_\perp^2 + p_+P^+(\tfrac{\pi}{2}) + \tilde{L}_0|_0 + L_0^{gh} - 1\right]|\Psi\rangle + gb_0\langle\Psi|\langle\Psi||V_3\rangle = 0 \tag{2.9}$$

In this paper we will always be looking for time dependent solutions in Siegel gauge, so eq.2.9 will be sufficient for our purposes. These equations specify the dynamics for an *infinite* number of local spacetime fields, so it seems impossible to generate a numerical solution which includes all of them. So we are forced to truncate the theory, including only a few of the lightest mass fields and setting the remaining ones to zero by fiat. The usual approach is to expand the equations in the mode basis α_n, b_n, c_n where position x corresponds to the string center of mass, and keep fields in the first few low-lying levels. As we've seen, the resulting approximate equations of motion contain an infinite number of time derivatives, and the dynamical solutions behave very strangely. Instead, we will expand the string field in the lightcone basis $\tilde{\alpha}_n, b_n, c_n$, and keep only the first few levels. As a matter of notation we will use $(\tilde{n}, \tilde{i})$ to denote the truncation of the theory to include fields up to level n and interactions up to level i in the lightcone basis, and reserve the notation (n, i) (no tildes) for the corresponding truncation in the center of mass basis. The equations of motion are then first order on lightcone time, and we can hope that the solutions will be more reasonable.

One further comment: A solution to eq.2.9 does not, in itself, necessarily represent a full solution to the equations of motion in open string field theory. Eq.2.9 must be supplemented by constraints on the initial conditions, which ensure that dynamics only proceeds on a physical submanifold in phase space where the Hamiltonian is positive (modulo the tachyon). Unfortunately, the consistency of the constraints with time evolution at finite level turns out to be a somewhat difficult issue. We will discuss this more in section 5.

3. The Linear Problem

The first step in constructing time dependent solutions in the lightcone basis is to understand what level truncation does to the theory at the linear level. To facilitate discussion, let's introduce a little notation. Consider a projection operator, $\tilde{P}_n$, projecting onto states at level n (in the lightcone basis). It satisfies some basic properties,

$$\begin{aligned} &\tilde{P}_m\tilde{\alpha}_n = \tilde{\alpha}_n\tilde{P}_{m+n} \qquad \tilde{P}_m b_n = b_n\tilde{P}_{m+n} \qquad \tilde{P}_m c_n = c_n\tilde{P}_{m+n} \\ &\tilde{P}_n|k\rangle' = \delta_{n0}|k\rangle' \qquad [\tilde{P}_n,\tilde{x}] = [\tilde{P}_n,p] = [\tilde{P}_n,b_0] = [\tilde{P}_n,c_0] = 0 \end{aligned} \tag{3.1}$$

Similarly we have P_n (without a tilde) which projects onto level n in the center of mass basis. Now define,

$$\tilde{\Pi}_n = \sum_{i=-\infty}^{n}\tilde{P}_i$$

$\tilde{\Pi}_n$ truncates any state up to level n in the lightcone basis (or for Π_n in the center of mass basis). Truncating the theory up to level $(\tilde{n},\tilde{3n})$ amounts to finding some state $|\Psi\rangle = \tilde{\Pi}_n|\Psi\rangle$ which satisfies the approximate field equations,

$$\tilde{\Pi}_n\Big[(L_0 + L_0^{gh} - 1)|\Psi\rangle + gb_0\langle\Psi|\langle\Psi||V_3\rangle\Big] = 0$$

In principle, sending $n\to\infty$ we recover an exact solution.

Let's consider level truncation in the free theory ($g = 0$). In the center of mass basis, this amounts to solving the equations,

$$\Pi_n(L_0 + L_0^{gh} - 1)|\Psi\rangle = (L_0 + L_0^{gh} - 1)|\Psi\rangle = 0 \tag{3.2}$$

where above we noted that Π_n commutes with L_0, L_0^{gh}. Of course, eq.3.2 is just the exact equations of motion. Apparently, level truncation at the free level is trivial in the center of mass basis. However, this is not what happens in the lightcone basis. In this case, the truncated linear equations are,

$$\left[\tfrac{1}{2}p^2 + \tilde{L}_0|_0 + L_0^{gh} - 1\right]|\Psi\rangle + p_+\tilde{\Pi}_n\sum_{n=1}^{\infty}(-1)^n(\tilde{\alpha}_{2n}^+ + \tilde{\alpha}_{-2n}^+)|\Psi\rangle = 0 \tag{3.3}$$

Since the operator multiplying $|\Psi\rangle$ in the second term does not preserve level number, $\tilde{\Pi}_n$ does not commute with it, and eq.3.3 is not equivalent to the exact equations of motion.

Since eq.3.2 and eq.3.3 are not the same, their solutions are inequivalent. This would seem problematic, since if we can't even get the correct time dependent solutions at the linear level, we certainly shouldn't expect to find reliable ones once we turn on the coupling.

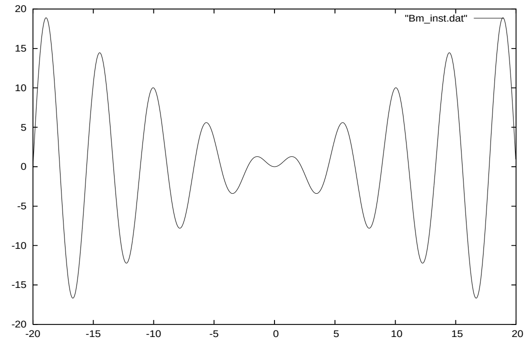


Figure 2: Spurious instability in B_+ at level 2.

To see the nature of this phenomenon, consider the first nontrivial case, level $\tilde{2}$. There, we have the string field,

$$|\Psi\rangle = \int dk \left[\phi(k) + \frac{i}{\sqrt{2}}(B_+(k)\tilde{\alpha}^+_{-2} + B_-(k)\tilde{\alpha}^-_{-2})\right] |k\rangle' \tag{3.4}$$

Of course, there are other fields at level 2, but they are less interesting since they only couple to fields at higher level. Plugging eq.3.4 into eq.3.3 for $n = 2$, we find the equations,

$$\begin{aligned} 0 &= (-\tfrac{1}{2}\,\partial^2 - 1)\phi + \sqrt{2}\partial_+ B_- \\ 0 &= (-\tfrac{1}{2}\,\partial^2 + 1)B_+ + \sqrt{2}\partial_+\phi \\ 0 &= (-\tfrac{1}{2}\,\partial^2 + 1)B_- \end{aligned} \tag{3.5}$$

For comparison, the analogous equations of motion in the center of mass basis for ϕ^{cm} (the tachyon) and $B_+^{\text{cm}}, B_-^{\text{cm}}$ are,

$$\begin{aligned} 0 &= (-\tfrac{1}{2}\,\partial^2 - 1)\phi^{\text{cm}} \\ 0 &= (-\tfrac{1}{2}\,\partial^2 + 1)B_+^{\text{cm}} \\ 0 &= (-\tfrac{1}{2}\,\partial^2 + 1)B_-^{\text{cm}} \end{aligned} \tag{3.6}$$

Indeed, these equations are similar except for the presence of extra linear couplings.

So, what are the solutions of eq.3.5? Consider for simplicity spatially homogenous time dependent solutions ($t = \frac{1}{\sqrt{2}}(\tilde{x}^+ + x^-)$). Making a plane wave ansatz for B_-, we can plug in and find the general solution,

$$\begin{aligned} B_-(t) &= Ae^{i\sqrt{2}t} + Be^{-i\sqrt{2}t} \\ \phi(t) &= Ce^{\sqrt{2}t} + De^{-\sqrt{2}t} + \frac{i}{\sqrt{2}}\left(Ae^{i\sqrt{2}t} - Be^{-i\sqrt{2}t}\right) \\ B_+(t) &= Ee^{i\sqrt{2}t} + Fe^{-i\sqrt{2}t} - \frac{1}{\sqrt{2}}\left(Ce^{\sqrt{2}t} + De^{-\sqrt{2}t}\right) - \frac{i}{\sqrt{2}}t\left(Ae^{i\sqrt{2}t} - Be^{-i\sqrt{2}t}\right) \end{aligned} \tag{3.7}$$

where $A, B, \ldots$ are arbitrary constants, subject to reality. The first pair of terms in each of these equations is the homogenous solution, corresponding to the oscillation (or decay, as it were) of the fields at their natural frequency. The remaining terms above come from the linear couplings in eq.3.5 which can be interpreted as external driving forces. Specifically, the field ϕ blows up exponentially at late and/or early times—corresponding to the decay of the unstable D-25 brane—but also oscillates because of its coupling to B_-. B_+ is more interesting. In addition to natural oscillation and tachyonic decay because of coupling to ϕ, we also see an awkward component where B_+ oscillates with linearly diverging amplitude (see figure 2). This behavior is directly because of B_-, which drives B_+ at its natural frequency. The appearance of this solution is interesting, since there does not seem to be any analogous behavior in the center of mass basis.

