UNIVERSITY OF CALIFORNIA, SANTA BARBARA
Department of Physics
2014 Winter Quarter

GALAXIES AND COSMOLOGY — Professor Treu

HOMEWORK #7
DARK MATTER

Problem 1: **Singular isothermal spheres**

A mass density profile of the form:

$$\rho(r) = \frac{\sigma_v^2}{2\pi G r^2}$$

is called a singular isothermal sphere. The Einstein radius of a singular isothermal sphere is given by the equation:

$$\theta_{\mathrm{E}} = 4\pi \left(\frac{\sigma_v}{c}\right)^2 \frac{D_{\mathrm{ls}}}{D_{\mathrm{s}}},$$

where D_{ls} and D_{s} are the angular size distance to between the lens and the background source and between the observer and the background source. The Einstein radius is observed to be 1.5 arcseconds, the lens is at redshift $z_l = 0.5$ and the velocity dispersion of the lens is observed to be 300 kms^{-1}. Considering for simplicity an Einstein de Sitter model ($\Omega_m = 1$, $\Omega_\Lambda = 0$) with H_0=70kms^{-1}Mpc^{-1}, infer the redshift of the source z_s.

Hint: the angular size distance is given by:

$$D_{\mathrm{A}}(z_l, z_s) = \frac{2c}{H_0}\frac{1}{1+z_s}\left[(1+z_l)^{-1/2} - (1+z_s)^{-1/2}\right]$$

Problem 2: **Lensing by a point mass**

Ryden 8.3

Problem 3: **The Hubble constant from gravitational time delays**

A gravitational lens consists of a source at $|\beta| = \theta_{\mathrm{E}}/2$. Assuming that the mass distribution of the lens is a singular isothermal sphere with Einstein Radius 1.5 arcseconds describe the morphology of the time delay surface *Hint: it helps to solve the lens equation.* The time delay between the two images is measured to be 1 ± 0.05 years. Determine the Hubble constant for an Einstein de-Sitter Universe, knowing the lens redshift is 0.5 and the source redshift is 2.14..