

Fall 2007 - PHYSICS 233 – Homework 4 - Due 11/09/07

Solving the level population equations generally requires numerical methods for systems involving more than three levels, as in the following case. You will need to write a short computer program to do this. You may choose any language or package you like. Do not share programs, please hand in a copy of your source code with your solution set.

Consider a 6 level model of the OIII ion. Values for the collisional rate coefficients γ_{ij} , Einstein A coefficients A_{ij} , statistical weights g_i and energy difference between levels E_{ij} are given below. If unspecified T and n_e are intended in K and cm^{-3} .

a) Write down the level population equations for this system as a function of electron density n_e and temperature T . Denote the level populations as n_i ($i = 1, 6$).

b) Show that these six equations are degenerate. Using the normalization $\sum_{i=1}^6 n_i = 1$ as a sixth independent equation solve this system for the following five pairs of values of n_e and T : $T = 10^4$ and $n_e = 10^3, 10^4, 10^5$; $n_e = 10^5$ $T = 5 \times 10^3, 2 \times 10^4$. For each n_e, T pair report the numerical values of n_i .

c) Ratios of the intensities of the $\lambda 5007$ to $\lambda 4363$ lines in the OIII ion (the 4-3 and 5-4 transitions, respectively) are particularly useful for measuring the temperature of ionized nebulae. Plot this ratio versus T over the range 6,000 - 20,000 K for $n_e = 100, 10^3, 10^4, 10^5$. Discuss why this is a useful ratio to measure T . Suppose the emission measure for an HII region indicates $n_e = 10^3$, while $I(5007)/I(4363)=800$ for the same nebula. What is the temperature?

Following are useful data for the OIII ion. The collision rate coefficients are $\gamma_{ij} = 8.629 \times 10^{-6} \Omega_{ij} / (g_i T^{1/2}) \text{ cm}^3 \text{ s}^{-1}$. The statistical weights of levels 1 thru 6 are 1,3,5,5,1,1 respectively. Use the tables in the following way. Consider the transition from $^1D_2 \rightarrow ^3P_2$ which gives the OIII 5007 line. This is level 4 to 3. Thus the energy separation is given by the element (4,3) of the matrix below, 2.4751 eV. The decay rate is $A_{4,3} = 0.0196 \text{ s}^{-1}$ and the collision rate is $\Omega_{3,4} = \Omega_{4,3} = 1.2056$. Each Ω_{ij} has a temperature dependance that is approximated as $(10^{-4} K^{-1} T)^{X_{ij}}$, with $X_{3,4} = 0.14$

levels	1	2	3	4	5	6
1	0	-	-	-	-	-
2	0.542	0	-	-	-	-
3	0.271	1.29	0	-	-	-
4	0.2411	0.7233	1.2056	0	-	-
5	0.0307	0.0902	0.1533	0.62	0	-
6	0.136	0.408	0.68	0	0	0

Table 1: Collision strengths at 10,000K (Ω_{ij})

levels	1	2	3	4	5	6
1	0	-	-	-	-	-
2	0	0	-	-	-	-
3	0	0	0	-	-	-
4	0.14	0.14	0.14	0	-	-
5	0.186	0.186	0.186	0	0	-
6	0	0	0	0	0	0

Table 2: Temperature dependance of cross-section (X_{ij})

levels	1	2	3	4	5	6
1	0	0	0	0	0	0
2	2.62e-5	0	0	0	0	0
3	3.02e-11	9.76e-5	0	0	0	0
4	2.74e-6	6.74e-3	0.0196	0	0	0
5	0	0.217	5.2e-4	1.78	0	0
6	0	212	522	0	0	0

Table 3: Einstein coefficients in s^{-1} (A_{ij})

levels	1	2	3	4	5	6
1	0	0.014	0.038	2.5131	5.3538	7.479
2	-	0	0.0239	2.4991	5.3395	7.465
3	-	-	0	2.4751	5.3157	7.441
4	-	-	-	0	2.8407	4.94
5	-	-	-	-	0	2.11
6	-	-	-	-	-	0

Table 4: Energy separation of levels in eV