Are these solutions correct? To answer this question, we have to describe some mechanism for mapping these solutions to the center of mass basis, and then ask whether they satisfy the exact linear equations of motion there. A natural mapping is,

$$|\Psi\rangle = \tilde{\Pi}_2|\Psi^{\rm cm}\rangle$$

where $|\Psi^{\rm cm}\rangle$ is the analogy of eq.3.4 in the center of mass basis. Expanding this out with the help of the transformation laws eqs.2.2-2.3 and matching terms, we find the relations[3],

$$\begin{aligned} \phi^{\rm cm} &= \phi - \frac{1}{\sqrt{2}}\partial_+ B_- \\ B_+^{\rm cm} &= B_+ + \frac{1}{\sqrt{2}}\partial_+\phi - \tfrac{1}{2}\,\partial_+^2 B_- \\ B_-^{\rm cm} &= B_- \end{aligned} \tag{3.8}$$

Plugging in our solutions eq.3.7,

$$\begin{aligned} B_-^{\rm cm} &= Ae^{i\sqrt{2}t} + Be^{-i\sqrt{2}t} \\ \phi^{\rm cm} &= Ce^{\sqrt{2}t} + De^{-\sqrt{2}t} \\ B_+^{\rm cm} &= (E - \tfrac{1}{2}\,A)e^{i\sqrt{2}t} + (F - \tfrac{1}{2}\,B)e^{-i\sqrt{2}t} - \frac{i}{\sqrt{2}}t\left(Ae^{i\sqrt{2}t} - Be^{-i\sqrt{2}t}\right) \end{aligned} \tag{3.9}$$

We see that $B_-^{\rm cm}$ behaves like a massive field and $\phi^{\rm cm}$ behaves like the tachyon, as they should. However, $B_+^{\rm cm}$ still has the diverging oscillatory part noted earlier. This piece obviously does not satisfy the equations of motion, so it must be an unphysical solution introduced by level truncation. The only way to get rid of this unwanted solution is to set $B_- = 0$, so the exact solution describing the natural oscillation of B_- has been lost.

What happens when we include higher levels? Basically, extra linear couplings emerge and both spurious and lost solutions proliferate. To see what happens, it is useful to proceed more abstractly. We make a two claims:

Claim 1: Given an exact solution to the linear equations at level n in the center of mass basis, $\Pi_n|\Psi^{\rm cm}\rangle = |\Psi^{\rm cm}\rangle$, the field $|\Psi\rangle = \tilde{\Pi}_n|\Psi^{\rm cm}\rangle$ satisfies

$$\tilde{\Pi}_n(L_0 + L_0^{gh} - 1)|\Psi\rangle = 0$$

[3]Note that we could have defined the mapping to be $\Pi_2|\Psi\rangle = |\Psi^{\rm cm}\rangle$. This turns out to give slightly different relations, differing from eq.3.8 by some extra terms proportional to B_-. However, the discussion goes through similarly regardless of these definitions.

if an only if,

$$p_+\tilde{\Pi}_n \sum_{j=1}^{n} \cos\frac{\pi j}{2}\tilde{\alpha}_j^+|\Psi\rangle = 0$$

Claim 2: Given an solution $\tilde{\Pi}_n|\Psi\rangle = |\Psi\rangle$ to the truncated equations of motion in the lightcone basis,

$$\tilde{\Pi}_n(L_0 + L_0^{gh} - 1)|\Psi\rangle = 0$$

the field $|\Psi^{\rm cm}\rangle = \Pi_n|\Psi\rangle$ satisfies the exact equations of motion if and only if,

$$p_+\Pi_n(1-\tilde{\Pi}_n)\sum_{j=1}^{n} \cos\frac{\pi j}{2}\tilde{\alpha}_{-j}^+|\Psi\rangle = 0$$

The proof of both of these statements is straightforward. Claim 1 tells us about lost solutions in the lightcone truncation scheme, and claim 2 tells us about spurious solutions. The first thing to notice about these claims is that the constraints are automatically satisfied if $|\Psi\rangle$ contains no excitations proportional to α_{-2n}^-; hence for such states there are no spurious or lost solutions. One might then wonder whether these constraints have any solutions with α_{-2n}^- excitations at all. Surprisingly, the answer is no. While it is fairly clear to us that this is the case, we do not know of a demonstration simple enough to be worth presenting here since for our purposes level $\tilde{2}$ will be sufficient.

If at no level in the truncation scheme do the correct α_{-2n}^- solutions emerge, in what sense can we say that the truncated equations of motion converge to the exact ones in the infinite level limit? The answer is that spurious solutions are always associated with fields at the highest levels in the particular truncation, and that by proceeding to a higher truncation these spurious solutions disappear, only to be replaced by new ones at even higher levels. To see how this happens, consider level $\tilde{4}$. There, we find that B_- sources the equations of motion of a new field associated with $\tilde{\alpha}_{-2}^+\tilde{\alpha}_{-2}^-$, and this field in turn sources B_+. These new couplings conspire to cancel the unwanted diverging oscillations we found in B_+ earlier. However, a new unphysical solution is introduced at level $\tilde{4}$ by the coupling of B_-, via ϕ, to the field associated with $\tilde{\alpha}_{-4}^+$. This unwanted solution, then, only disappears at level $\tilde{6}$ upon introduction of $\tilde{\alpha}_{-4}^+\tilde{\alpha}_{-2}^-$. Clearly, at any level the oscillation of B_- will introduce unphysical solutions for the fields at the highest levels, so at no stage is it consistent to turn on B_-. However, it is still clear that the correct solutions are, in some sense, emerging as the level of truncation is increased.

While at level $\tilde{2}$ it seems okay to simply set $B_- = 0$, at higher levels fields with $\tilde{\alpha}_{-2n}^-$ excitations proliferate and their effect on the equations of motion cannot be ignored. In this case, it is probably not adequate to do a straightforward level truncation. Rather, if one wanted physically correct linear equations up to level $\tilde{n}$, one would need to include select additional fields at higher levels, with their additional linear couplings, so that spurious solutions at lower levels would cancel out. At level $\tilde{2}$, for example, this could be achieved by simply including the field associated with $\tilde{\alpha}_{-2}^+\tilde{\alpha}_{-2}^-$.

4. Truncation in the α^+ oscillators

As a first look into level truncation in the lightcone basis, it is natural to ask whether we can find time dependent solutions resembling the known (though admittedly problematic) solutions constructed in the center of mass basis, where the interaction vertex contains an infinite number of time derivatives. Indeed, it seems curious that a simple change of basis could radically alter the apparent solution space of the theory and the qualitative behavior of the dynamics. Of course, there is no real paradox here, since string field theory truncated in the center of mass basis is not equivalent to the theory truncated in the lightcone basis. However, since both truncations are an approximation to the *same* exact theory, truncated solutions in one basis should emerge from a suitable approximation in the other basis. It is interesting to see how this comes about, and may help us appreciate the degree to which the erratic time dependent behavior in the center of mass basis should, or should not, be taken seriously.

As a start we investigate the rolling of the pure tachyon in the center of mass basis. The tachyon in the lightcone basis is an infinite linear combination of states excited by $\tilde{\alpha}^+_{-2n}$ oscillators:

$$|\phi^{\rm cm}\rangle = \int dk \phi^{\rm cm}(k)|k\rangle = \int dk\phi^{\rm cm}(k)\exp\left[-k_+\sum_{n=1}^{\infty}\frac{(-1)^n}{2n}\tilde{\alpha}^+_{-2n}\right]|k\rangle'$$

It is natural to suppose that if we truncate the theory in the lightcone basis in such a way that only fields corresponding to $\tilde{\alpha}^+_{-2n}$ excitations are included, we should recover solutions resembling the old solution in the center of mass basis. More precisely, the infinite level limits of α^+_{-2m} truncation in the center of mass and lightcone bases should agree with each other.

The first step is to find an appropriate time dependent solution for the pure tachyon at level 0 in the center of mass basis, which will serve as a standard for comparison when truncating $\tilde{\alpha}^+_{-2n}$ excitations in the lightcone basis. Since the level 0 action for the pure tachyon contains an infinite number of time derivatives, it does not have a well-defined initial value formulation and it is not a priori clear how to construct generic time dependent solutions. In the past[11, 10], a useful approach has been to start with a homogenous time dependent solution in the free theory, whose equations of motion are only second order in time derivatives, and add nonlinear corrections to this perturbatively. The level 0 equations of motion for the tachyon are,

$$(-\tfrac{1}{2}\,\partial^2 - 1)\phi^{\rm cm} + g\kappa e^{\frac{1}{2}V_{00}\partial^2}\left[e^{\frac{1}{2}V_{00}\partial^2}\phi^{\rm cm}\right]^2 = 0 \tag{4.1}$$

where $\kappa = \frac{3^{9/2}}{2^6}, V_{00} = \frac{1}{2}\ln\frac{27}{16}$ and g is the open string coupling, which we will set to one. At the linear level, an interesting homogenous time dependent solution is,

$$\phi^{\rm cm} = \phi_1^{\rm cm}e^{\sqrt{2}t}$$

This corresponds to placing the tachyon infinitesimally close to the unstable vacuum at $t \to -\infty$ and then letting it roll off the unstable maximum in the approximation that

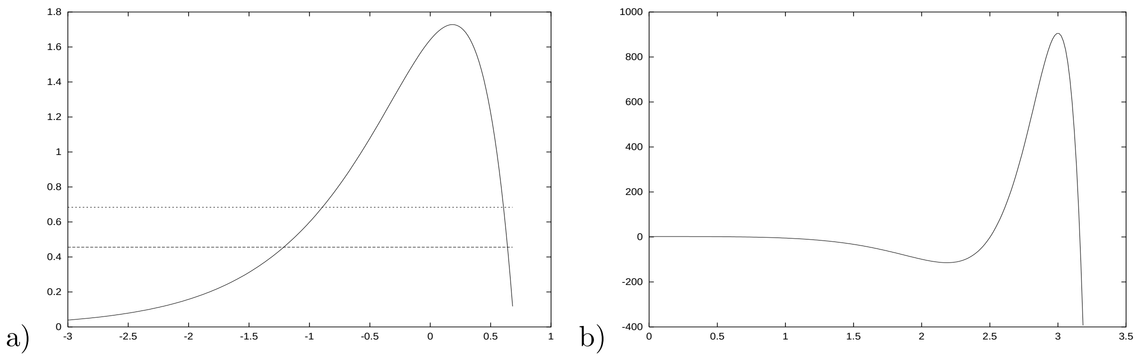


Figure 3: Time evolution of the pure tachyon at level 0 in the center of mass basis, from a) $t = -3, .6$ and b) $t = 0, 3.35$. These two pictures capture the first two oscillations of the tachyon about the closed string vacuum. The two horizontal lines in figure 3a represent the closed string vacuum and the classical turning point in ϕ^3 theory.

