Physics 133: Extragalactic Astronomy and Cosmology

Lecture 12; February 24 2014
Previously:

- There is dark matter
  - Galaxies – rotation curves
  - Clusters – virial theorem and hydrostatic equilibrium

- We do not know what it is:
  - It cannot be hidden baryons
  - It could be new exotic particles..
Outline:

• Gravitational lensing (intro)

• Gravitational lensing (theory):
  – Strong
  – Weak
  – Micro (Ryden 8.4)

• Cool things you can do with lensing (applications):
  – Detect dark matter
  – Test gravity
  – Cross section of dark matter
Detecting dark matter. Gravitational lensing!

- Mass concentrations perturb spacetime, altering the propagation of light

![Gravitational lensing diagram](image-url)
Detecting dark matter. Strong gravitational lensing!

- Under special circumstances the distortion is so strong that creates two images of a background object. This is called strong lensing
Detecting dark matter. Examples of strong lenses

Einstein Ring Gravitational Lenses
*Hubble Space Telescope* • Advanced Camera for Surveys

NASA, ESA, A. Bolton (Harvard-Smithsonian CfA), and the SLACS Team

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Detecting dark matter. Why is it called lensing?

- The physics is very similar to that of common optical lenses
- In fact many of the features of gravitational lensing can be reproduced by common optical devices.

Figure courtesy of Phil Marshall
Detecting dark matter.
Strong lensing by clusters

- Clusters are also strong lenses
- The blue objects here are distorted images of the same object
Detecting dark matter. Why do we care about lensing?

• The image separation gives us a direct measurement of the mass enclosed by the images.
• It is arguably the most precise measurement of mass that we can make.
• And there are other applications too (we’ll see later..)
Detecting dark matter. Weak lensing

- Even when the gravitational field is not strong enough to produce multiple images, the large scale structure perturbs space time.
- This alters the shape of observed galaxies in the sky, shearing and magnifying them in a measurable way.
Detecting dark matter. Weak lensing mass maps

Optical image

Dark matter mass
Lensing... a little math
**Lensing Basics.**  
**I: Thin Screen Approximation**

Surface mass density

\[ \Sigma(\xi) = \int \rho(\xi, z) \, dz \]

Deflection angle

\[ \tilde{\alpha}(\xi) = \frac{4G}{c^2} \int \frac{(\xi - \xi')\Sigma(\xi')}{|\xi - \xi'|^2} \, d^2\xi' \]

\[ \hat{\alpha}(\xi) = \frac{4GM(\xi)}{c^2\xi} \]

\[ M(\xi) = 2\pi \int_0^\xi \Sigma(\xi')\xi' \, d\xi' \]

Reduced deflection angle

\[ \tilde{\alpha} = \frac{D_{\text{ds}}}{D_s} \hat{\alpha} \]

\[ \tilde{\beta} = \tilde{\theta} - \tilde{\alpha}(\tilde{\theta}) \]

\[ \Sigma_{\text{cr}} = \frac{c^2}{4\pi G} \frac{D_s}{D_{\text{ds}}} = 0.35 \, \text{g cm}^{-2} \left( \frac{D}{1 \, \text{Gpc}} \right)^{-1} \]

Critical density

Lens equation
Lensing Basics.

II: useful general relations

2D potential

\[ \psi(\vec{\theta}) = \frac{D_{ds}}{D_d D_s} \frac{2}{c^2} \int \Phi(D_d \vec{\theta}, z) \, dz \]

\[ \vec{\nabla}_{\vec{\theta}} \psi = D_d \vec{\nabla}_\xi \psi = \frac{2}{c^2} \frac{D_{ds}}{D_s} \int \vec{\nabla}_\perp \Phi \, dz = \vec{\alpha} \]

2D Poisson Equation

\[ \nabla^2_\vec{\theta} \psi = \frac{2}{c^2} \frac{D_d D_{ds}}{D_s} \int \nabla^2_\xi \Phi \, dz = \frac{2}{c^2} \frac{D_d D_{ds}}{D_s} 4\pi G \Sigma = 2 \frac{\Sigma(\vec{\theta})}{\Sigma_{cr}} = 2\kappa(\vec{\theta}) \]

