Homework 4 Solutions

Prepared by: William Wolf

Below are suggested solutions for the homework problems assigned. Other solutions may exist. If you have any question about them, feel free to consult Bill in his office hours.

(26.1) Why did Isaac Newton conclude that the universe was static? Was he correct?

Solution: Newton noted that if the universe were not static, gravitational forces must necessarily (after sufficient time) cause all matter to collect into a compact blob. For no apparent reason, this was an undesirable conclusion, so Newton surmised that the universe must be infinite in extent and size, so that gravitational forces cannot cause an eventual collapse, since all matter would experience gravitational forces in all directions to roughly the same degree. Olbers's paradox posed a problem for this theory, and it was eventually completely debunked by Hubble's discovery of the expanding universe.

(26.2) What is Olbers's paradox? How can it be resolved?

Solution: If we assume the universe is infinite in time and space with stars more or less uniformly distributed, any and all lines of sight into space should eventually hit a star. Thus, we would expect the entire night sky to be as bright as the average star. This is obviously not the case, and this contradiction *is* Olbers's paradox.

Olbers first thought to remedy this by adding in some sort of cosmic fog that absorbs light. This doesn't really solve the problem since absorbed light would be re-emitted, albeit at different wavelengths. The problem was eventually solved (at least tentatively) by the big bang model, which posits that the universe is finite in time, so we can only see up to a distance in which light has had time to travel since the beginning of time.

(26.35) Estimate the age of the universe for a Hubble constant of 50km/s/Mpc, 75 km/s/Mpc, and 100km/s/Mpc. On the basis of your answers, explain how the ages of globular clusters could be used to place limit on the maximum value of the Hubble constant.

Solution: The age of the universe is given approximately by $1/H_0$. Thus, let us first convert the units of H_0^{-1} to billions of years:

$$\begin{aligned}
\text{Mpc/km/s} &= \frac{3.08\times 10^{19}\ \text{km}}{1\ \text{km/s}} \\
&= 3.08\times 10^{19}\ \text{s}\times \frac{1\ \text{year}}{\pi\times 10^{7}\ \text{seconds}} \\
&= 9.8\times 10^{11}\ \text{years} \\
&= 980\ \text{Gyrs}
\end{aligned}$$

Now we compute the possible ages for the universe

$$T(50 \text{ km/s/Mpc}) = \frac{1}{50} 980 \text{ Gyrs} = 19.6 \text{ Gyrs}$$
$$T(75 \text{ km/s/Mpc}) = \frac{1}{75} 980 \text{ Gyrs} = 13.1 \text{ Gyrs}$$
$$T(100 \text{ km/s/Mpc}) = \frac{1}{100} 980 \text{ Gyrs} = 9.8 \text{ Gyrs}$$

So the results are

$$\boxed{T(50 \text{ km/s/Mpc}) = 19.6 \text{ Gyrs}}$$

$$\boxed{T(75 \text{ km/s/Mpc}) = 13.1 \text{ Gyrs}}$$

$$\boxed{T(100 \text{ km/s/Mpc}) = 9.8 \text{ Gyrs}}$$

Looking at the main-sequence cutoffs on the H-R diagram of the oldest globular clusters can help us put an upper bound on H_0. Note that higher values of H_0 give shorter universe ages. For instance, if we find a globular cluster with a main sequence cutoff indicating that it is 11 billion years old, it is immediately obvious that $H_0 < 100$ km/s/Mpc, for otherwise the cluster would be older than the universe itself.