the potential is quadratic. Different choices of $\phi_1^{\rm cm}$ corresponds to time translations of the solution; we will set it to a convenient value in a moment. To solve the full nonlinear equations of motion, we make a perturbative ansatz, expanding the tachyon in modes $e^{n\sqrt{2}t}$:

$$\phi^{\rm cm} = \sum_{n=1}^{\infty} \phi_n^{\rm cm} e^{n\sqrt{2}t}.$$

Plugging this in to eq.4.1 implies a recursive formula for the ϕ_ns,

$$\phi_n^{\rm cm} = -\frac{g\kappa e^{-\frac{1}{2}V_{00}n^2}}{n^2-1}\sum_{k=1}^{n-1}\phi_k^{\rm cm} e^{-\frac{1}{2}V_{00}k^2}\phi_{n-k}^{\rm cm} e^{-\frac{1}{2}V_{00}(n-k)^2} \tag{4.2}$$

The factors $e^{-\frac{1}{2}V_{00}n^2}$, originate from the infinite number of time derivatives in the interaction. The suppression of these factors will make the higher ϕ_ns much smaller than they would be in, say, ϕ^3 theory. Since the higher modes are responsible for turning the tachyon around after it rolls through the closed string vacuum and the ensuing oscillations, the fact that the ϕ_ns are small means that the tachyon will turn around in each oscillation comparatively late; so we would expect the tachyon to roll back and forth about the vacuum in a sequence of diverging oscillations. This is indeed what we find upon explicit summation of the series as shown in figure 3. In figure 3a, we see that at the first peak of oscillation the tachyon rolls on the static potential $V(\phi) = -\frac{1}{2}\phi^2 + \frac{g\kappa}{3}\phi^3$ to a height 64 times the energy difference $\Delta E = .0347$ between the stable and unstable vacua. Figure 3b shows the peak of the second oscillation of the tachyon, which reaches a staggering height of 1.6×10^{10} times ΔE! Clearly, if such time dependent behavior was characteristic of string theory we would be in big trouble. In ref.[11] a similar time dependent solution for the tachyon was constructed by expanding in $\cosh n\sqrt{2}t$ modes. This is slightly more complicated but has the advantage of giving control over the position of the tachyon (and the velocity, which vanishes) at $t = 0$. The solution eq.4.2, however, will be sufficient for our purposes.

Now we turn towards constructing analogous time dependent solutions in the lightcone basis. First consider level $\tilde{0}$, where we have only the tachyon-like field ϕ,

$$|\Psi_0\rangle = \int dk\phi(k)|k\rangle'$$

As explained in the introduction, the action for ϕ at level zero is simply that of ϕ^3 theory if we restrict to spatially constant solutions. In this case, we can actually find an exact solution for the decay process,

$$\phi^{(\text{level }0)} = \frac{3}{2g\kappa}\frac{1}{\cosh^2\frac{t}{\sqrt{2}}} \tag{4.3}$$

This solution describes ϕ rolling off the unstable maximum, bouncing off the steep side of the cubic potential, and then approaching the unstable maximum again at late times. We have placed the "bounce" at $t = 0$ corresponding to the choice,

$$\phi_1 = \phi_1^{\text{cm}} = \frac{6}{g\kappa}$$

fixing the arbitrary constant encountered earlier.

To find a solution more closely resembling eq.4.2 in the lightcone basis, we need to include additional fields corresponding to higher $\tilde{\alpha}^+_{-2n}$ mode excitations. Keeping $\tilde{\alpha}^+_{-2n}$ excitations up to level $\tilde{2}$ requires us to include the extra field B_+, which we met in eq.3.4:

$$|\Psi_2\rangle = |\Psi_0\rangle + \frac{i}{\sqrt{2}}\int dkB_+(k)\tilde{\alpha}^+_{-2}|k\rangle'$$

What are the relevant truncated equations of motion for ϕ, B_+? The correct prescription is to write the cubic action including *all* fields up to level $\tilde{2}$ (including B_-). Then, we vary the action with respect to the *center of mass* fields $\phi^{\text{cm}}, B_-^{\text{cm}}$, and then set everything in the resulting equations of motion to zero except ϕ, B_+. It is crucial that we vary the action with respect to the center of mass fields since ultimately we are interested in the approximate equations of motion for the pure tachyon, which is obtained by varying the action with respect to ϕ^{cm}. Thus, the relevant equations of motion are,

$$\begin{aligned}
0 =& \left.\frac{\delta S}{\delta\phi^{\text{cm}}(x)}\right|_{\phi,B_+\neq 0;B_-\ldots=0} \\
=& \left.\left(\frac{\delta S}{\delta\phi(x)} + \frac{1}{\sqrt{2}}\partial_+\frac{\delta S}{\delta B_+(x)}\right)\right|_{\phi,B_+\neq 0;B_-\ldots=0} \\
0 =& \left.\frac{\delta S}{\delta B_-^{\text{cm}}(x)}\right|_{\phi,B_+\neq 0;B_-\ldots=0} = \left.\frac{\delta S}{\delta B_-(x)}\right|_{\phi,B_+\neq 0;B_-\ldots=0}
\end{aligned} \tag{4.4}$$

Here we used the chain rule and the transformation law eq.3.8 relating fields in the center of mass basis to the lightcone basis. Explicitly including interactions up to level $\tilde{4}$ and setting the transverse momenta to zero, these equations of motion are,

$$\begin{aligned}
0 =& (\partial_+\partial_- - 1)\phi + g\kappa\Big[\phi^2 + G_{\phi\phi B_+}\phi\partial_- B_+ + G^1_{\phi B_+B_+}\partial_-^2 B_+B_+ + G^2_{\phi B_+B_+}(\partial_- B_+)^2 \\
&+\frac{1}{\sqrt{2}}\partial_+\Big(G_{B_+\phi\phi}\phi\partial_-\phi + G^1_{B_+\phi B_+}\partial_-^2\phi B_+ + G^2_{B_+\phi B_+}\partial_-\phi\partial_- B_+ + G^3_{B_+\phi B_+}\phi\partial_-^2 B_+\Big)\Big] \\
0 =& -(\partial_+\partial_- + 1)B_+ - \sqrt{2}\partial_+\phi + g\kappa G_{B_-\phi B_+}\phi B_+
\end{aligned} \tag{4.5}$$

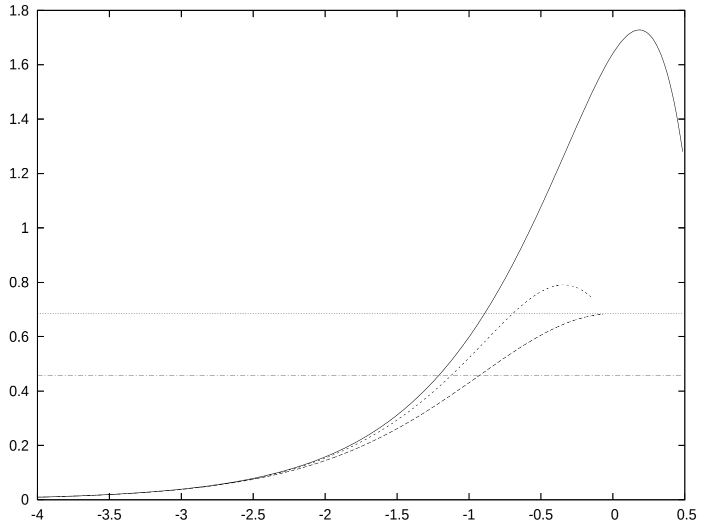


Figure 4: The level $\tilde{2}$ approximation to the rolling of the pure tachyon in the center of mass basis, sandwiched between the level $\tilde{0}$ profile in the lightcone basis and the level 0 profile in the center of mass basis. The perturbative series eq.4.6 does not converge past $t \approx -.22$. The horizontal lines represent the level 0 closed string vacuum and the turning point in the ϕ^3 potential.

The eight Gs above are various powers and linear combinations of the Neumann coefficients. We can easily solve these equations by making a perturbative ansatz:

$$\phi = \sum_{n=1}^{\infty} \phi_n e^{n\sqrt{2}t} \qquad\qquad B_+ = \sum_{n=1}^{\infty} B_{+,n} e^{n\sqrt{2}t} \tag{4.6}$$

where for consistency at the linear level we have,

$$B_{+,1} = -\frac{1}{\sqrt{2}}\phi_1 \tag{4.7}$$

Plugging this ansatz into eq.4.5 we derive recursive formulas for the various coefficients in the usual way. Summing the series, we find the tachyon profile shown in figure 4, which for comparison we have graphed alongside the level $\tilde{0}$ profile of ϕ^3 theory and the level 0 profile in the center of mass basis. We see that, already at level $\tilde{2}$, the rolling of the tachyon looks much more like it does at level 0 in the center of mass basis. Strictly speaking, however, we should be comparing this result to the analogous computation including B_+^{cm} in the center of mass basis. However, at least in the region of convergence of eq.4.6, including B_+^{cm} has negligible effect on the level 0 profile.

We emphasize that the equations of motion we have just solved,

$$0 = \left.\frac{\delta S}{\delta\phi^{\mathrm{cm}}(x)}\right|_{\phi^{\mathrm{cm}},B_+^{\mathrm{cm}}\neq 0;B_-^{\mathrm{cm}}...=0} = \left.\frac{\delta S}{\delta B_-^{\mathrm{cm}}(x)}\right|_{\phi^{\mathrm{cm}},B_+^{\mathrm{cm}}\neq 0;B_-^{\mathrm{cm}}...=0} = ... \tag{4.8}$$

are *not* equivalent to

$$0 = \left.\frac{\delta S}{\delta\phi(x)}\right|_{\phi,B_+\neq 0;B_-\ldots=0} = \left.\frac{\delta S}{\delta B_-(x)}\right|_{\phi,B_+\neq 0;B_-\ldots=0} = \ldots \tag{4.9}$$

even when both are expressed in terms of fields in the same basis. The second set of equations, in particular, contain no ∂_+ derivatives in the interaction when written in the lightcone basis. These equations differ from each other by terms proportional the B_- equation of motion—and, at higher levels, those for other $\tilde{\alpha}^-_{-2n}$ fields. If we allow the $\tilde{\alpha}^-_{-2n}$ fields to be nonzero and separately solve their equations of motion, then eq.4.8 and eq.4.9 become equivalent.