Jacobian matrix

\[ \vec{\alpha}(\vec{\theta}) = \vec{\nabla} \psi = \frac{1}{\pi} \int \kappa(\vec{\theta}) \frac{\vec{\theta} - \vec{\theta}'}{|\vec{\theta} - \vec{\theta}'|^2} \, d^2 \theta' \]

\[ A \equiv \frac{\partial \vec{\beta}}{\partial \vec{\theta}} = \left( \delta_{ij} - \frac{\partial \alpha_i(\vec{\theta})}{\partial \theta_j} \right) = \left( \delta_{ij} - \frac{\partial^2 \psi(\vec{\theta})}{\partial \theta_i \partial \theta_j} \right) = \mathcal{M}^{-1} \]
Lensing Basics.

III: Caustics and Critical Lines

Saha & Williams 2003

Koopmans & Treu 2003

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The time delay surface and Fermat’s principle
The time delay surface and Fermat’s principle

\[(\bar{\theta} - \bar{\beta}) - \nabla_\theta \psi = 0\]  
Lens equation

\[\nabla_\theta \left[ \frac{1}{2} (\bar{\theta} - \bar{\beta})^2 - \psi \right] = 0\]  
Extrema of the time delay surface

Time delay surface  
\[t(\bar{\theta}) = \frac{1 + z_d}{c} \frac{D_d D_s}{D_{ds}} \left[ \frac{1}{2} (\bar{\theta} - \bar{\beta})^2 - \psi(\bar{\theta}) \right] = t_{geom} + t_{grav}\]

adimensional

\[T = \frac{\partial^2 t(\bar{\theta})}{\partial \theta_i \partial \theta_j} \propto (\delta_{ij} - \psi_{ij}) = A\]

Multiple images form at the extrema of the time delay surface
Gravitational lensing.
Meaning of the time delay surface

• Gravitational fields not only bend light, but also “slow” down time.
• For this reason, as light travels close to a lens, it takes longer than it should in normal geometry.
• As we will see, we can use this effect to measure distances and the Hubble constant.
Summary. Lensing basics

- Mass concentrations distort the images of objects in on the sky in a way similar to that of optical images.
- Strong and weak lensing provide very accurate mass maps of objects (in projection).
- Gravitational fields also “slow down” light. Strong lensing can be formulated in terms of Fermat’s principle.
Dark Matter. Smooth...
A simple example.

Singular isothermal sphere

- The singular isothermal sphere is the simplest model that provides a decent description of galaxies.
- Let's see its properties and how it can be used to measure masses.

\[ \rho_{\text{SIS}} = \frac{\sigma_v^2}{2\pi Gr^2} \]

\[ \Sigma(\xi) = \int_{-\infty}^{+\infty} d\xi \rho_{\text{SIS}} = \frac{\sigma_v^2}{2G\xi} \]

\[ \theta_E \equiv 4\pi \left( \frac{\sigma_v}{c} \right)^2 \frac{D_{ds}}{D_s} \]

\[ \Delta \theta = 2\theta_E \]

\[ M_E = \pi \theta_E^2 \Sigma_{\text{crit}} \]
...and clumpy
Gravitational lensing. Detecting substructure..

- Substructure problem
- Two alternatives:
  - 1) Cold dark matter is wrong
  - 2) Satellites are present but not visible
- Lensing can detect them through their effects on multiple images. There have been claims that this has been detected. The jury is still out..
Lensing and dark matter
A case study: the bullet cluster

MOVIE!  

Clowe et al. 2006; Bradac et al. 2006
Inferences from the bullet cluster

• There is mass where there are no baryons
  – Non baryonic dark matter
  – Mond is wrong

• Dark matter is collisionless
  – Limits on self interaction cross section <0.7cm²/g
The End

See you on wednesday!