UNIVERSITY OF CALIFORNIA, SANTA BARBARA
Department of Physics
2014 Winter Quarter

GALAXIES AND COSMOLOGY — Professor Treu

HOMEWORK #2
FOUNDATIONS OF THE BIG BANG

Problem 1: **Positive curvature**
Ryden, Problem 3.2

Problem 2: **The cosmological constant and the solar system**
Ryden, Problem 4.1

Problem 3: **Geometry of curved spaces:**
In ordinary three dimensional space, we can write the distance between two nearby points $\mathrm{R}, \theta, \mathrm{Z})$ and $(\mathrm{R}+\mathrm{dR}, \theta+\mathrm{d}\theta, \mathrm{Z}+\mathrm{dZ})$ as $\mathrm{ds}^2 = \mathrm{dR}^2 + \mathrm{R}^2\mathrm{d}\theta^2 + \mathrm{dZ}^2$ using cylindrical coordinates. The equation $\mathrm{R}^2 + \mathrm{Z}^2 = \mathrm{R_c^2}$ describes a sphere of radius $\mathrm{R_c}$.

Show that if the two points lie on this sphere, then the distance between them is

$$ds^2 = dR^2(1 + \frac{R^2}{Z^2}) + R^2 d\theta^2 = R_c^2[\frac{d\sigma^2}{1-\sigma^2} + \sigma^2 d\theta^2] \quad (1)$$

where $\sigma = \mathrm{R}/\mathrm{R_c}$

For what value of the curvature parameter k does the Robertson-Walker metric, resemble the equation in part above? Do we call this a closed, open, or flat geometry? Notice that a surface of constant ϕ would look like a sphere of radius $\mathrm{R_c}$.

Problem 4: **Propagation of light in Newton's gravity:**
Answer the following questions assuming that photons travel in the gravitational field following Newton's law.

a) A laser beam is pointed upwards from the surface of the Earth. Will the photons escape Earth's gravity? What would the radius of the Earth need to be in order to prevent the photon from escaping? *This is the same result that it is obtained in General Relativity.*

b) A photon travels towards the Sun with impact parameter b Compute the deflection angle due to the Sun's gravity. Assume that the deflection angle is small. *This is half the value obtained using General Relativity.*