However, this suggests the following: Since the second set of equations, eq.4.9, contain *no* lightcone time derivatives in the interaction, we would expect their solutions to be more nicely behaved— resembling that of ϕ^3 theory at level $\tilde{0}$. This suggests that it might be possible to approximate these more well-behaved solutions by truncating α^+_{-2n} modes in the *center of mass basis*. At level 2 we must consider $\phi^{\rm cm}, B_+^{\rm cm}$. By a similar prescription to eq.4.4, the relevant equations of motion are,

$$\begin{aligned}
0 &= \left.\frac{\delta S}{\delta\phi(x)}\right|_{\phi^{\rm cm},B_+^{\rm cm}\neq 0;B_-^{\rm cm}\ldots=0} \\
&= \left.\left(\frac{\delta S}{\delta\phi^{\rm cm}(x)} - \frac{1}{\sqrt{2}}\partial_+\frac{\delta S}{\delta B_+^{\rm cm}(x)}\right)\right|_{\phi^{\rm cm},B_+^{\rm cm}\neq 0;B_-^{\rm cm}\ldots=0} \\
0 &= \left.\frac{\delta S}{\delta B_-(x)}\right|_{\phi^{\rm cm},B_+^{\rm cm}\neq 0;B_-^{\rm cm}\ldots=0} = \left.\frac{\delta S}{\delta B_-^{\rm cm}(x)}\right|_{\phi^{\rm cm},B_+^{\rm cm}\neq 0;B_-^{\rm cm}\ldots=0}
\end{aligned} \tag{4.10}$$

As usual, we solve these equations by making a perturbative ansatz. The resulting tachyon profile is shown in figure 5a, along with the profiles at level 0, eq.4.2, and at level $\tilde{2}$ with ϕ, B_+ in the lightcone basis. We can see that including $B_+^{\rm cm}$ makes the tachyon turn around well before it does at level 0. Figure 5b shows the evolution of $\phi^{\rm cm}$ at later times. While the subsequent oscillations are still pronounced, they are much less virulent then they were at level 0; the peak of the second oscillation reaches a potential height of only $2\times 10^7\times\Delta E$, about 1000 times smaller than the height of the second peak at level 0. Remarkably, even computing in the center of mass basis, truncating the theory in a particular way can yield time dependent solutions whose qualitative behavior is substantially improved over the level 0 solution.

Thus it seems possible to approximate solutions in one basis by making an appropriate truncation in the other. However, this brings up a natural question: Precisely in what sense does level truncation in $\tilde{\alpha}^+_{-2n}$ modes in the lightcone basis converge to the equations of motion eq.4.1 for the tachyon in the center of mass basis? To explain this, consider the level $\tilde{2}$ equations of motion for ϕ, B_+, eq.4.5. To compare this with eq.4.1, we substitute in center of mass fields via the transformation law $\phi = \phi^{\rm cm}, B_+ = B_+^{\rm cm} - \frac{1}{\sqrt{2}}\partial_+\phi^{\rm cm}$ and suppose $B_+^{\rm cm}$ can be ignored. Then eq.4.5 results in the following approximate equations for the pure tachyon $\phi^{\rm cm}$:

$$0 = (\partial_+\partial_- - 1)\phi^{\rm cm} + g\kappa\left[(\phi^{\rm cm})^2 - \frac{1}{\sqrt{2}}(G_{\phi\phi B_+} - G_{B_+\phi\phi})\phi^{\rm cm}\partial_+\partial_-\phi^{\rm cm}\right. \tag{4.11}$$

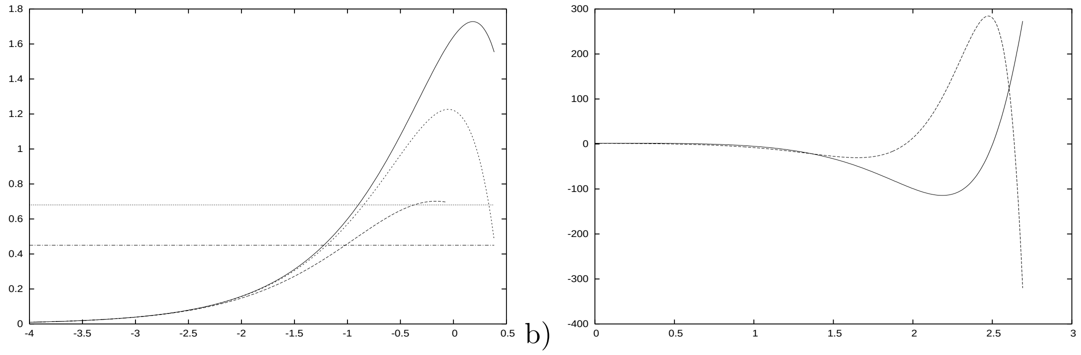
a)  b)

Figure 5: a) The level 2 approximation to the rolling of ϕ in the lightcone basis, graphed in comparison to the ϕ, B_+ solution at level $\tilde{2}$, at the bottom, and the level 0 tachyon solution the center of mass basis, at the top. b) The level 2 approximation to ϕ (dotted line) for later times $0 < t < 2.7$, graphed in comparison to level 0 (solid line). We have not graphed the ϕ, B_+ solution out here since the series does not converge.

$$+\frac{1}{\sqrt{2}}G_{B_+\phi\phi}\partial_+\phi^{\rm cm}\partial_-\phi^{\rm cm} + ...\Big] \tag{4.12}$$

where $+...$ are nonlinear terms with two ∂_+ derivatives, which we will not need for the sake of the present discussion. Since the nonlinear terms in eq.4.12 contain only up to 2 lightcone derivatives in the interaction, whereas eq.4.1 contains an infinite number, it is clear that they must be related by some sort of polynomial expansion of the infinite differential operator $\exp(-V_{00}\partial_+\partial_-)$. Expanding the derivatives in the interaction term out to first order in ∂_+, eq.4.2 becomes,

$$0 = (\partial_+\partial_- - 1)\phi^{\rm cm} + g\kappa\left[(\phi^{\rm cm})^2 - 4V_{00}\phi^{\rm cm}\partial_+\partial_-\phi^{\rm cm} - 2V_{00}\partial_+\phi^{\rm cm}\partial_-\phi^{\rm cm} + ...\right]$$

This leads to the identifications,

$$4V_{00} \sim \frac{1}{\sqrt{2}}(G_{\phi\phi B_+} - G_{B_+\phi\phi}) \quad 2V_{00} \sim -\frac{1}{\sqrt{2}}G_{B_+\phi\phi}$$

Noting that

$$G_{\phi\phi B_+} = \frac{1}{\sqrt{2}}(2V_{02}^{33} - V_{02}^{13} - V_{02}^{23}) \qquad G_{B_+\phi\phi} = -G_{\phi\phi B_+}$$

these equations amount to the identification,

$$V_{00} \sim \frac{1}{2\sqrt{2}}(2V_{02}^{33} - V_{02}^{13} - V_{02}^{23})$$

Noting that $V_{0n}^{33} + V_{0n}^{13} + V_{0n}^{23} = 0$, this can be rewritten,

$$\ln\frac{27}{16} \sim -3\left(-\frac{1}{\sqrt{2}}V_{02}^{33}\right)$$

This is the first approximation to the exact identity[16],

$$\ln\frac{27}{16} = -3\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{2n}}V_{0,2n}^{33} \tag{4.13}$$

n, $(\phi_n^{\rm cm})$	2	3	4	5	6	7	...	n
level $2k$	4	8	14	22	32	44	...	$n(n-1)+2$

Table 1: This table shows what level in $\tilde{\alpha}^+_{-2k}$ excitations we need to include to get $\phi_n^{\rm cm}$s, up to some n, resembling those in the center of mass basis within 10%.

So it seems there are two levels of approximation going on. The first is polynomial approximation to the infinite order differential operator $\exp(-V_{00}\partial_+\partial_-)$; the second is an approximation to the coefficients of this polynomial through eq.4.13. Using this knowledge we can get an estimate for how a solution including $\tilde{\alpha}^+_{-2n}$ modes up to level $\tilde{n}$ converges to the center of mass solution, eq.4.1. Define $P_k(x)$ to be the kth order polynomial approximation to e^x, expanded around $x=0$. Roughly speaking, the level $\tilde{2k}$ approximation to eq.4.1 is,

$$0 = (\partial_+\partial_- - 1)\phi^{\rm cm} + g\kappa P_k(-V_{00}\partial_+\partial_-)\left[P_k(-V_{00}\partial_+\partial_-)\phi^{\rm cm}\right]^2 + ...$$

Here $+...$ includes interactions with fields at higher levels and some higher derivative $(\phi^{\rm cm})^2$ nonlinear terms, which we will ignore. Also, we have assumed that V_{00} has been adequately approximated by the identity eq.4.13. Making a perturbative ansatz, the solution is,

$$\phi_n^{\rm cm} = -\frac{g\kappa P_k(-\frac{1}{2}V_{00}n^2)}{n^2-1}\sum_{j=1}^{n-1}\phi_j^{\rm cm}P_k(-\tfrac{1}{2}V_{00}j^2)\phi_{n-j}^{\rm cm}P_k(-\tfrac{1}{2}V_{00}(n-j)^2)$$

It is clear that for large enough n the coefficients in this solution will look very different from eq.4.2. Only for the first few modes, where $P_k(-\frac{1}{2}V_{00}n^2)$ is a good approximation to $e^{\frac{1}{2}V_{00}n^2}$ do the coefficients look similar. So, if we want a good approximation to ϕ_n up to some integer n, how many $\tilde{\alpha}^+_{-2n}$ modes do we need to include in the lightcone basis? Assuming we want to reproduce the $\phi_n^{\rm cm}$ to an accuracy of 10%, the answer is shown in Table 1. Roughly speaking, as we consider higher modes we must increase the level of truncation quadratically to get answers similar to those in the center of mass basis. Similar comments apply if we want to approximate solutions in the lightcone basis by level truncation in the center of mass basis.

So we have studied two types of time dependent solutions describing the decay of an unstable D-brane: those arising naturally in the center of mass basis and those arising naturally from the lightcone basis. We have seen that, with a suitable truncation and definition of the equations of motion, it is possible to approximate the "natural" solution in one basis by fields in the other. However, this brings up an obvious question: which solution is more representative of the *actual* time dependent behavior of the string field as the D-brane decays? At this stage that we have seen no reason to favor one type of solution over the other. The difference between the lightcone and center of mass solutions really comes down to the fact that they solve *different* equations of motion, despite the fact that the field content of the solutions is identical. Indeed there is an intrinsic ambiguity in defining the equations of motion in the $\tilde{\alpha}^+_{-2n}$ truncation scheme. In order to derive equations of motion for $\tilde{\alpha}^+_{-2n}$ fields, we must vary the action with respect to $\tilde{\alpha}^-_{-2n}$ fields, but we do not separately account for the equations that the $\tilde{\alpha}^-_{-2n}$ fields must satisfy—rather, we have simply set $\tilde{\alpha}^-_{-2n}$ fields to zero. This means that the equations of motion in the $\tilde{\alpha}^+_{-2n}$ truncation scheme are really only defined up to terms proportional to the $\tilde{\alpha}^-_{-2n}$ equations of motion. This ambiguity accounts of the profound difference between the solution at level 0 in the center of mass basis and that at level $\tilde{0}$ in the lightcone basis.

To resolve this ambiguity one might propose to include $\tilde{\alpha}^-_{-2n}$ fields and solve their equations of motion. We will do essentially this (also including other fields out to level $\tilde{2}$) in the next section. However, we do not believe this is likely to give a more realistic picture of D-brane decay. The reasons are twofold: First, $\tilde{\alpha}^-_{-2n}$ excitations create spurious solutions at the linear level in the lightcone basis—as we saw in section 3. Second, more fundamentally, $\tilde{\alpha}^-_{-2n}$ fields infect solutions with negative energies. We will have more to say on this in the next section, but it suffices to remark that these unphysical negative energies can only be projected out with a consistent imposition of constraints. That being said, we believe that the ϕ^3 solution at level $\tilde{0}$ probably more closely resembles the true decay process. The reason is that eq.1.1 possesses two features we know must be true of the exact theory: it is local in lightcone time and has a Hamiltonian free of all instabilities except the physical one associated with the tachyon.

5. Solutions at level $(\tilde{2}, \tilde{4})$

We now turn towards finding time dependent solutions representing the decay of the D-25 brane vacuum, including all relevant fields up to level $(\tilde{2}, \tilde{4})$. For our purposes it will be sufficient to restrict to solutions which depend only on $\tilde{x}^+$ (time) and x^-, but not on the remaining 24 transverse directions. To find the most efficient consistent truncation at level $\tilde{2}$, we require that the string field $|\Psi\rangle$ satisfy the following properties:

1) $|\Psi\rangle$ is real and has ghost number 1.
2) $b_0|\Psi\rangle = 0$
3) $|\Psi\rangle$ is twist even, i.e. only has excitations at even levels.
4) $|\Psi\rangle$ is a singlet under the transverse subgroup $SO(24)$ of the Lorentz group.

Condition 1 is simply a property of a physical string field. Condition 2 is our choice of gauge. Condition 3 follows from the well-known fact that twist odd fields only appear quadratically in the string field theory action, so setting them equal to zero solves their equations of motion identically. Condition 4 follows from symmetry; if we require $|\Psi\rangle$ to be a singlet initially, it will remain a singlet under time evolution since the solutions we study have zero transverse momentum. Imposing these conditions, we find that the string field at level $\tilde{2}$ takes the form,

$$|\Psi\rangle = \int dk \Big[\phi(k) + D_\parallel(k)\tilde{\alpha}^+_{-1}\tilde{\alpha}^-_{-1} + \frac{1}{\sqrt{48}}D_\perp(k)\tilde{\alpha}^i_{-1}\tilde{\alpha}^i_{-1} + \frac{1}{\sqrt{2}}(D_{++}(k)\tilde{\alpha}^+_{-1}\tilde{\alpha}^+_{-1} + D_{--}(k)\tilde{\alpha}^-_{-1}\tilde{\alpha}^-_{-1})$$
$$+ \frac{i}{\sqrt{2}}(B_+(k)\tilde{\alpha}^+_{-2} + B_-(k)\tilde{\alpha}^-_{-2}) + \beta(k)c_{-1}b_{-1}\Big]\,|k\rangle' \tag{5.1}$$

The eight spacetime fields above are expressed in the momentum representation and are real in the momentum space sense, e.g. $\overline{\phi(k)} = \phi(-k)$. Actually, there is another simplification coming from the fact that we include interactions only up to level $\tilde{4}$: we can consistently set $D_{++} = 0$, and as a result D_{--} decouples from the dynamics. We can choose $D_{++} = 0$ since the only nonlinear term in the D_{++} equation of motion is proportional to ϕD_{++}; the other possible term B_+B_+ only enters at level $(\tilde{2}, \tilde{6})$. By simple matching of Lorentz indices, it is easy to see that all other couplings are ruled out by the absence of ∂_+ derivatives in the vertex. Also by matching Lorentz indices it is possible to see that D_{--} decouples when $D_{++} = 0$, since the only way D_{--} effects the dynamics is through a $D_{--}D_{++}$ term in the equation of motion for ϕ; other couplings do not enter until $(\tilde{2}, \tilde{6})$. Hence we can set $D_{++} = 0$ and ignore D_{--}, bringing the total to six relevant spacetime fields at level $(\tilde{2}, \tilde{4})$.

A natural approach to finding a solution is to expand the fields in modes $e^{\sqrt{2}nt}$, as we did in the last section. This method has the advantage of giving a spatially homogenous solution, like Sen's rolling tachyon[1]. However, we have found that the radius of convergence of this solution at level $(\tilde{2}, \tilde{4})$ is not sufficiently large to follow the interesting dynamics—the series diverges before the tachyon turns around for the first time. One might attempt to perform some sort of Padé resummation, but we have found this to be unreliable.

However, in the lightcone basis we have an obvious alternative: we can make use of the fact that the theory possesses an initial value formulation. In particular, we can specify initial conditions for the fields at $\tilde{x}^+ = 0$, and then integrate the equations of motion to generate a full time dependent solution. To see how this is done, let us write the equations of motion for the eight fields eq.5.1 in an abbreviated notation,

$$\begin{aligned} 0 &= (\partial_+\partial_- - 1)\phi + \sqrt{2}\partial_+ B_- + g\phi^{\rm int}[\phi, B_+, F_i] \\ 0 &= (\partial_+\partial_- + 1)B_+ + \sqrt{2}\partial_+\phi + gB_+^{\rm int}[\phi, B_+, F_i] \\ 0 &= (\partial_+\partial_- + 1)F_i + gF_i^{\rm int}[\phi, B_+, F_i] \end{aligned} \tag{5.2}$$

where $F_i = B_-, D_\perp, D_\|, \beta, D_{--}, D_{++}$, and $\phi^{\rm int}$... denote the nonlinear terms in the equations of motion. There are over a hundred of these terms at level $(\tilde{2}, \tilde{4})$—we will not write them explicitly—but we emphasize that they contain no lightcone time derivatives ∂_+. We can integrate the equations of motion eq.5.2 stepwise on a lattice, defining,

$$\tilde{x}^+ = n\epsilon \qquad \phi_n(x^-) = \phi(n\epsilon, x^-) \qquad B_{+,n}(x^-) = B_+(n\epsilon, x^-) \qquad F_{i,n}(x^-) = F_i(n\epsilon, x^-)$$

where ϵ is a sufficiently small number. We can then use eq.5.2 to generate the fields at the $n+1$st time from the fields at the nth time:

$$\begin{aligned} F_{i,n+1}(x^-) &= F_{i,n}(x^-) - \epsilon \int_{x_0^-}^{x^-} da \left[F_{i,n}(a) + gF^{\rm int}[\phi_n, B_{+,n}, F_{i,n}](a)\right] \\ \phi_{n+1}(x^-) &= \phi_n(x^-) - \epsilon \int_{x_0^-}^{x^-} da \left[\sqrt{2}\frac{B_{-,n+1}(a) - B_{-,n}(a)}{\epsilon} + g\phi^{\rm int}[\phi_n, B_{+,n}, F_{i,n}](a) - \phi_n(a)\right] \\ B_{+,n+1}(x^-) &= B_{+,n}(x^-) - \epsilon \int_{x_0^-}^{x^-} da \left[\sqrt{2}\frac{\phi_{n+1}(a) - \phi_n(a)}{\epsilon} + gB_+^{\rm int}[\phi_n, B_{+,n}, F_{i,n}](a) + B_{+,n}(a)\right] \end{aligned} \tag{5.3}$$

In these expressions we have already fixed a boundary condition which requires the fields to be *constant* in time $\tilde{x}^+$ along the light-like surface $x^- = x_0^-$. This choice of boundary condition corresponds to the familiar $p_- = 0$ ambiguity occurring in any relativistic field theory whose initial value formulation is constructed in the lightcone frame. For most of our discussion we will take x_0^- to be sufficiently far in the past that we can assume that the fields vanish. Note that these formulas must be solved in the order presented: The first formula gives the F_i at $n+1$st time given the fields at n; the second gives ϕ at $n+1$ given the fields at n and B_- at $n+1$ (which we found in the first formula); the third gives B_+ at $n+1$ given the fields at n and ϕ at $n+1$ (which we found from the second formula). Thus, taking ϵ to zero, we generate a solution by specifying the field configuration at $\tilde{x}^+ = 0$.

This technique gives us a way of generating a wide class of solutions from generic initial conditions. Unfortunately, except for very special cases, the resulting solutions are not homogenous, i.e. depend on both $\tilde{x}^+$ and x^-. Nevertheless, we have found that generally these types of solutions look similar to the homogenous (perturbative) solution when the radius of convergence of the latter allow them to be usefully compared—at least for initial conditions that are a sufficiently "small" perturbation of the open string vacuum.

For definiteness, in most of our discussion we will choose the initial conditions,

$$\phi(x^-) = \frac{H}{4\cosh^2 \frac{1}{2}x^-} \qquad B_+(x^-) = -\frac{H}{2\sqrt{2}}(1+\tanh\tfrac{1}{2}x^-) \qquad F_i(x^-) = 0 \tag{5.4}$$

where H is "sufficiently small" (for our purposes we choose $H = .1$). For most purposes, this choice of initial conditions seems irrelevant; any small perturbation of the D-brane generates a similar decay as long as the tachyon is pushed towards the closed string vacuum. Of course, if the initial conditions are too "large," the resulting solution will depend more sensitively on the exact profile of the fields at $\tilde{x}^+ = 0$. For the most part, however, we are interested in generic features of the decay resulting from small perturbations of the D-25 brane, and not on detailed features of the time evolution of certain large field excitations. Therefore "small" initial conditions will serve us well.

That being said, eq.5.4 has some noteworthy advantages as a choice of initial conditions. First, for early times $t = \frac{1}{\sqrt{2}}(\tilde{x}^+ + x^-)$ eq.5.4 generates a homogenous solution in the linear theory, and as a result the first wave-crest in the nonlinear solution coincides roughly with $t = 0$. If, for example, we had chosen gaussian initial conditions, the first wave crest would advance at the speed of light along the line $x^- = 0$. Still, however, the solution would possess all of the principal qualitative features we will discuss. The second advantage of eq.5.4 is that they solve the truncated constraints for string field theory in Siegel gauge. As mentioned earlier, a time dependent solution of the Siegel gauge equations of motion eq.2.9 does not *a priori* represent a full solution to the string field equations $Q_B\Psi + g\Psi * \Psi = 0$. The other "half" of the string field equations, not accounted for by the Siegel gauge equations of motion, amount to a set of constraints which the string field must satisfy at all times (particularly $\tilde{x}^+ = 0$), like Gauss's law in electrodynamics. In ref.[16] the constraints were found to take the form,

$$0 = b_0(c(\tfrac{\pi}{2})P^+(\tfrac{\pi}{2}) - i\pi_b(\tfrac{\pi}{2})x^+(\tfrac{\pi}{2})')Q_B\Psi \tag{5.5}$$

where $c(\frac{\pi}{2}), P(\frac{\pi}{2}), \pi_b(\frac{\pi}{2}), x(\frac{\pi}{2})'$ are the c ghost, string momentum, b ghost momentum, and worldsheet derivative of the string position evaluated at the string midpoint, respectively. In principle, the constraints eq.5.5 play an extremely important role in the theory: they restrict dynamics to a submanifold in phase space where the Hamiltonian is positive (modulo the tachyon). This essentially amounts to a field-theoretic restatement of the no-ghost theorem of first quantized strings. It is not difficult to see that constraints are necessary to give physically acceptable solutions. Even at the linear level and ignoring the tachyon, we can see from the Siegel gauge action (at level $\tilde{2}$),

$$S_{\text{free}} = \int dx \, \Big[\tfrac{1}{2}\,\phi(\tfrac{1}{2}\,\partial^2 - 1)\phi + \tfrac{1}{2}\,D_{\|}(\tfrac{1}{2}\,\partial^2 + 1)D_{\|} + \tfrac{1}{2}\,D_{\perp}(\tfrac{1}{2}\,\partial^2 + 1)D_{\perp} \\ + D_{++}(\tfrac{1}{2}\,\partial^2 + 1)D_{--} - B_{+}(\tfrac{1}{2}\,\partial^2 + 1)B_{-} - \tfrac{1}{2}\,\beta(\tfrac{1}{2}\,\partial^2 + 1)\beta + \sqrt{2}\phi\partial_{+}B_{-} \Big]$$

that the Hamiltonian is unbounded from below, containing unphysical "ghost" degrees of freedom like β and $\frac{1}{\sqrt{2}}(B_+ + B_-)$ whose kinetic and potential energies are negative definite. Some extra conditions are necessary to ensure that the dynamics generated by the Siegel gauge equations of motion is not completely unstable. Thus it seems important that the initial conditions eq.5.4 satisfy the constraints and therefore set the solution on the right track. Unfortunately, however, we will find that the qualitative behavior of our solutions, in particular the unphysical negative energy instabilities we will encounter, are largely insensitive to the fact that eq.5.4 satisfies the constraints. The reason is that, at finite level, the constraints are inconsistent with the equations of motion; even if the string field satisfies the constraints initially, it will in general fail to upon time evolution at finite level. Indeed, the consistency of the constraints eq.5.5 with time evolution in the exact theory is a subtle issue relying on certain nontrivial identities between the Neumann coefficients[16] which only emerge properly in the infinite level limit. Later, we will have to deal with unphysical instabilities in a more direct fashion, "freezing out" problematic negative energy fields by hand.

So let us present the level $(\tilde{2}, \tilde{4})$ time dependent solution derived from these initial conditions. Figure 6a shows the dynamics of ϕ as a function of $\tilde{x}^+, x^-$, and figure 6b shows more specifically the profile at $\tilde{x}^+ = 2.5$. At early times the solution does the expected thing; it rolls off the open string vacuum, bounces off the ϕ^3 potential wall and turns around. What happens after this is more disturbing: the tachyon rolls back over the open string vacuum into the unbounded side of the ϕ^3 potential, but somehow feels the urge to turn around again and shoots quickly back towards the open string vacuum. At later times the solution becomes increasingly unstable, oscillating back and forth with increasing frequency and amplitude as shown in figure 6c. After a certain point the gradients become too large to follow the solution reliably, but we have no reason to believe that the dynamics ever stabilizes.

We hope, of course, that this solution does not in any way approximate the dynamics of D-brane decay. This erratic behavior clearly indicates that we are dealing with a system of equations which couple to unbounded negative energies. What mechanism is responsible for this erratic behavior? The problem seems closely related to the unphysical linear instability we discovered in section 3, where the linear motion of B_- drives B_+ at its

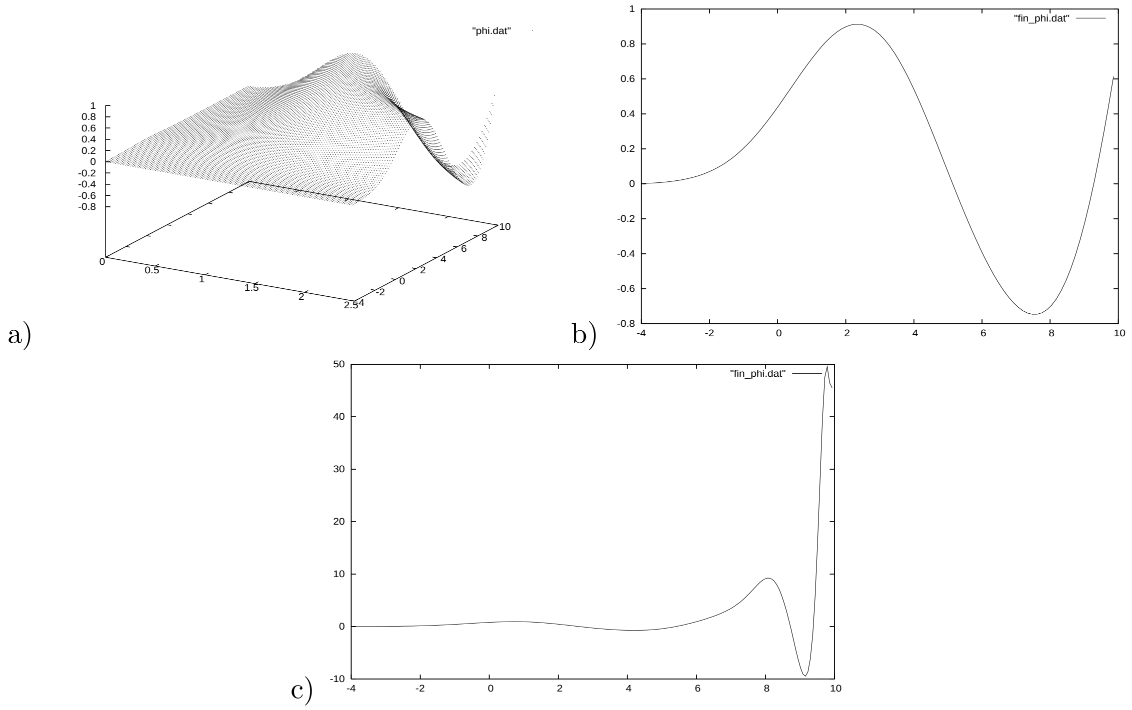


Figure 6: a) The level $(\tilde{2},\tilde{4})$ approximation to the rolling of ϕ in the lightcone basis, graphed as a function of $\tilde{x}^+, x^-$ for $-4 < x^- < 10$ and $0 < \tilde{x}^+ < 2.5$. b) The profile of ϕ at $\tilde{x}^+ = 2.5$, c) at $\tilde{x}^+ = 3.6$.

resonant frequency. In figure 7 we have graphed the profiles of B_+, ϕ, B_- at $\tilde{x}^+ = 2.5$; at its peak B_+ acquires a value approximately ten times that of other fields, strongly suggesting that B_+ is the principal victim of the instability. The fact that the linear motion of B_- is the cause of the instability is supported by two bits of evidence. First, if the linear coupling of B_- to the ϕ equations of motion is turned off, B_+ is not driven to large values and the resulting solution is much more well-behaved. Second, if the nonlinear couplings in the B_- equations of motion are turned off, but we set nonvanishing initial conditions for B_-, the resulting purely linear motion of B_- still causes the instability. However, nonlinear effects do seem to play an important role. For example, we saw in section 3 that including an extra field at level $\tilde{4}$, $\tilde{\alpha}^+_{-2}\tilde{\alpha}^-_{-2}|k\rangle'$, makes the linear instability of B_+ disappear. However, adding $\tilde{\alpha}^+_{-2}\tilde{\alpha}^-_{-2}|k\rangle'$ onto our solution does not make it more well-behaved. The reason, we think, is that by the time the instability sets in, the tachyon is far off shell and the cancellation of the unphysical B_+ instability no longer occurs. Another important nonlinear effect comes from the $\phi\partial_- B_+$ coupling in the ϕ equation of motion. If this term is set to zero, the instability does not occur in its present form.

Since the linear instability introduced by B_- is an artifact of the level truncation scheme, it seems clear that the instability of the level $\tilde{2}$ solution is also an unphysical artifact of level truncation. However, limited at present to level $\tilde{2}$, it seems important to explore possible modifications of the equations of motion at this level which have a chance of painting a more realistic picture of D-brane decay. We propose to set all nonlinear

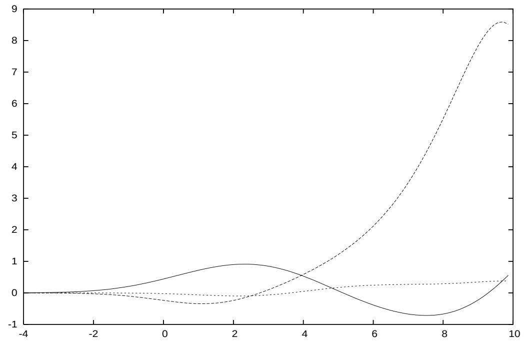


Figure 7: The level $(\tilde{2}, \tilde{4})$ profiles for B_+ (dashed line), ϕ (solid line), and B_- (dotted line) at $\tilde{x}^+ = 2.5$.

couplings in the B_- equation of motion to zero. Then, given our initial conditions eq.5.4, this has the effect of setting $B_- = 0$. Remarkably, with this modification alone the solutions we find will paint a consistent and believable picture of D-brane decay beyond level $\tilde{0}$.

With $B_- = 0$, we have plotted in figure 8 the profile of ϕ at $\tilde{x}^+ = 2$ generated from the initial conditions eq.5.4. For the purposes of comparison, we have graphed it alongside another profile generated from the equations of motion obtained upon setting $B_+ = 0$. When setting $B_+ = 0$, the equations of motion simplify to that of an ordinary cubic theory coupling three scalars $\phi, \beta, A = -\frac{1}{2}D_{\|} = \frac{1}{\sqrt{48}}D_\perp$ with no derivative dependence in the interaction. The linear and nonlinear couplings of B_+ can be thought of as more characteristic of string theory, so the $B_+ = 0$ profile provides a useful gauge for identifying uniquely stringy behavior. We see two characteristic features emerge from figure 8: First, initially ϕ rolls somewhat more quickly towards the closed string vacuum relative to the $B_+ = 0$ profile. Second, ϕ does not become as large before turning around. Relative to the $B_+ = 0$ profile, the first peak is 6% closer to the closed string vacuum. Both of these features are fairly insensitive to the choice of initial conditions.

However, at this point it is worth mentioning that even having set $B_- = 0$ there is another point of possible concern with the equations of motion: The field β has negative kinetic energy and may introduce further unphysical instabilities into solutions. It is worth seeing the nature of these instabilities, though we will see that they do not significantly effect the qualitative features we identified in figure 8. Suppose that we set initial conditions so that the string field is very close to the closed string vacuum,

$$\phi^{\text{vac}} = .542, \quad D_{\|}^{\text{vac}} = -.0519 \quad D_\perp^{\text{vac}} = .180 \quad \beta^{\text{vac}} = -.173 \tag{5.6}$$

but we displace ϕ away from the vacuum by a small amount $\Delta\phi = .01$. The resulting time dependent solution is plotted in figure 9. The tachyon oscillates back and forth around the stable vacuum, as expected, but the oscillations grow with time. This behavior is directly caused by the dynamics of β; fixing β to its value at the closed string vacuum, the oscillations no longer grow with time.

We can try two things in an effort to deal with the negative energy instabilities of β. First, we can simply fix $\beta = 0$, which is valid at least in the linear approximation.

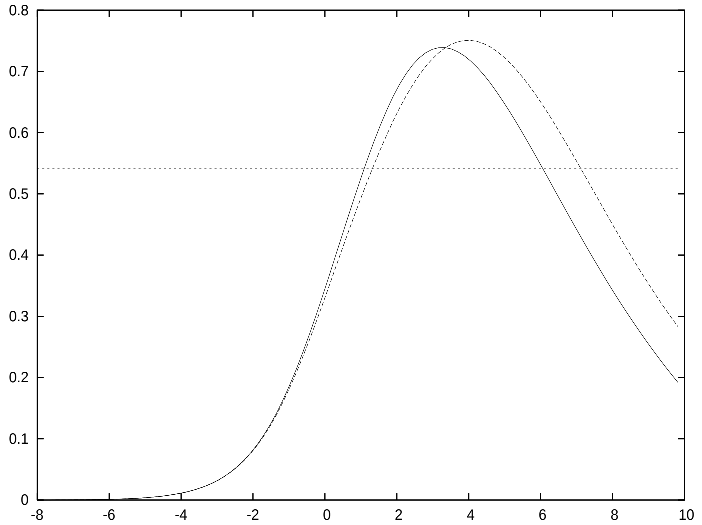


Figure 8: Profile for ϕ (solid line) at $\tilde{x}^+ = 2$ including all fields except B_- starting from the initial conditions eq.5.4. The dashed line shows the analogous profile generated upon setting $B_+ = 0$. The horizontal lines denote the vev of ϕ at the closed string vacuum.

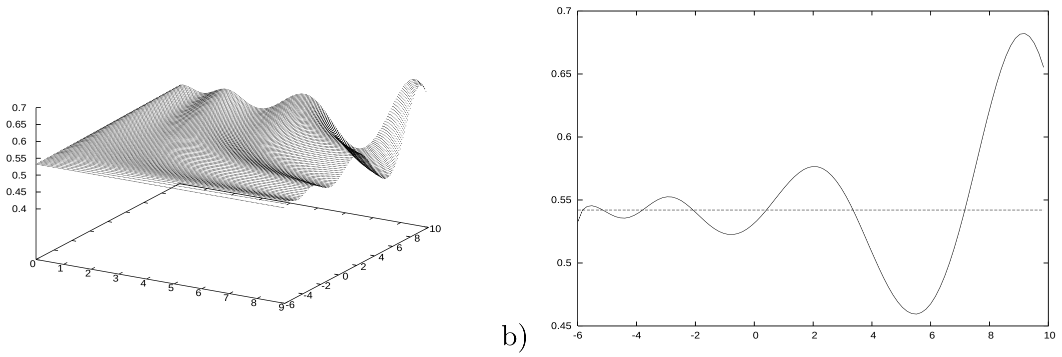
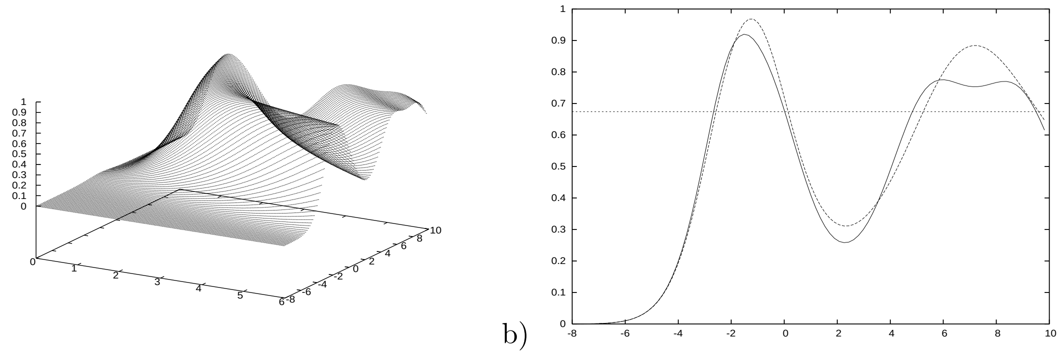


Figure 9: Dynamics of ϕ at level $(\tilde{2}, \tilde{4})$ (excluding B_-) if we place the string field close to the closed string vacuum, plotted a) as a function of $0 < \tilde{x}^+ < 9$, $-4 < x^- < 10$ and b) at $\tilde{x}^+ = 9$. The horizontal line denotes the vev of ϕ at the closed string vacuum.

Truncating away β however changes the location of the closed string vacuum,

$$\phi = .674 \quad D_\perp = .174 \qquad D_\parallel = -.0503$$

and the vacuum energy density at this point is -1.37 times that of the D-25 brane tension. Starting from the initial conditions eq.5.4 we have generated a solution with $\beta = B_- = 0$ as shown in figure 10a. Figure 10b shows the profile of ϕ at $\tilde{x}^+ = 6$, graphed for comparison to the corresponding profile with $B_+ = 0$. Again we see that ϕ initially rolls faster to the closed string vacuum, but turns around 17% closer to the closed string vacuum than it does with $B_+ = 0$. The second approach to dealing with β is to set $\beta = -.173$, its value at the closed string vacuum, and look at small time dependent fluctuations around



a) b)

Figure 10: Dynamics of ϕ at level $(\tilde{2}, \tilde{4})$ excluding B_- and β a) as a function of $0 < \tilde{x}^+ 6$, $-8 < x^- < 10$ and b) at $\tilde{x}^+ = 6$ (shown as solid line). The dashed line shows the same profile generated from the equations after setting $B_+ = 0$. The horizontal line denotes the vev of ϕ at the closed string vacuum.

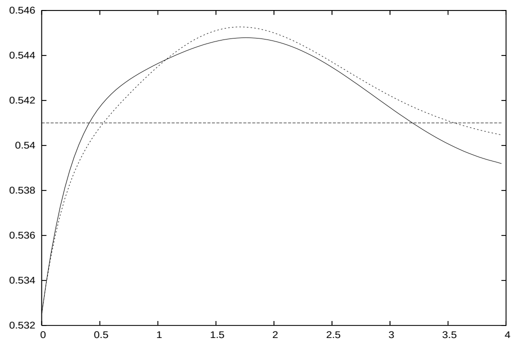


Figure 11: Fixing $\beta = -.173, B_- = 0$, the solid line shows small time dependent fluctuations around the closed string vacuum graphed at $\tilde{x}^+ = 4$ starting from the initial conditions eq.5.6. The dashed line shows the analogous profile obtained upon setting $B_+ = 0$.

the vacuum. Using the initial conditions eq.5.6, at $\tilde{x}^+ = 4$ we get the ϕ profile shown in figure 11, alongside the analogous profile with $B_+ = 0$. Again ϕ falls faster to the closed string vacuum, but turns around early. The fact that these qualitative features emerge from three inequivalent truncations strongly suggests that this behavior not an artifact of our approximation scheme, and is largely independent of the negative energies of β.

Since ϕ turns around early in all of these solutions, it is tempting to hope that in the infinite level limit ϕ may not cross the closed string vacuum at all. Thus, the picture of D-brane decay in open string field theory would resemble Sen's "rolling tachyon" boundary conformal field theory solution, where the tachyon falls homogeneously off the unstable maximum towards the closed string vacuum, but does not cross over in finite time. Indeed, there are preliminary indications that at higher levels the height of the first peak of ϕ is further suppressed. For example, in figure 12 we have graphed the profile of ϕ at $\tilde{x}^+ = 6$

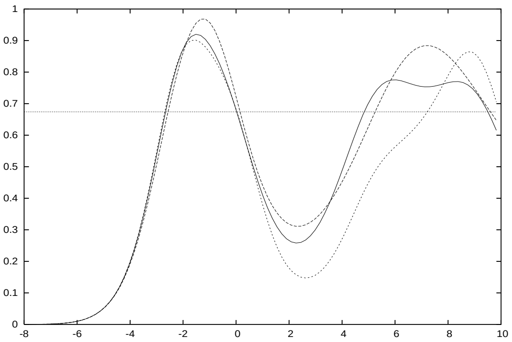


Figure 12: Three profiles of ϕ at $\tilde{x}^+ = 6$ generated from the initial conditions eq.5.4. The solid line includes all fields at level $\tilde{2}$ except β, B_-. The dotted line includes all fields at level $\tilde{2}$ except β, B_-, but also includes the level $\tilde{4}$ field C_+. The dashed line is the profile one obtains setting $B_+ = C_+ = 0$.

(starting from eq.5.4) generated if we include the extra level $\tilde{4}$ field,

$$\int dk \frac{i}{2} C_+(k) \tilde{\alpha}^+_{-4} |k\rangle$$

and set $\beta = B_- = 0$. Now ϕ turns around even earlier, 7% closer to the closed string vacuum than of we had set $C_+ = 0$ and 23% closer than if we had set $B_+ = C_+ = 0$. However, before we jump to conclusions, it is worth mentioning that, from the perspective of open string field theory, it is not clear what it *means* for the string field to monotonically approach the closed string vacuum but not "cross over" in finite time. In string field theory the closed string vacuum corresponds to a single point in an infinite dimensional space of fields at zero momentum, and in the process of decay the string field might oscillate many times around the closed string vacuum but never actually cross over this point. To illustrate this, we have generated two parametric plots following the path of $\phi, D_\perp$, as a function of x^- at $\tilde{x}^+ = 6$, as the D-brane decays. Setting the negative energy fields B_-, β to zero, figure 13a shows the path followed if we truncate away B_+ and figure 13b shows the path including B_+. In 13a the string field oscillates regularly around the closed string vacuum; in 13b, ϕ does not become as large (as we saw before) but the oscillations around the vacuum seem much more chaotic. Indeed, it is not clear from these pictures how our solutions might be approaching Sen's "monotonic" rolling tachyon solution, or even how we would know it if they were. Still, the fact that $D_\perp$ oscillates more wildly for our solutions could have been expected, since the kinetic energy which would otherwise allow ϕ to roll further up the potential wall must be absorbed by more massive fields, causing them to oscillate more strongly. If the rolling tachyon solution exists in open string field theory, this could be the mechanism allowing the field to approach the closed string vacuum asymptotically—the energy originally stored in the tachyon and lightest mass fields gets absorbed by fields of arbitrarily large mass.

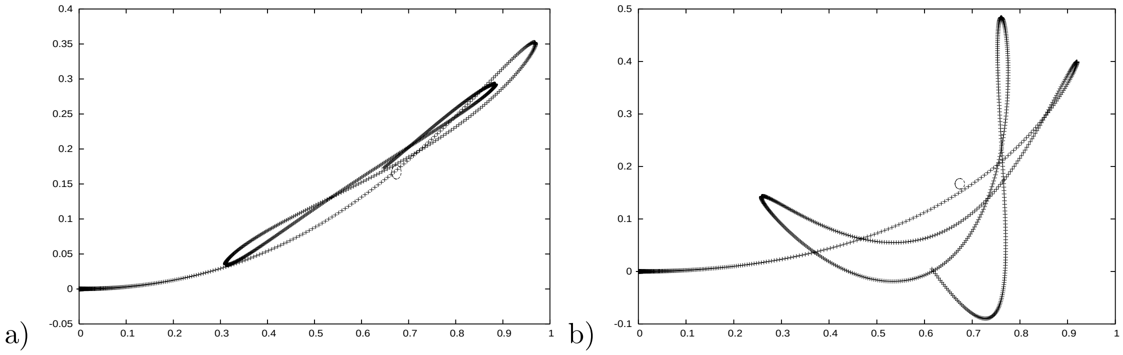


Figure 13: Plots showing the dynamics of ϕ (horizontal axis) and $D_\perp$ (vertical axis) as parametric functions of x^- at $x^+ = 6$. In both plots we truncated away the negative energy fields B_-, β. 12a shows the dynamics with $B_+ = 0$ and 12b shows the dynamics with $B_+ \neq 0$. The little circle in these figures shows the position of the closed string vacuum.

6. Conclusion

In this paper we have studied level truncation and time dependent solutions representing the decay of an unstable D25-brane in open string field theory. We introduced a basis of spacetime fields, the "lightcone basis," for which the equations of motion are local in lightcone time and therefore admit a well-defined initial value formulation and a Hamiltonian free of higher derivative instabilities. We studied the effect of level truncation in the linear theory and investigated a modified level expansion which allowed us to understand the relationship between time dependent solutions in the lightcone basis and the center of mass basis, where the interaction vertex possesses an infinite number of time derivatives. Finally, we investigated time dependent solutions including all spacetime fields up to level $\tilde{2}$. We found that, due to spurious instabilities introduced by truncating the theory at the linear level, the solutions at level $\tilde{2}$ are unstable. However, by looking at a slightly modified set of equations of motion, we found much more regular solutions. The picture that emerges seems to indicate that, in comparison to solutions generated by only including the scalars at level $\tilde{2}$, the tachyon rolls faster towards the closed string vacuum initially but then rapidly decelerates and does not become as large before turning around. Though these results can only be regarded as a first step, they seem to indicate that it is at least possible that Sen's rolling tachyon solution can be realized in open string field theory.

Of course, it would seem highly desirable to take our analysis further, perhaps by including higher levels, to see of a more refined picture of the D-brane decay emerges. Unfortunately, at higher levels the equations of motion become exponentially more complicated and unphysical instabilities proliferate. In principle, if the constraints are consistently imposed and we go to high enough levels, the unphysical instabilities should work themselves out, but we have no understanding of how far we need to go for this to occur. Probably it is unreasonable to hope that a brute force approach to higher levels will yield a clearer picture of the decay. What is needed is a clearer analytic understanding of the constraints eq.5.5, particularly how they exorcise unphysical modes in the linear and nonlinear theory. The consistency of the constraints with time evolution is subtle, relying on nontrivial properties of the vertex in the infinite level limit. Indeed, it is not even clear that the closed string vacuum satisfies the constraints—though, barring some sort of anomaly, naively it does. Armed with a better understanding of constraints, it may be easier to identify the relevant approximations and take our analysis further.

Of course, there are other techniques which may be worth exploring. One might be to follow the approach of ref.[10], inverse Wick rotating a marginal solution interpolating between a D1 and D0-brane compactified on a unit circle along $\tilde{x}^1 = \frac{1}{\sqrt{2}}(\tilde{x}^+ - x^-)$. Indeed, constructing time independent marginal and lump solutions carrying nonzero momentum along $\tilde{x}^1$ in the lightcone basis is of interest in its own right. Another possibly interesting approach is to find time dependent solutions using the technique of ref.[19] based on the regulated butterfly state, though so far this has only been applied towards approximating the closed string vacuum solution at zero momentum. One might also try to investigate time dependent solutions in the context of vacuum string field theory[20, 21], where direct analytic understanding is possible. Unfortunately, it seems doubtful that the lightcone basis could be usefully applied in this context, since naively the action would contain no ∂_+ derivatives at all, rendering the dynamics trivial. Still, it may be interesting to reconcile the lightcone basis with some of the time dependent solutions constructed so far in vacuum string field theory.

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